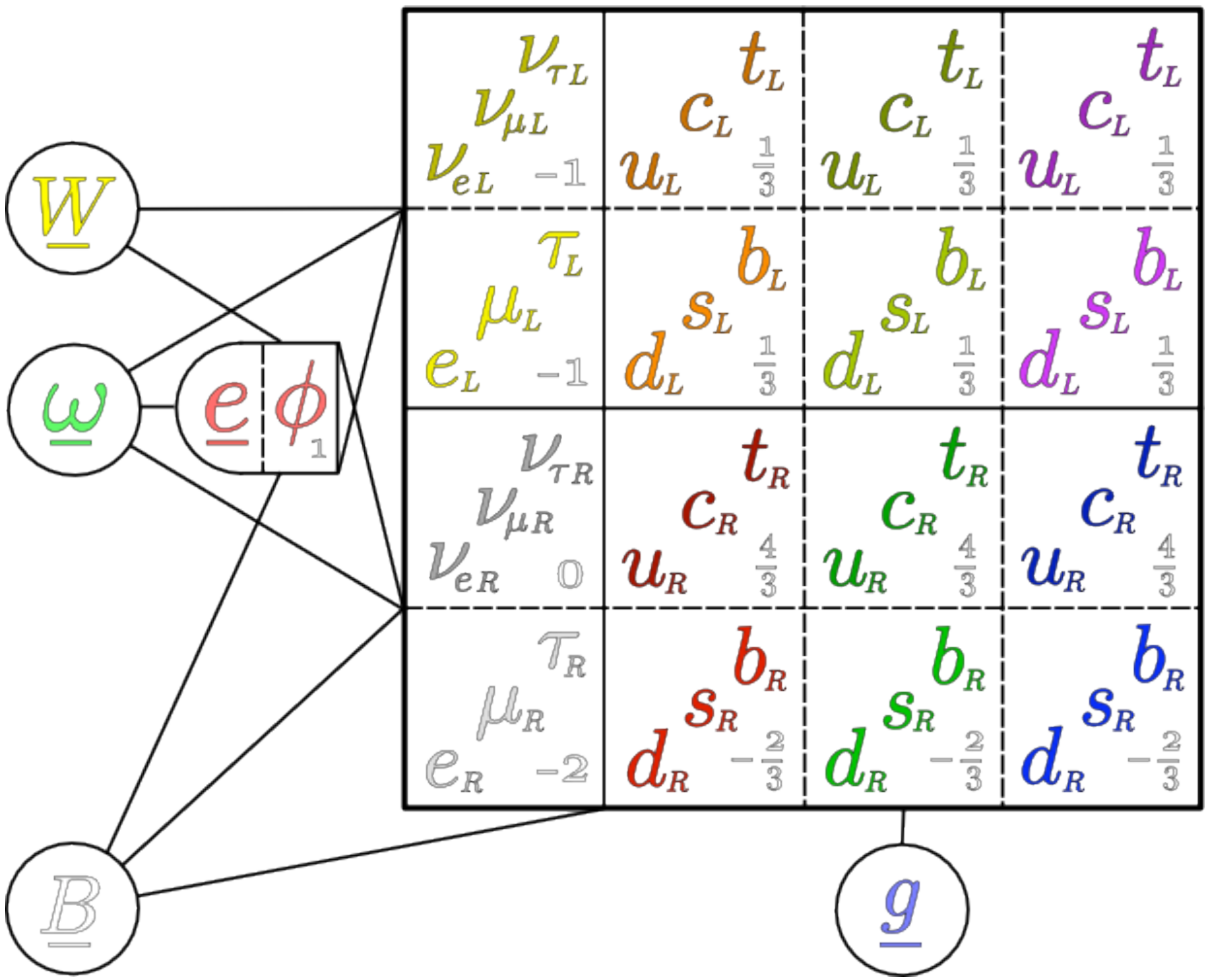
Periodic table of the standard model



An Exceptionally Simple Theory of Everything

http://arxiv.org/abs/0711.0770

Garrett Lisi
FQXi

A connection with everything

$$\underline{\omega} = d\underline{x}^k \tfrac{1}{2}\omega_k{}^{\mu\nu}\gamma_{\mu\nu} \in \underline{C}l^2(3,1) \qquad \underline{e} = d\underline{x}^k (e_k)^\mu \gamma_\mu \in \underline{C}l^1(3,1) \qquad \begin{bmatrix} e_L^\wedge \\ e_L^\vee \\ e_R^\wedge \\ e_R^\vee \end{bmatrix}$$

$$\underline{W} = d\underline{x}^k W_k{}^\pi \tfrac{i}{2}\sigma_\pi \in s\underline{u}(2) \qquad \begin{bmatrix} \phi_+ \\ \phi_0 \end{bmatrix} \qquad \begin{bmatrix} \nu_{eL} \\ e_L \end{bmatrix}$$

$$\underline{B} = d\underline{x}^k B_k i \in \underline{u}(1) \qquad Y$$

$$\underline{g} = d\underline{x}^k g_k{}^A \tfrac{i}{2}\lambda_A \in s\underline{u}(3) \qquad [u^r, u^g, u^b]$$

$$\updownarrow$$

$$\underset{\cdot}{\underline{A}} = \tfrac{1}{2}\underline{\omega} + \tfrac{1}{4}\underline{e}\phi + \underline{W} + \underline{B} + \underline{g} + (\underset{\cdot}{\nu}_e + \underset{\cdot}{e} + \underset{\cdot}{u} + \underset{\cdot}{d})$$
$$+ (\underset{\cdot}{\nu}_\mu + \underset{\cdot}{\mu} + \underset{\cdot}{c} + \underset{\cdot}{s}) + (\underset{\cdot}{\nu}_\tau + \underset{\cdot}{\tau} + \underset{\cdot}{t} + \underset{\cdot}{b})$$

$$\underset{\cdot}{\underline{\underline{F}}} = \underline{d}\,\underset{\cdot}{\underline{A}} + \frac{1}{2}\left[\underset{\cdot}{\underline{A}}, \underset{\cdot}{\underline{A}}\right]$$

Review of some representation theory

Cartan subalgebra: $C = C^a T_a \subset \mathrm{Lie}(G)$

Built from a maximal commuting set of R generators,

$$\left[T_a, T_b\right] = T_a T_b - T_b T_a = 0 \qquad \forall \quad 1 \leq a, b \leq R$$

Root vectors, V_β, are eigenvectors of C in the Lie bracket,

$$[C, V_\beta] = \alpha_\beta V_\beta = \sum_a i C^a \alpha_{a\beta} V_\beta$$

Roots, $\alpha_{a\beta}$, are the eigenvalue coefficients. The pattern of roots in R dimensions corresponds to the Lie algebra,

$$[V_\beta, V_\gamma] = V_\delta \quad \Leftrightarrow \quad \alpha_\beta + \alpha_\gamma = \alpha_\delta$$

Weight vectors and **weights** are eigenvectors and eigenvalue coefficients of C acting on some representation space,

$$C\, V_\beta = \alpha_\beta V_\beta$$

Weight vectors are particles, weights are their quantum numbers.

Gluon and quark weights

$$g = g^A T_A = g^A \frac{i}{2}\lambda_A = \frac{i}{2}\begin{bmatrix} g^3+\frac{1}{\sqrt{3}}g^8 & g^1-ig^2 & g^4-ig^5 \\ g^1+ig^2 & -g^3+\frac{1}{\sqrt{3}}g^8 & g^6-ig^7 \\ g^4+ig^5 & g^6+ig^7 & -\frac{2}{\sqrt{3}}g^8 \end{bmatrix}$$

Cartan subalgebra: $C = g^3 T_3 + g^8 T_8$ (the diagonal)

Roots and root vectors:

$$\left[C, V_{g^{g\bar{b}}}\right] = i\left(\left(-\frac{1}{2}\right)g^3 + \left(\frac{\sqrt{3}}{2}\right)g^8\right) V_{g^{g\bar{b}}} \qquad V_{g^{g\bar{b}}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

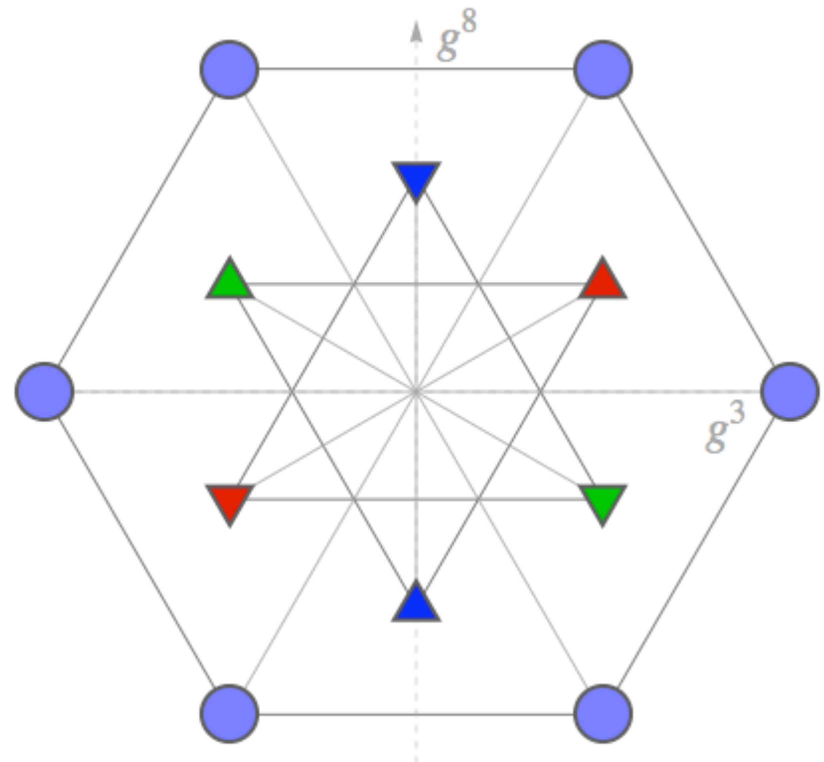
for the $g^{g\bar{b}}$ gluon. Weights and weight vectors:

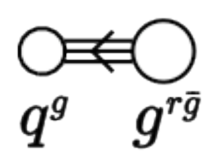
$$C\, V_{q^r} = i\left(\left(\frac{1}{2}\right)g^3 + \left(\frac{1}{2\sqrt{3}}\right)g^8\right) V_{q^r} \qquad V_{q^r} = [1, 0, 0]$$

for a red quark, q^r, and for their duals acted on by $-C^T$, the anti-quarks.

Strong G2

$G2$		V_β	g^3	g^8
●	$g^{r\bar{g}}$	$(T_2 - iT_1)$	1	0
●	$g^{\bar{r}g}$	$(-T_2 - iT_1)$	-1	0
●	$g^{r\bar{b}}$	$(T_5 - iT_4)$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
●	$g^{\bar{r}b}$	$(-T_5 - iT_4)$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
●	$g^{\bar{g}b}$	$(-T_7 - iT_6)$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
●	$g^{g\bar{b}}$	$(T_7 - iT_6)$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
▲	q^r	$[1,0,0]$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$
▲	q^g	$[0,1,0]$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$
▲	q^b	$[0,0,1]$	0	$-\frac{1}{\sqrt{3}}$
▼	$\bar{q}^r$	$[1,0,0]$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$
▼	$\bar{q}^g$	$[0,1,0]$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$
▼	$\bar{q}^b$	$[0,0,1]$	0	$\frac{1}{\sqrt{3}}$





Exceptional Lie brackets

The 14 Lie algebra elements of the smallest exceptional Lie group, $G2$:

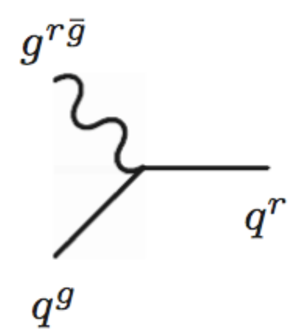
$$g2 = su(3) + 3 + \bar{3}$$
$$\underline{g} \quad + \underset{\cdot}{q} + \underset{\cdot}{\bar{q}} \in \underline{g}\underset{\cdot}{2}$$

Structure of $G2$ implies Lie bracket equivalent to fundamental action,

$$[g, q] = \left[g^A T_A, q^B T_B\right] = g\, q = \begin{bmatrix} \frac{i}{2} g^3 + \frac{i}{2\sqrt{3}} g^8 & g^{r\bar{g}} & g^{r\bar{b}} \\ g^{\bar{r}g} & -\frac{i}{2} g^3 + \frac{i}{2\sqrt{3}} g^8 & g^{g\bar{b}} \\ g^{\bar{r}b} & g^{\bar{g}b} & -\frac{i}{\sqrt{3}} g^8 \end{bmatrix} \begin{bmatrix} q^r \\ q^g \\ q^b \end{bmatrix}$$

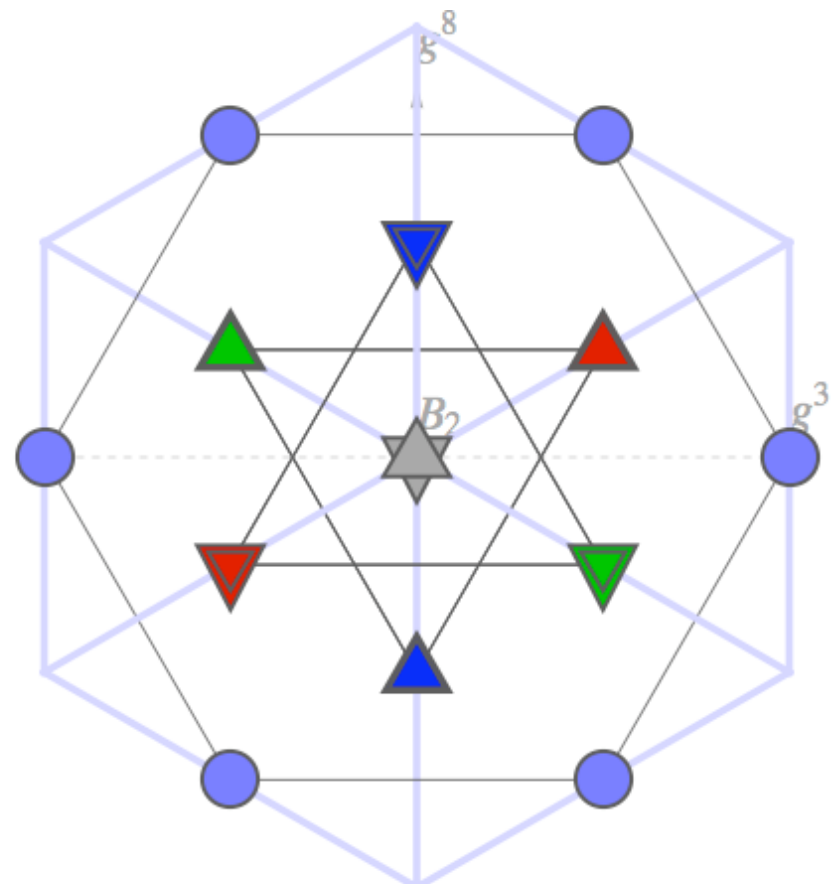
corresponding to the strong interactions, such as

$$\left[g^{r\bar{g}}, q^g\right] = q^r$$



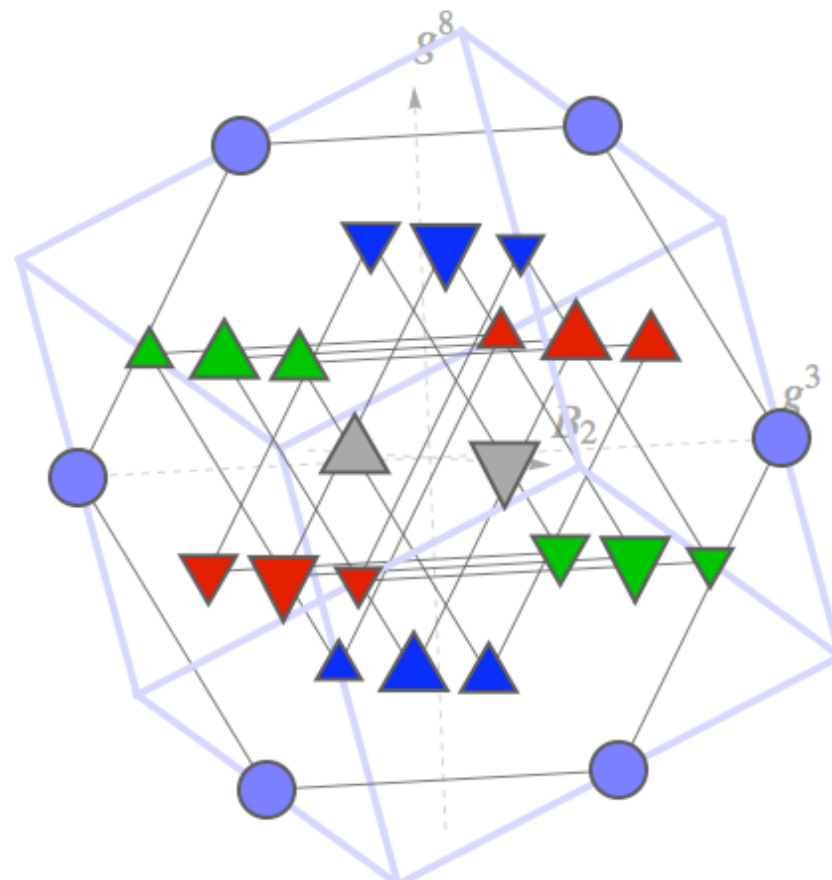
G2 in SO(7) .

$G2+$	x	y	z	g^3	g^8	$\frac{\sqrt{8}}{\sqrt{3}}B_2$
$g^{r\bar{g}}$	-1	1	0	1	0	0
$g^{\bar{r}g}$	1	-1	0	-1	0	0
$g^{r\bar{b}}$	-1	0	1	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0
$g^{\bar{r}b}$	1	0	-1	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0
$g^{\bar{g}b}$	0	1	-1	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0
$g^{g\bar{b}}$	0	-1	1	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0
q_I^r	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$
q_I^g	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$
q_I^b	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{3}$
$\bar{q}_I^r$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
$\bar{q}_I^g$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
$\bar{q}_I^b$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{3}$
q_{II}	∓ 1			"	"	$\pm\frac{2}{3}$
q_{III}	$\pm 1 \quad \pm 1$			"	"	$\mp\frac{4}{3}$
l	$\mp\frac{1}{2}$	$\mp\frac{1}{2}$	$\mp\frac{1}{2}$	0	0	± 1



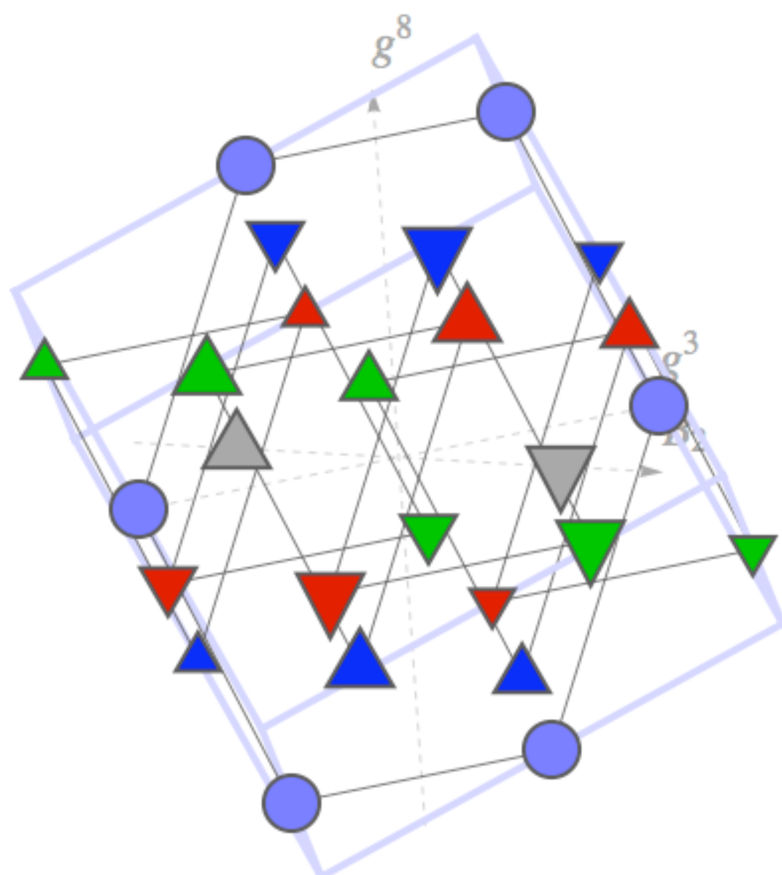
G2 in SO(7) ..

	$G2+$	x	y	z	g^3	g^8	$\frac{\sqrt{8}}{\sqrt{3}}B_2$
	$g^{r\bar{g}}$	-1	1	0	1	0	0
	$g^{\bar{r}g}$	1	-1	0	-1	0	0
	$g^{r\bar{b}}$	-1	0	1	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0
	$g^{\bar{r}b}$	1	0	-1	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0
	$g^{\bar{g}b}$	0	1	-1	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0
	$g^{g\bar{b}}$	0	-1	1	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0
	q_I^r	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$
	q_I^g	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$
	q_I^b	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{3}$
	$\bar{q}_I^r$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
	$\bar{q}_I^g$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
	$\bar{q}_I^b$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{3}$
	q_{II}	∓ 1			"	"	$\pm\frac{2}{3}$
	q_{III}	$\pm 1 \quad \pm 1$			"	"	$\mp\frac{4}{3}$
	l	$\mp\frac{1}{2}$	$\mp\frac{1}{2}$	$\mp\frac{1}{2}$	0	0	± 1



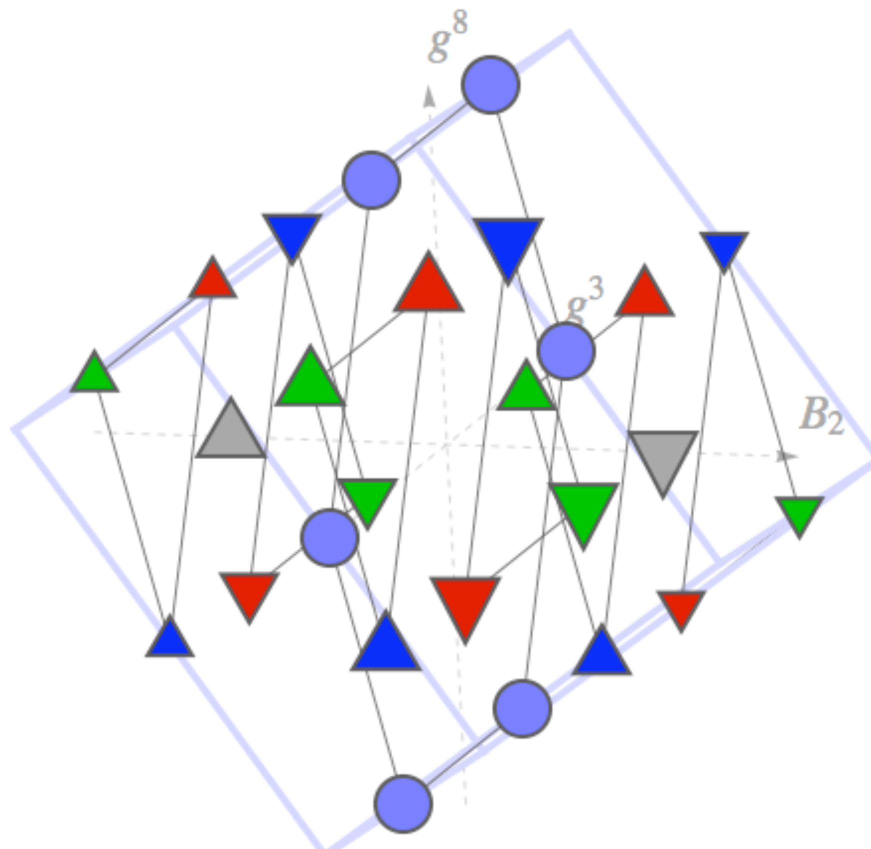
G2 in SO(7) ...

$G2+$		x	y	z	g^3	g^8	$\frac{\sqrt{8}}{\sqrt{3}}B_2$
	$g^{r\bar{g}}$	-1	1	0	1	0	0
	$g^{\bar{r}g}$	1	-1	0	-1	0	0
	$g^{r\bar{b}}$	-1	0	1	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0
	$g^{\bar{r}b}$	1	0	-1	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0
	$g^{\bar{g}b}$	0	1	-1	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0
	$g^{g\bar{b}}$	0	-1	1	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0
	q_I^r	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$
	q_I^g	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$
	q_I^b	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{3}$
	$\bar{q}_I^r$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
	$\bar{q}_I^g$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
	$\bar{q}_I^b$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{3}$
	q_{II}	∓ 1			"	"	$\pm\frac{2}{3}$
	q_{III}	$\pm 1 \quad \pm 1$			"	"	$\mp\frac{4}{3}$
	l	$\mp\frac{1}{2}$	$\mp\frac{1}{2}$	$\mp\frac{1}{2}$	0	0	± 1



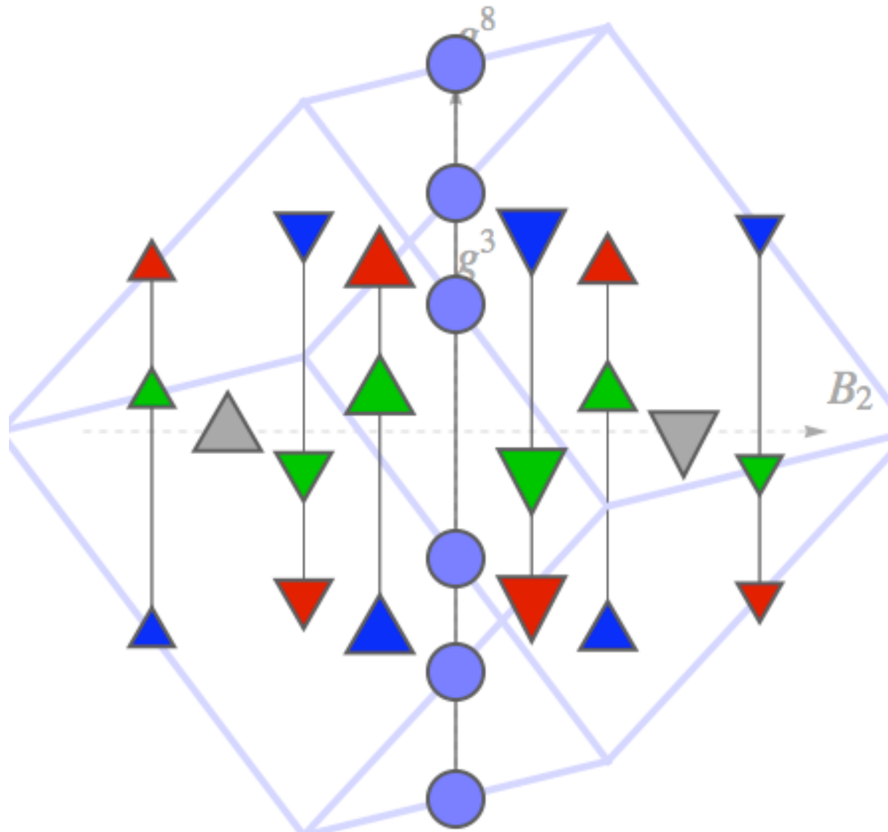
G2 in SO(7) ....

$G2+$	x	y	z	g^3	g^8	$\frac{\sqrt{8}}{\sqrt{3}}B_2$
$g^{r\bar{g}}$	-1	1	0	1	0	0
$g^{\bar{r}g}$	1	-1	0	-1	0	0
$g^{r\bar{b}}$	-1	0	1	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0
$g^{\bar{r}b}$	1	0	-1	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0
$g^{\bar{g}b}$	0	1	-1	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0
$g^{g\bar{b}}$	0	-1	1	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0
q_I^r	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$
q_I^g	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$
q_I^b	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{3}$
$\bar{q}_I^r$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
$\bar{q}_I^g$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
$\bar{q}_I^b$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{3}$
q_{II}	∓ 1			"	"	$\pm\frac{2}{3}$
q_{III}	± 1	± 1		"	"	$\mp\frac{4}{3}$
l	$\mp\frac{1}{2}$	$\mp\frac{1}{2}$	$\mp\frac{1}{2}$	0	0	± 1

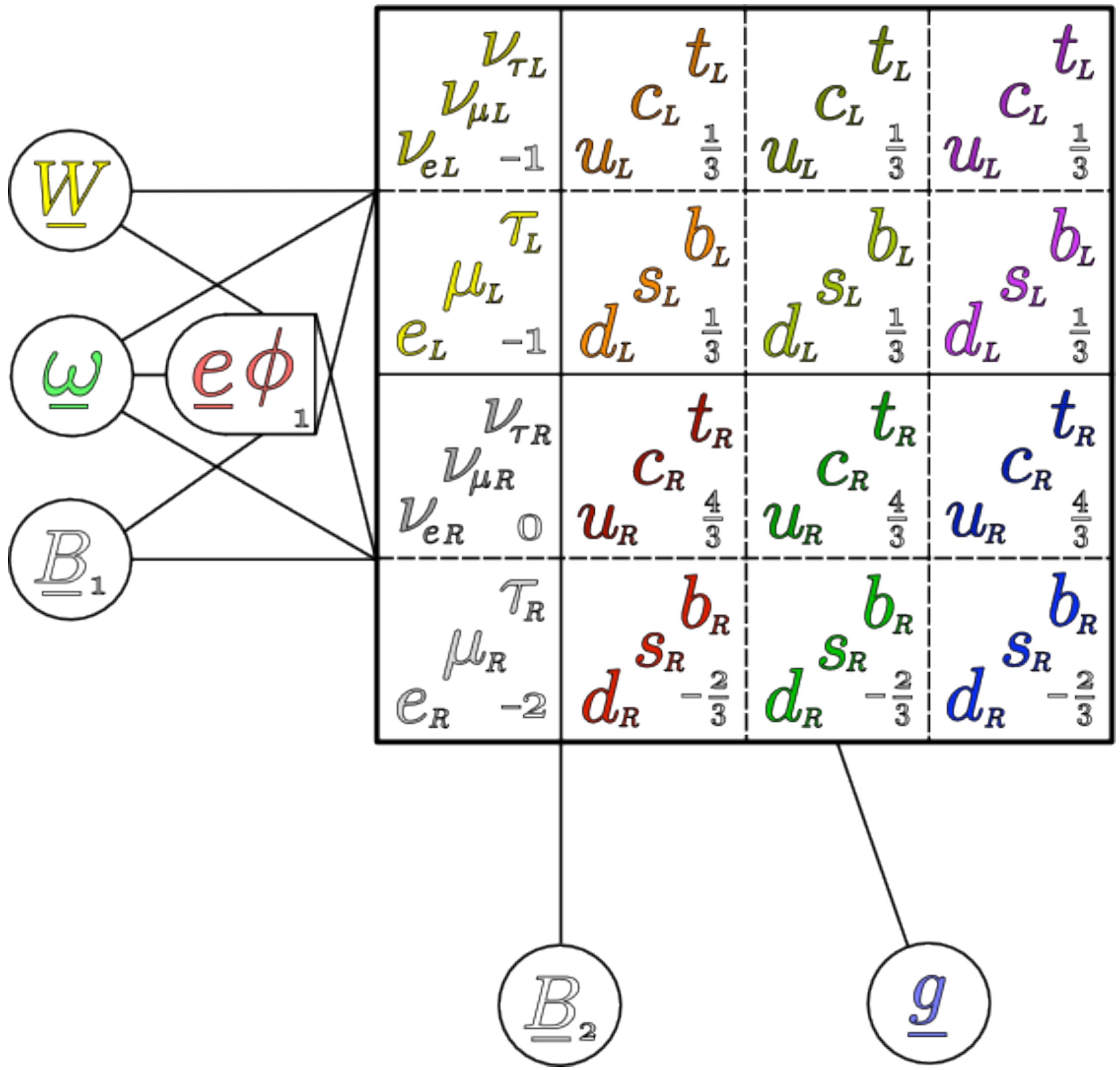


G2 in SO(7) x

	$G2+$	x	y	z	g^3	g^8	$\frac{\sqrt{8}}{\sqrt{3}}B_2$
	$g^{r\bar{g}}$	-1	1	0	1	0	0
	$g^{\bar{r}g}$	1	-1	0	-1	0	0
	$g^{r\bar{b}}$	-1	0	1	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0
	$g^{\bar{r}b}$	1	0	-1	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0
	$g^{\bar{g}b}$	0	1	-1	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0
	$g^{g\bar{b}}$	0	-1	1	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0
	q_I^r	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$
	q_I^g	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{3}$
	q_I^b	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{3}$
	$\bar{q}_I^r$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
	$\bar{q}_I^g$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
	$\bar{q}_I^b$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{3}$
	q_{II}	∓ 1			"	"	$\pm\frac{2}{3}$
	q_{III}	± 1	± 1		"	"	$\mp\frac{4}{3}$
	l	$\mp\frac{1}{2}$	$\mp\frac{1}{2}$	$\mp\frac{1}{2}$	0	0	± 1



Pati-Salam model plus gravity



$$\big(SO(3,1) + 4 \times 4 + SU(2)_L + SU(2)_R\big) + \big(U(1) + SU(3)\big)$$

Gravitational SO(3,1)

$$\omega = \tfrac{1}{2}\omega^{\mu\nu}\gamma_{\mu\nu} = \begin{bmatrix} \omega_L & \\ & \omega_R \end{bmatrix}$$

$$\omega_{L/R} = \begin{bmatrix} i\omega^3_{L/R} & \omega^{\wedge}_{L/R} \\ \omega^{\vee}_{L/R} & -i\omega^3_{L/R} \end{bmatrix}$$

$$e = e^{\mu}\gamma_{\mu} = \begin{bmatrix} & e_R \\ e_L & \end{bmatrix}$$

$$e_{L/R} = \begin{bmatrix} e_T^{\wedge/\vee} & \mp e_S^{\wedge} \\ \mp e_S^{\vee} & e_T^{\vee/\wedge} \end{bmatrix}$$

$$f = \begin{bmatrix} f_L \\ f_R \end{bmatrix} \qquad f_{L/R} = \begin{bmatrix} f^{\wedge}_{L/R} \\ f^{\vee}_{L/R} \end{bmatrix}$$

$SO(3,1)$		$\frac{1}{2}\omega^3_L$	$\frac{1}{2}\omega^3_R$
●	$\omega_L^{\wedge}$	1	0
●	$\omega_L^{\vee}$	-1	0
●	$\omega_R^{\wedge}$	0	1
●	$\omega_R^{\vee}$	0	-1
■	$e_S^{\wedge}$	$\frac{1}{2}$	$\frac{1}{2}$
■	$e_S^{\vee}$	$-\frac{1}{2}$	$-\frac{1}{2}$
■	$e_T^{\wedge}$	$-\frac{1}{2}$	$\frac{1}{2}$
■	$e_T^{\vee}$	$\frac{1}{2}$	$-\frac{1}{2}$
▲	$f_L^{\wedge}$	$\frac{1}{2}$	0
▲	$f_L^{\vee}$	$-\frac{1}{2}$	0
▲	$f_R^{\wedge}$	0	$\frac{1}{2}$
▲	$f_R^{\vee}$	0	$-\frac{1}{2}$

Electroweak SU(2) and U(1)

$$W = \begin{bmatrix} \frac{i}{2}W^3 & W^+ \\ W^- & -\frac{i}{2}W^3 \end{bmatrix} \quad B_1 = \begin{bmatrix} \frac{i}{2}B_1^3 & B_1^+ \\ B_1^- & -\frac{i}{2}B_1^3 \end{bmatrix}$$

$$[\begin{bmatrix} W & \\ & B_1 \end{bmatrix}, \begin{bmatrix} & \phi_B \\ \phi_W & \end{bmatrix}]$$

$$\phi_{W/B} = \begin{bmatrix} -\phi_{0/1} & \phi_+ \\ \phi_- & \phi_{1/0} \end{bmatrix}$$

$$\begin{bmatrix} W & \\ & B_1 \end{bmatrix} \quad \begin{bmatrix} \nu_{eL} \\ e_L \\ \nu_{eR} \\ e_R \end{bmatrix} \quad \begin{bmatrix} u_L \\ d_L \\ u_R \\ d_R \end{bmatrix}$$

$$(\frac{\sqrt{3}}{\sqrt{5}}B_1^3 - \frac{\sqrt{2}}{\sqrt{5}}B_2) = (\frac{\sqrt{3}}{\sqrt{5}})\frac{1}{2}Y \ \rightarrow \ g_1 = \frac{\sqrt{3}}{\sqrt{5}}$$

$SO(4)$	W^3	B_1^3	$\frac{\sqrt{2}}{\sqrt{3}}B_2$	$\frac{1}{2}Y$	Q
W^+	1	0	0	0	1
W^-	-1	0	0	0	-1
B_1^+	0	1	0	1	1
B_1^-	0	-1	0	-1	-1
ϕ_+	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	1
ϕ_-	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1
ϕ_0	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	0
ϕ_1	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0
ν_{eL}	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	0
e_L	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	-1
ν_{eR}	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0
e_R	0	$-\frac{1}{2}$	$\frac{1}{2}$	-1	-1
u_L	$\frac{1}{2}$	0	$-\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
d_L	$-\frac{1}{2}$	0	$-\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
u_R	0	$\frac{1}{2}$	$-\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
d_R	0	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$

Graviweak SO(7,1)

$$H_1 = (\tfrac{1}{2}\omega + \tfrac{1}{4}e\phi + W + B_1)$$

$$\in so(3,1) + 4\times 4 + \big(su(2) + su(2)\big)$$

$$= Cl^2(7,1) = so(7,1) = d4$$

$$8_{S+} \quad \rightarrow \qquad H_1\,(\nu_e + e)$$

$$=$$

$$\begin{bmatrix} \frac{1}{2}\omega_L + \frac{i}{2}W^3 & W^+ & -\frac{1}{4}e_R\phi_1 & \frac{1}{4}e_R\phi_+ \\ W^- & \frac{1}{2}\omega_L - \frac{i}{2}W^3 & \frac{1}{4}e_R\phi_- & \frac{1}{4}e_R\phi_0 \\ -\frac{1}{4}e_L\phi_0 & \frac{1}{4}e_L\phi_+ & \frac{1}{2}\omega_R + \frac{i}{2}B_1^3 & B_1^+ \\ \frac{1}{4}e_L\phi_- & \frac{1}{4}e_L\phi_1 & B_1^- & \frac{1}{2}\omega_R - \frac{i}{2}B_1^3 \end{bmatrix} \begin{bmatrix} \nu_{eL} \\ e_L \\ \nu_{eR} \\ e_R \end{bmatrix}$$

$D4$	$\frac{1}{2}\omega_L^3$	$\frac{1}{2}\omega_R^3$	W^3	B_1^3
$\omega_L^{\wedge/\vee}$	± 1	0	0	0
$\omega_R^{\wedge/\vee}$	0	± 1	0	0
$W^\pm$	0	0	± 1	0
$B_1^\pm$	0	0	0	± 1
$e_T^{\wedge/\vee}\phi_+$	$\mp\frac{1}{2}$	$\pm\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$e_S^{\wedge/\vee}\phi_+$	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$e_T^{\wedge/\vee}\phi_-$	$\mp\frac{1}{2}$	$\pm\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$e_S^{\wedge/\vee}\phi_-$	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$e_T^{\wedge/\vee}\phi_0$	$\mp\frac{1}{2}$	$\pm\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$
$e_S^{\wedge/\vee}\phi_0$	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$
$e_T^{\wedge/\vee}\phi_1$	$\mp\frac{1}{2}$	$\pm\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
$e_S^{\wedge/\vee}\phi_1$	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
$\nu_{eL}^{\wedge/\vee}$	$\pm\frac{1}{2}$	0	$\frac{1}{2}$	0
$e_L^{\wedge/\vee}$	$\pm\frac{1}{2}$	0	$-\frac{1}{2}$	0
$\nu_{eR}^{\wedge/\vee}$	0	$\pm\frac{1}{2}$	0	$\frac{1}{2}$
$e_R^{\wedge/\vee}$	0	$\pm\frac{1}{2}$	0	$-\frac{1}{2}$

Graviweak F4

A **triality** rotation, T, of $D4$:

$$\begin{bmatrix} \frac{1}{2}\omega'^3_L \\ \frac{1}{2}\omega'^3_R \\ W'^3 \\ B'^3_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2}\omega^3_L \\ \frac{1}{2}\omega^3_R \\ W^3 \\ B^3_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\omega^3_R \\ B^3_1 \\ W^3 \\ \frac{1}{2}\omega^3_L \end{bmatrix}$$

$$T\,T\,T\,\omega^{\wedge}_R = T\,T\,\omega^{\wedge}_L = T\,B^{+}_1 = \omega^{\wedge}_R$$

8_V	$\frac{1}{2}\omega^3_L$	$\frac{1}{2}\omega^3_R$	W^3	B^3_1
tri	$\frac{1}{2}\omega^3_R$	B^3_1	W^3	$\frac{1}{2}\omega^3_L$
$\nu^{\wedge/\vee}_{\mu L}$	0	0	$\frac{1}{2}$	$\pm\frac{1}{2}$
$\mu^{\wedge/\vee}_{L}$	0	0	$-\frac{1}{2}$	$\pm\frac{1}{2}$
$\nu^{\wedge/\vee}_{\mu R}$	$\pm\frac{1}{2}$	$\frac{1}{2}$	0	0
$\mu^{\wedge/\vee}_{R}$	$\pm\frac{1}{2}$	$-\frac{1}{2}$	0	0

Roots invariant under this T:

$$\{W^+,\, W^-,\, e^{\wedge}_S\phi_+,\, e^{\wedge}_S\phi_0,\, e^{\vee}_S\phi_-,\, e^{\vee}_S\phi_1\}$$

Rotations to triality-equivalent vector and negative chiral spinor representation spaces:

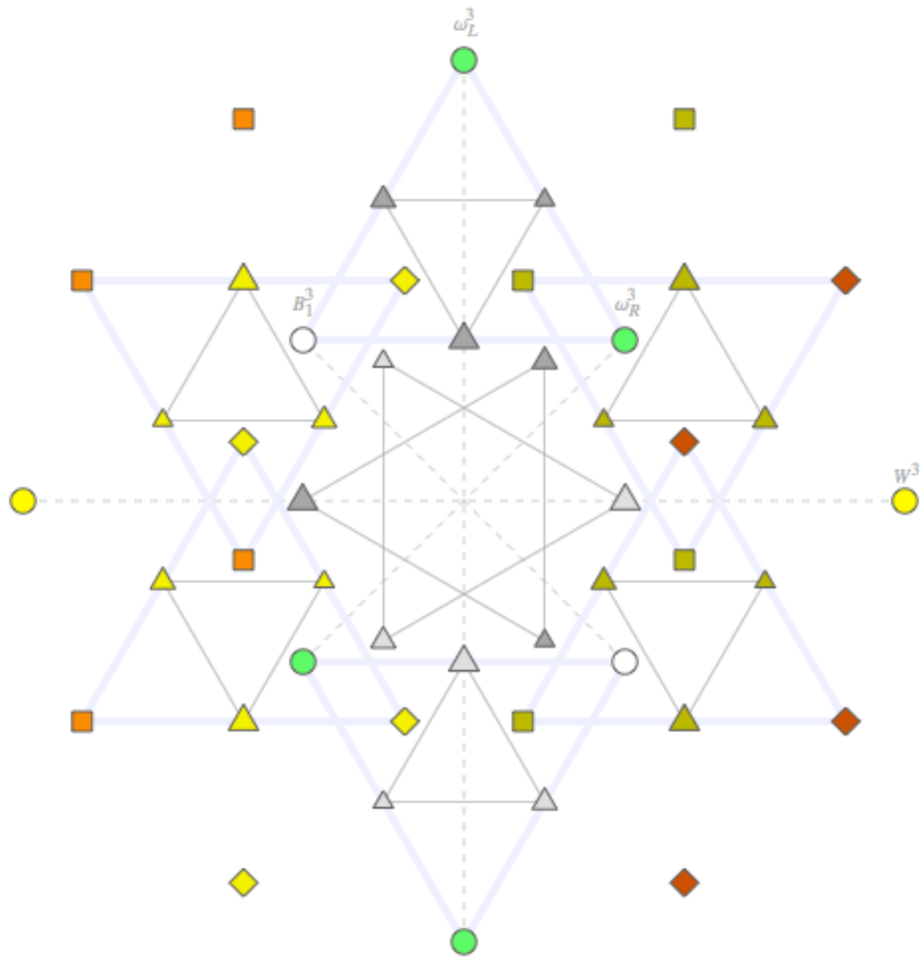
$$T\,8_{S+} = 8_V \quad T\,8_V = 8_{S-} \quad T\,8_{S-} = 8_{S+}$$

8_{S-}	$\frac{1}{2}\omega^3_L$	$\frac{1}{2}\omega^3_R$	W^3	B^3_1
tri	B^3_1	$\frac{1}{2}\omega^3_L$	W^3	$\frac{1}{2}\omega^3_R$
$\nu^{\wedge/\vee}_{\tau L}$	0	$\pm\frac{1}{2}$	$\frac{1}{2}$	0
$\tau^{\wedge/\vee}_{L}$	0	$\pm\frac{1}{2}$	$-\frac{1}{2}$	0
$\nu^{\wedge/\vee}_{\tau R}$	$\frac{1}{2}$	0	0	$\pm\frac{1}{2}$
$\tau^{\wedge/\vee}_{R}$	$-\frac{1}{2}$	0	0	$\pm\frac{1}{2}$

Three generations, related by triality:

$$T\,e^{\wedge}_L = \mu^{\wedge}_L \quad T\,\mu^{\wedge}_L = \tau^{\wedge}_L \quad T\,\tau^{\wedge}_L = e^{\wedge}_L$$

F4 root system



F4 and G2 together

$$
\begin{array}{lll}
F4 & : & (\frac{1}{2}\omega^3_L, \frac{1}{2}\omega^3_R, W^3, B^3_1) \quad \begin{cases}\text{graviweak interactions}\\ \text{three generations}\end{cases} \\
G2 & : & (B_2, g^3, g^8) \quad \begin{cases}\text{strong interactions}\\ \text{anti-particles}\end{cases} \\
\hline
E8 & : & (\frac{1}{2}\omega^3_L, \frac{1}{2}\omega^3_R, W^3, B^3_1, w, B_2, g^3, g^8) \quad \{ \text{ everything}
\end{array}
$$

Breakdown of E8 to the standard model and gravity:

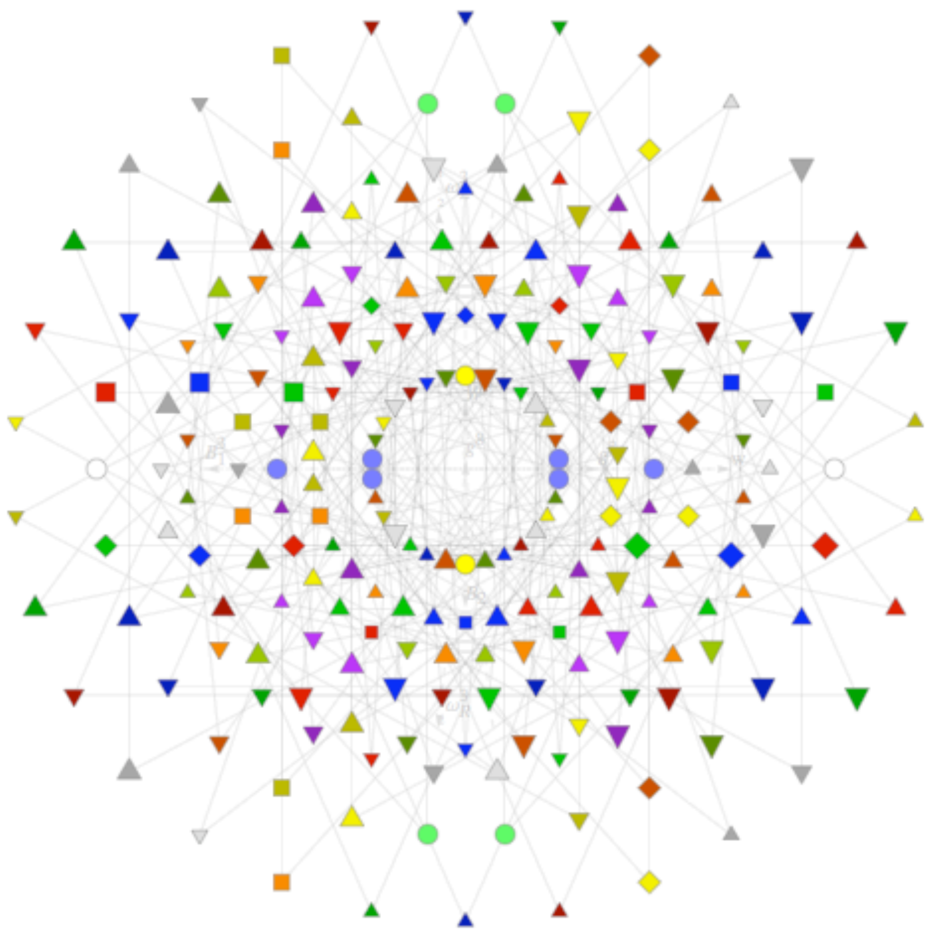
$$
\begin{aligned}
e8 &= f4 + g2 + 26\times 7 \\
&= so(7,1) + su(3) + (8_{S+}+8_V+8_{S-})\times(1+1+3+\bar{3}) + 3\times(3+\bar{3}) + 2
\end{aligned}
$$

$$
A = \left(\frac{1}{2}\omega + \frac{1}{4}e\phi + W + B_1\right) + g + 3\times\Psi + x\Phi + B_2 + w
$$

Two new quantum numbers and some non-standard particles:

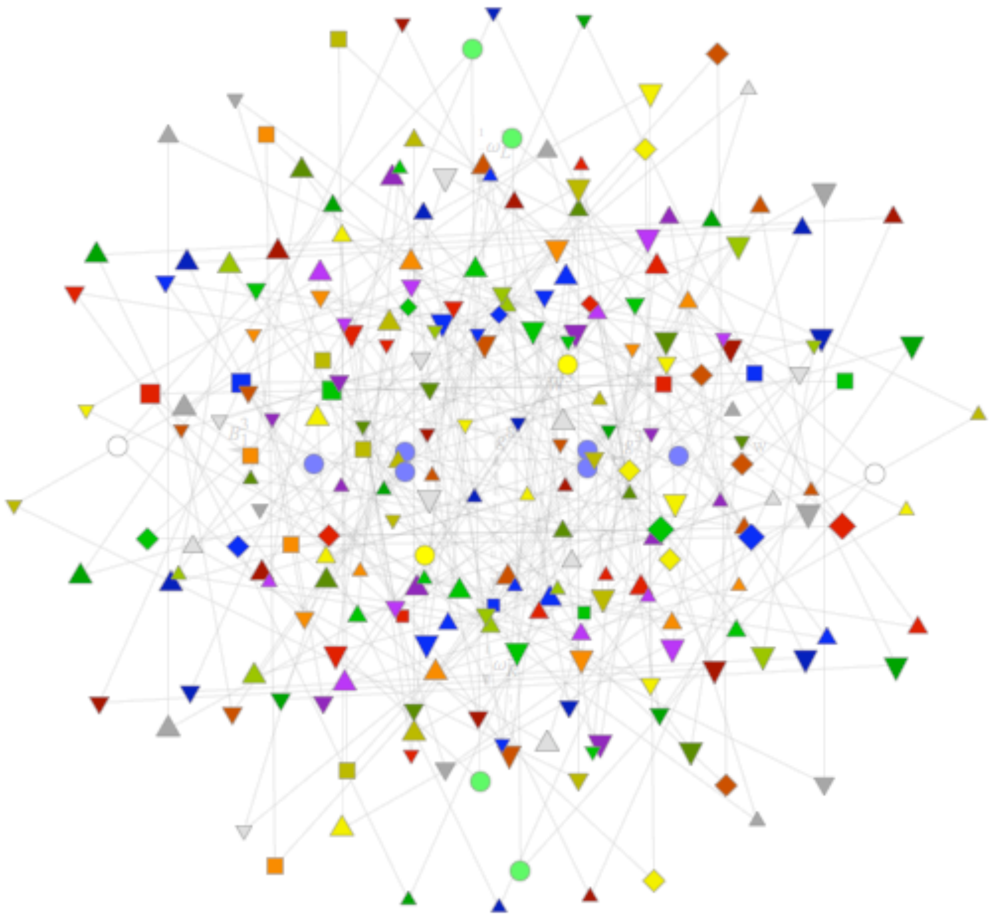
$$
\{ \; w \quad (B^3_1+B_2) \quad B^{\pm}_1 \quad x_{1/2/3}\Phi^{r/g/b} \quad x_{1/2/3}\Phi^{\bar{r}/\bar{g}/\bar{b}} \; \}
$$

E8 root system .



E.S.T.o.E.: Each E8 vertex corresponds to an elementary particle field.

E8 root system ..



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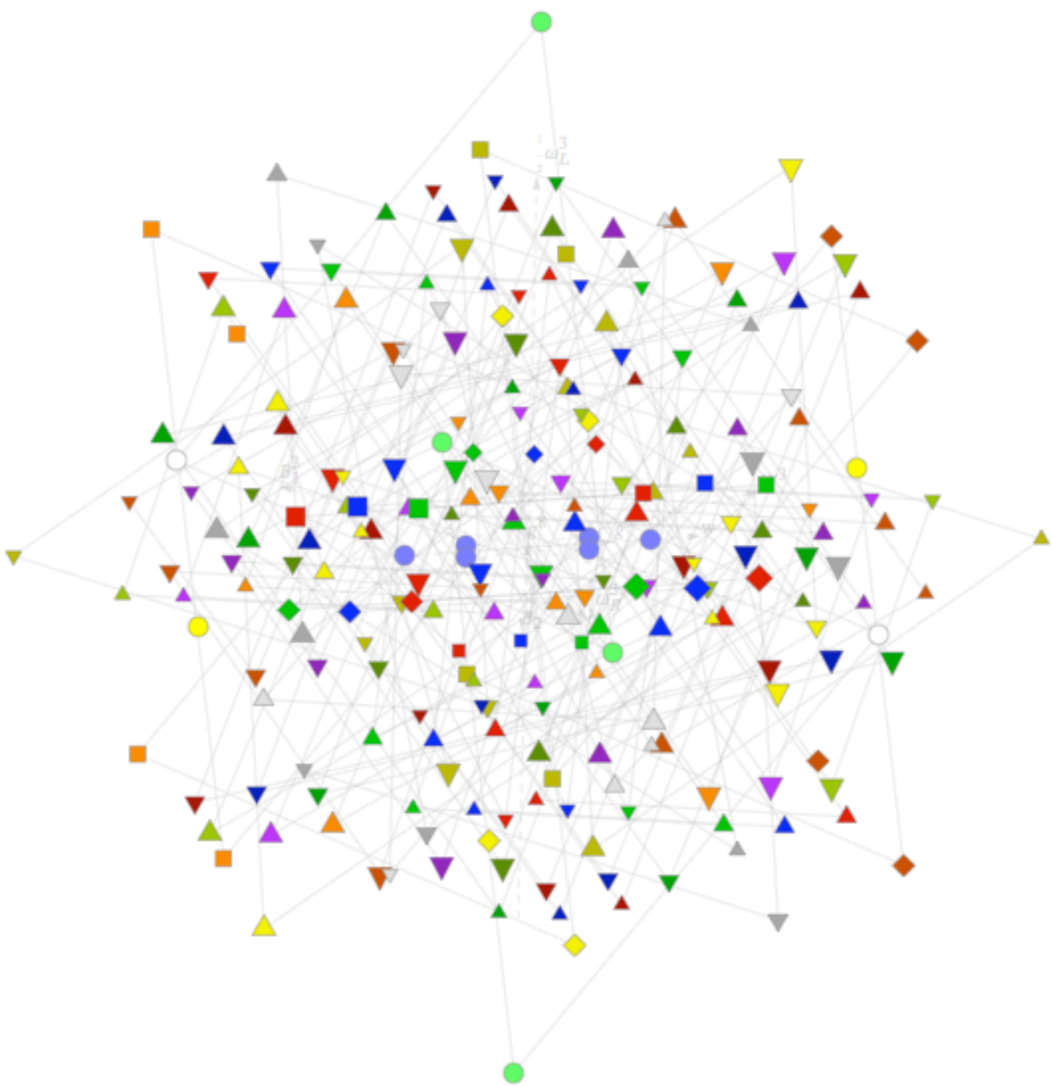

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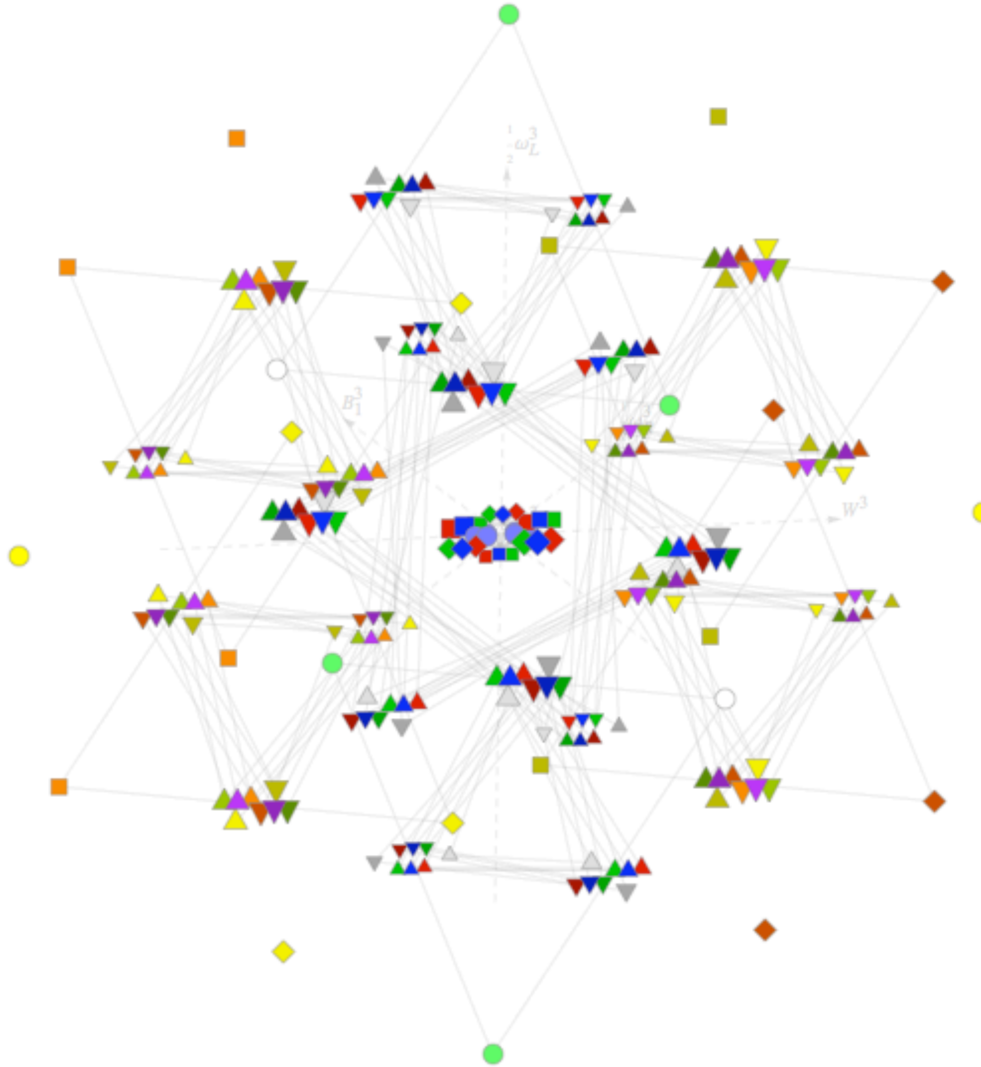
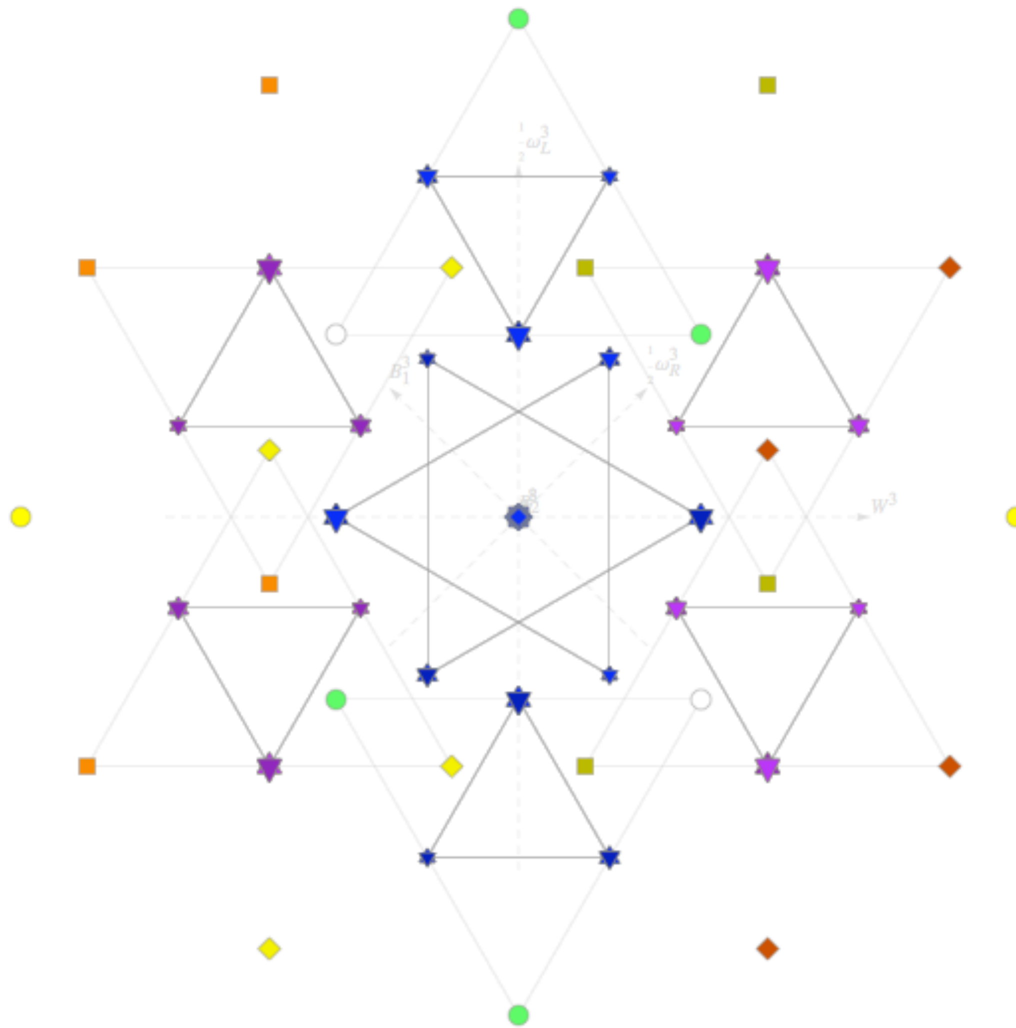
E8 root system ....



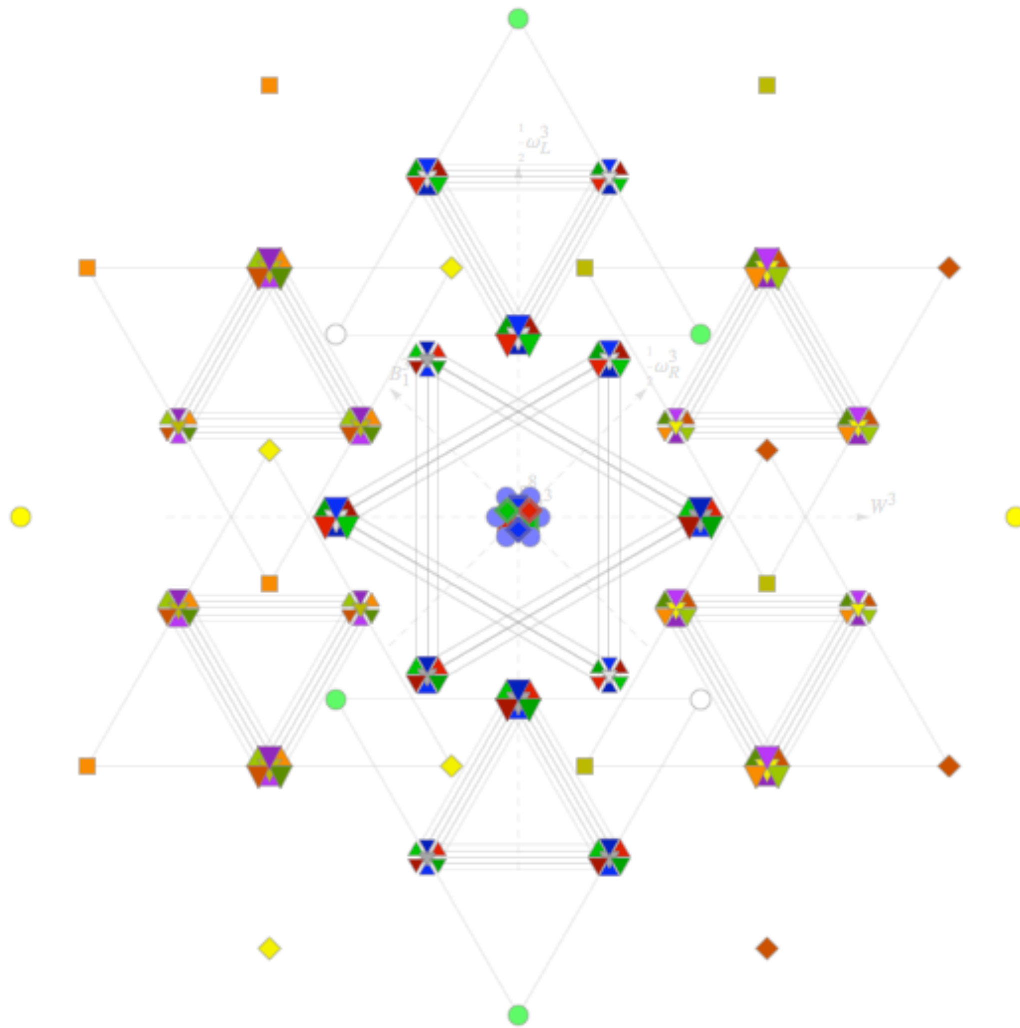
E.S.T.o.E.: Each E8 vertex corresponds to an elementary particle field.

E8 root system x



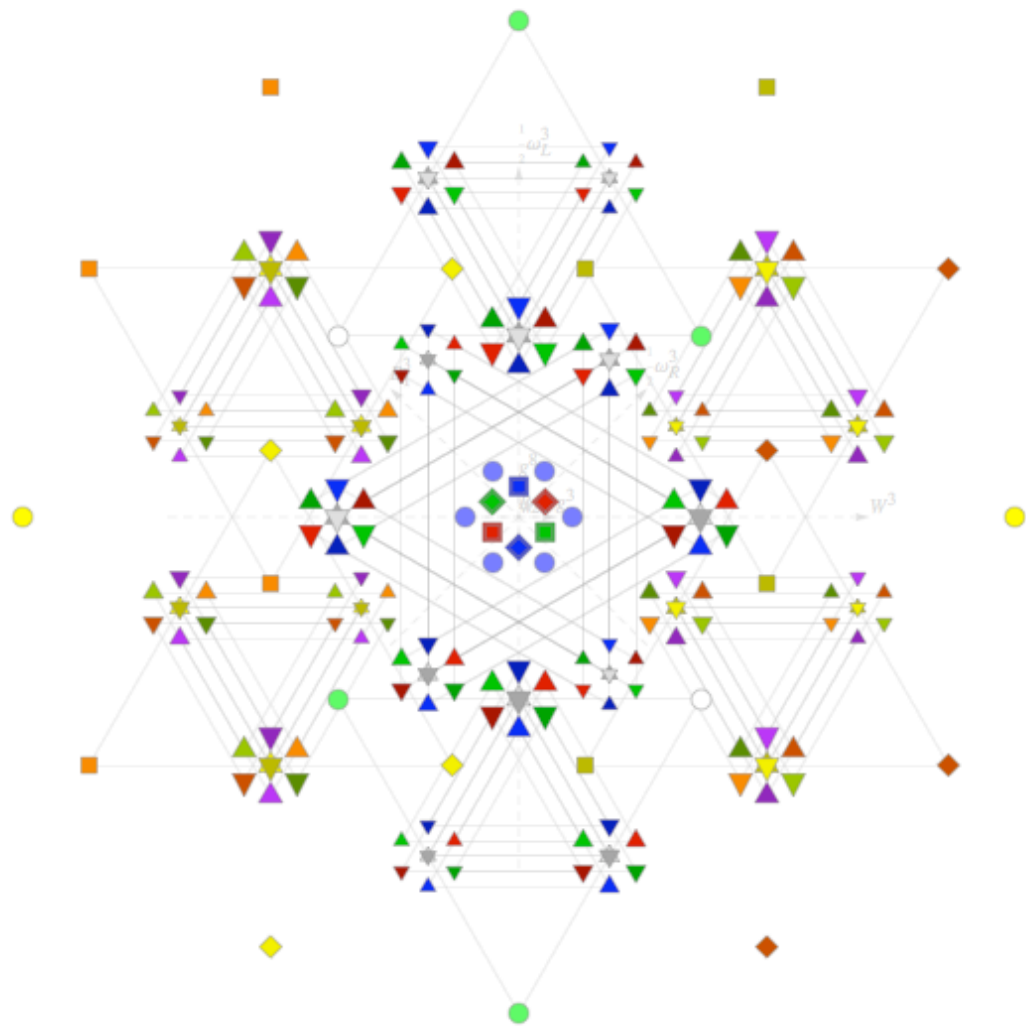
E.S.T.o.E.: Each E8 vertex corresponds to an elementary particle field.

E8 root system ....x.



E.S.T.o.E.: Each E8 vertex corresponds to an elementary particle field.

E8 root system ....x.x



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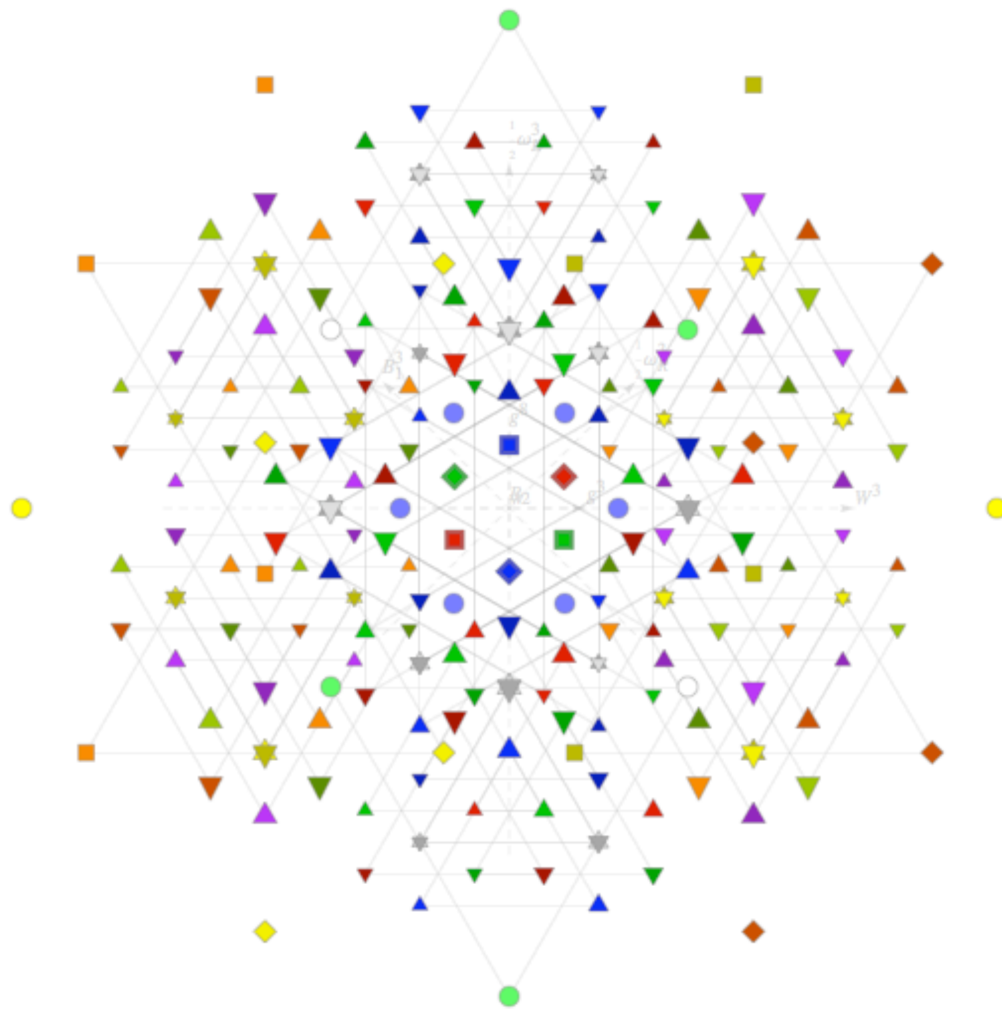
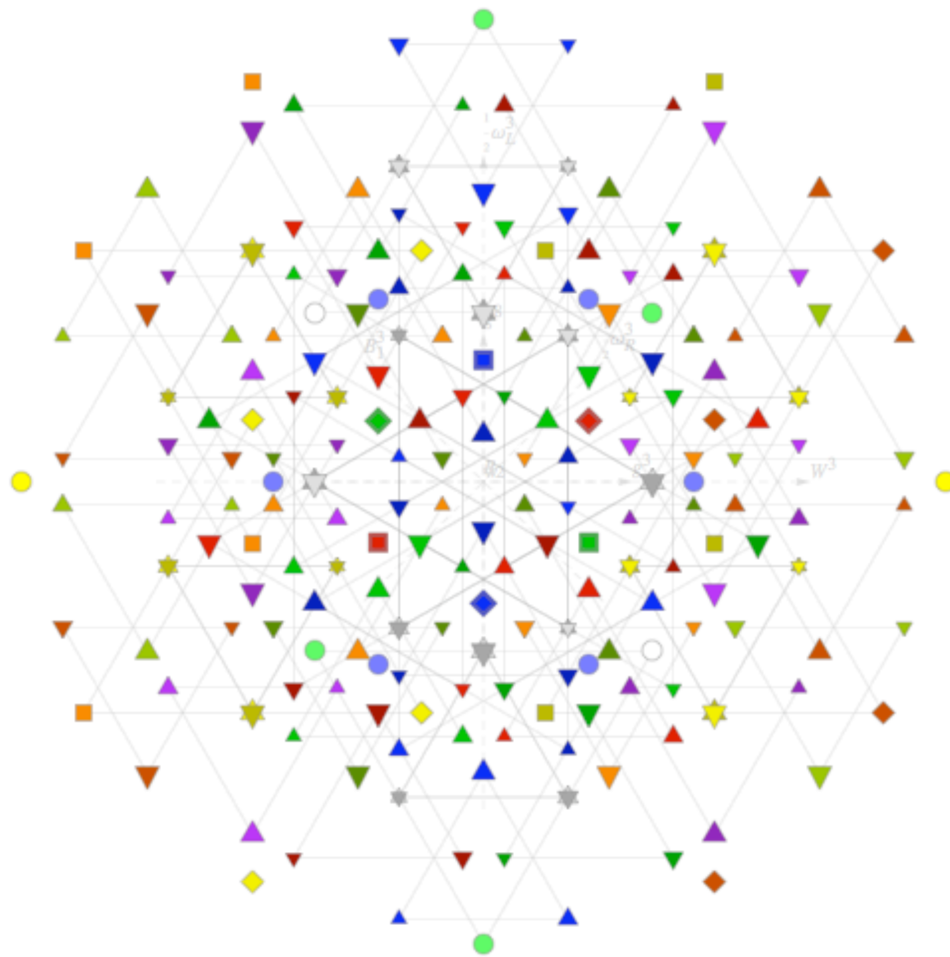
E8 root system ....x.x.



E.S.T.o.E.: Each E8 vertex corresponds to an elementary particle field.

E8 root system ....x.x..



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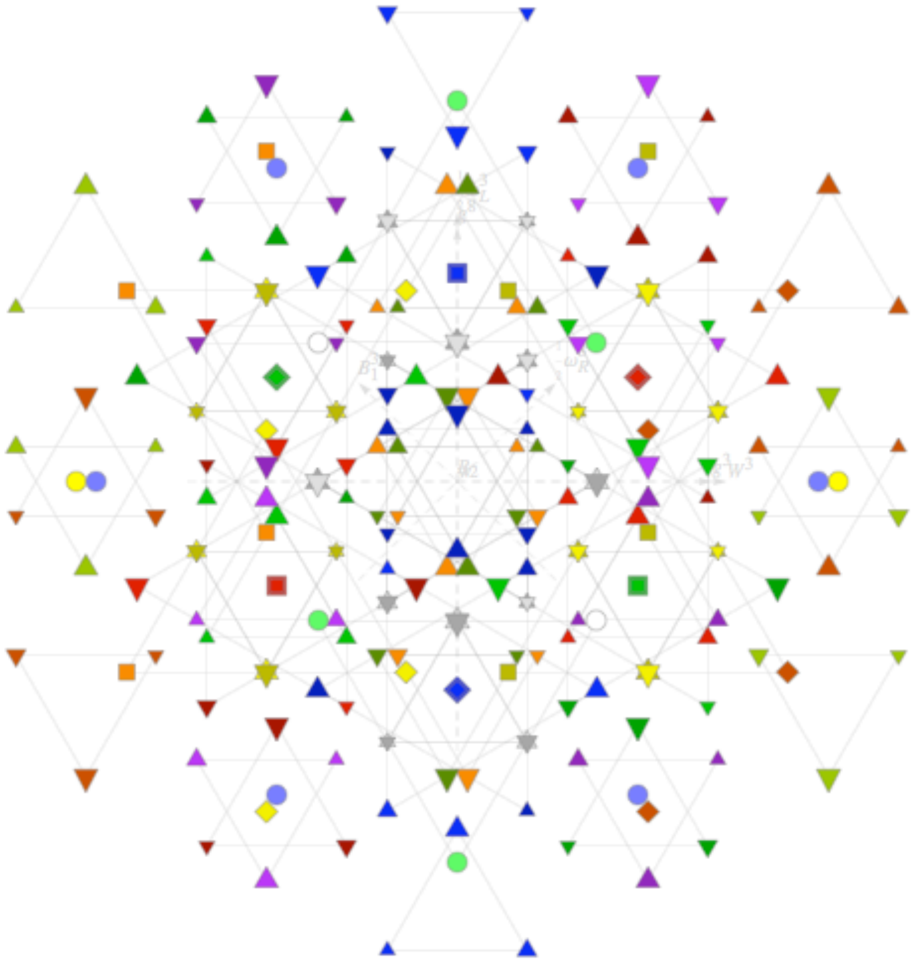
E8 root system ....x.x...



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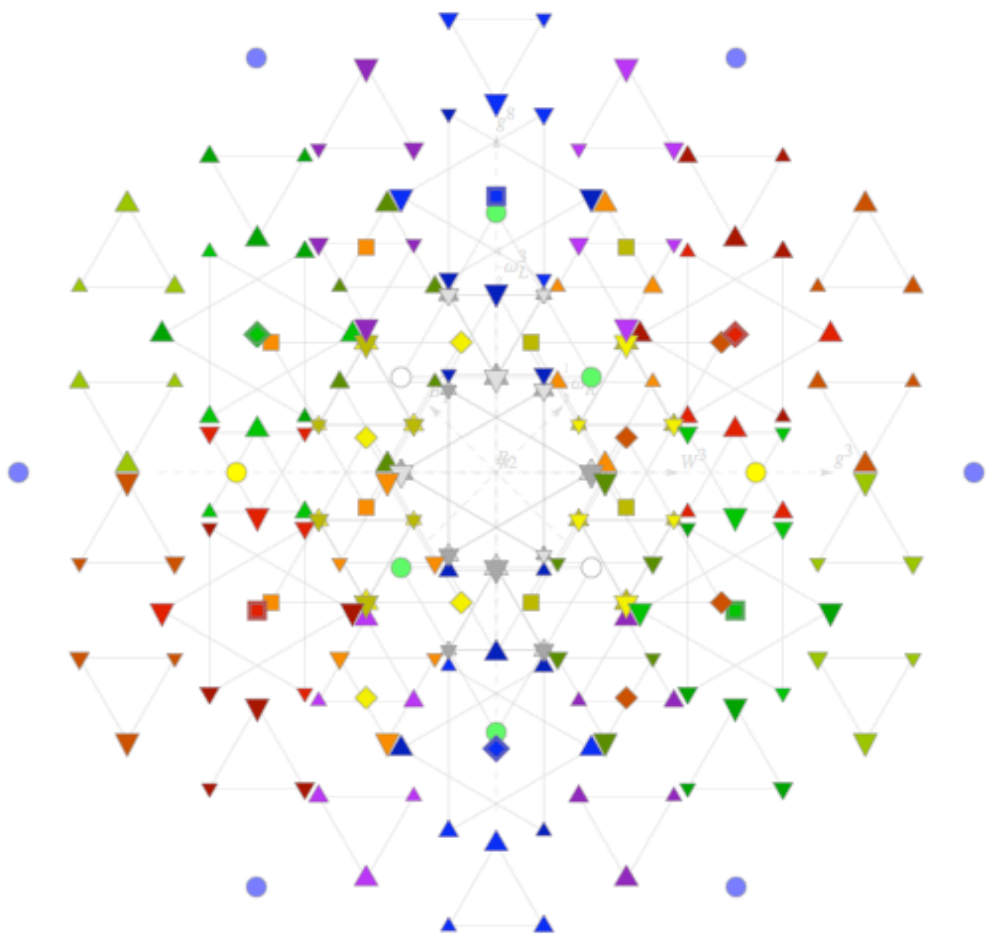
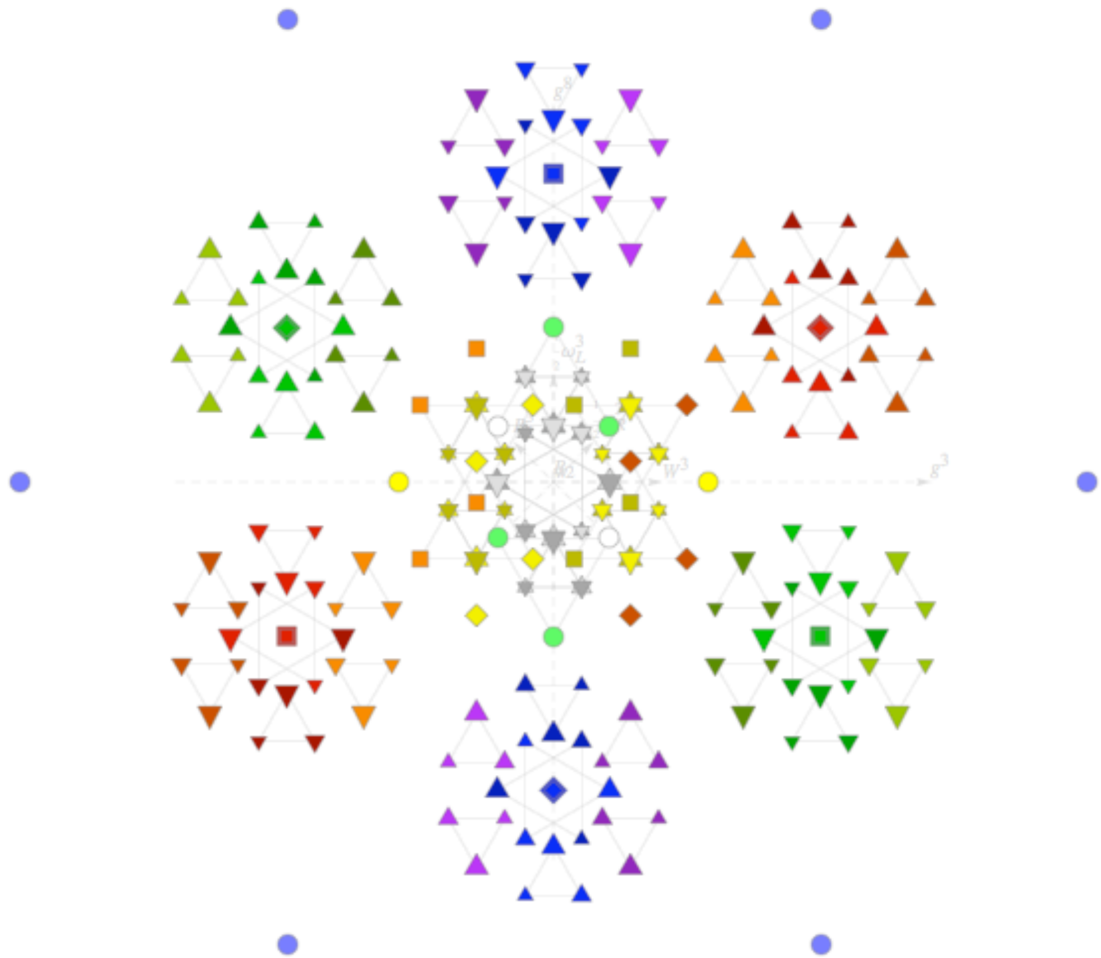
E8 root system ....x.x....



E.S.T.o.E.: Each E8 vertex corresponds to an elementary particle field.

E8 root system ....x.x....x



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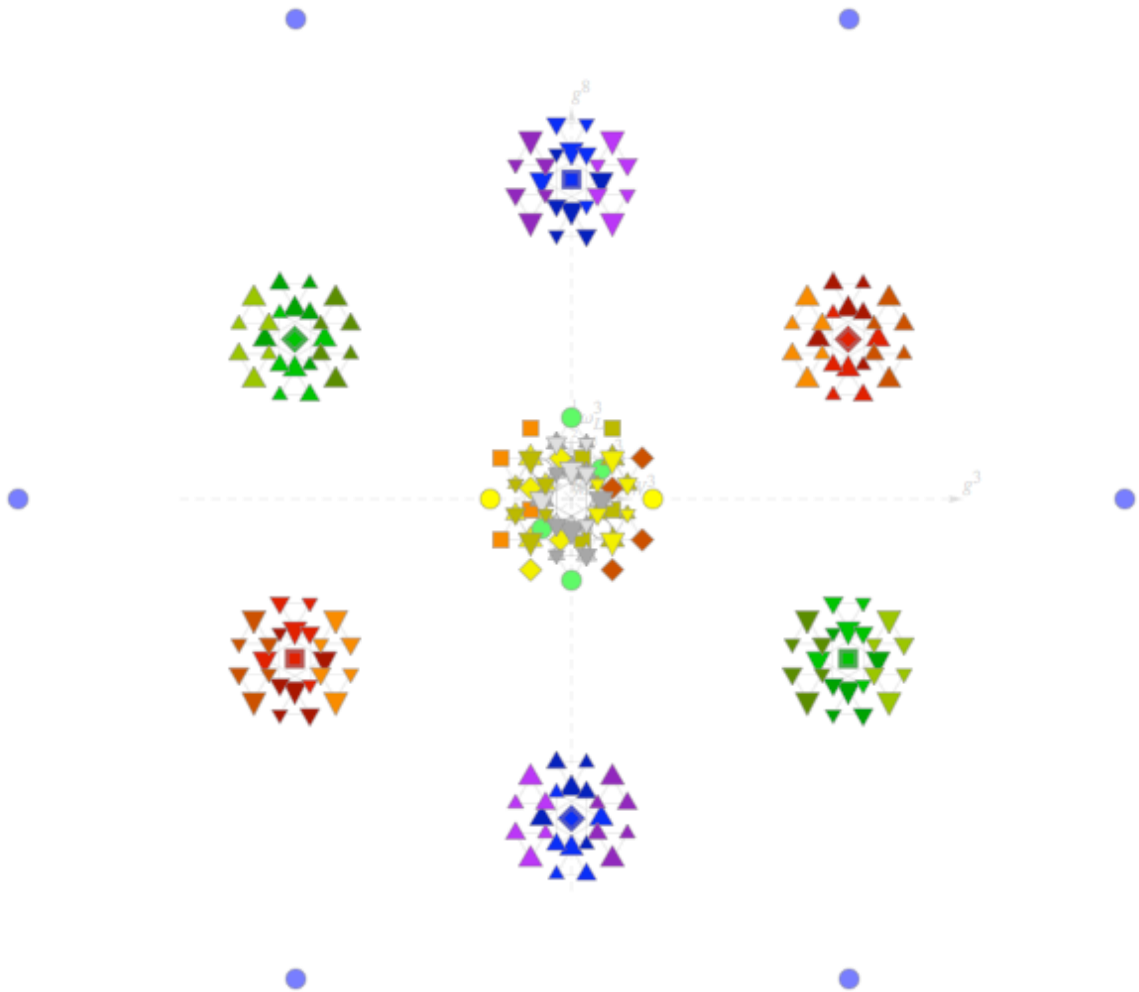
E8 root system ....x.x....x.



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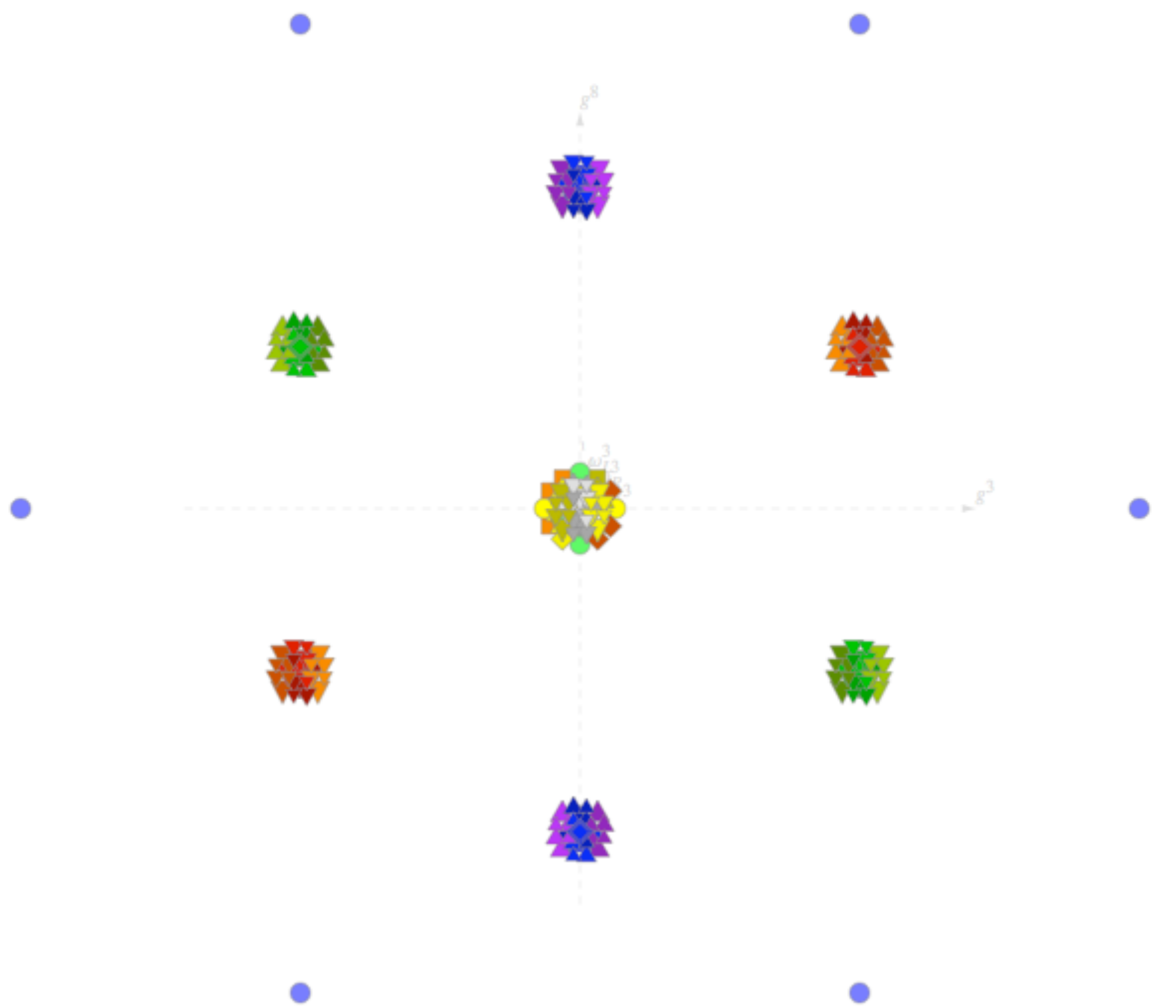
E8 root system ....x.x....x..



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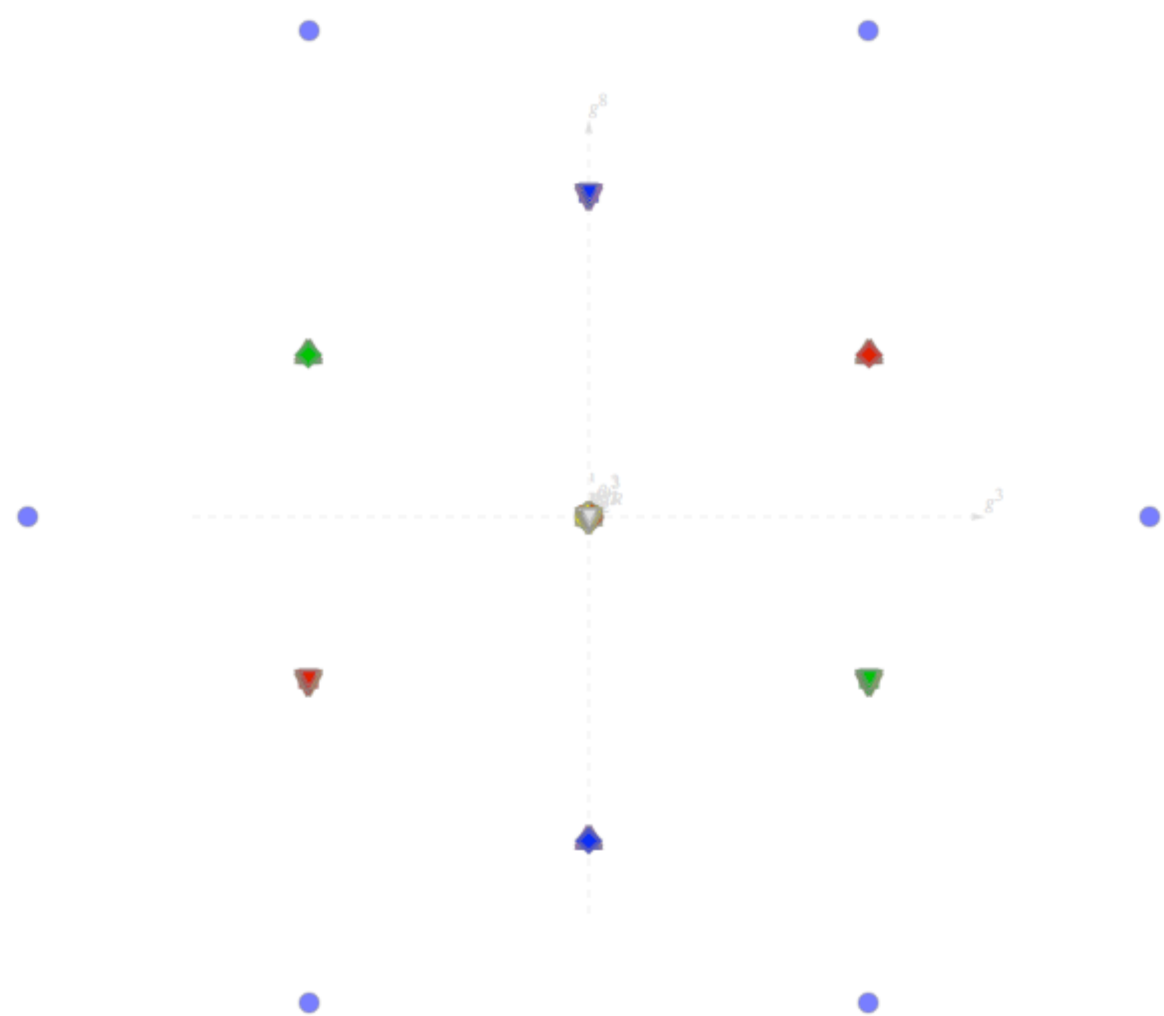
E8 root system ....x.x....x..x



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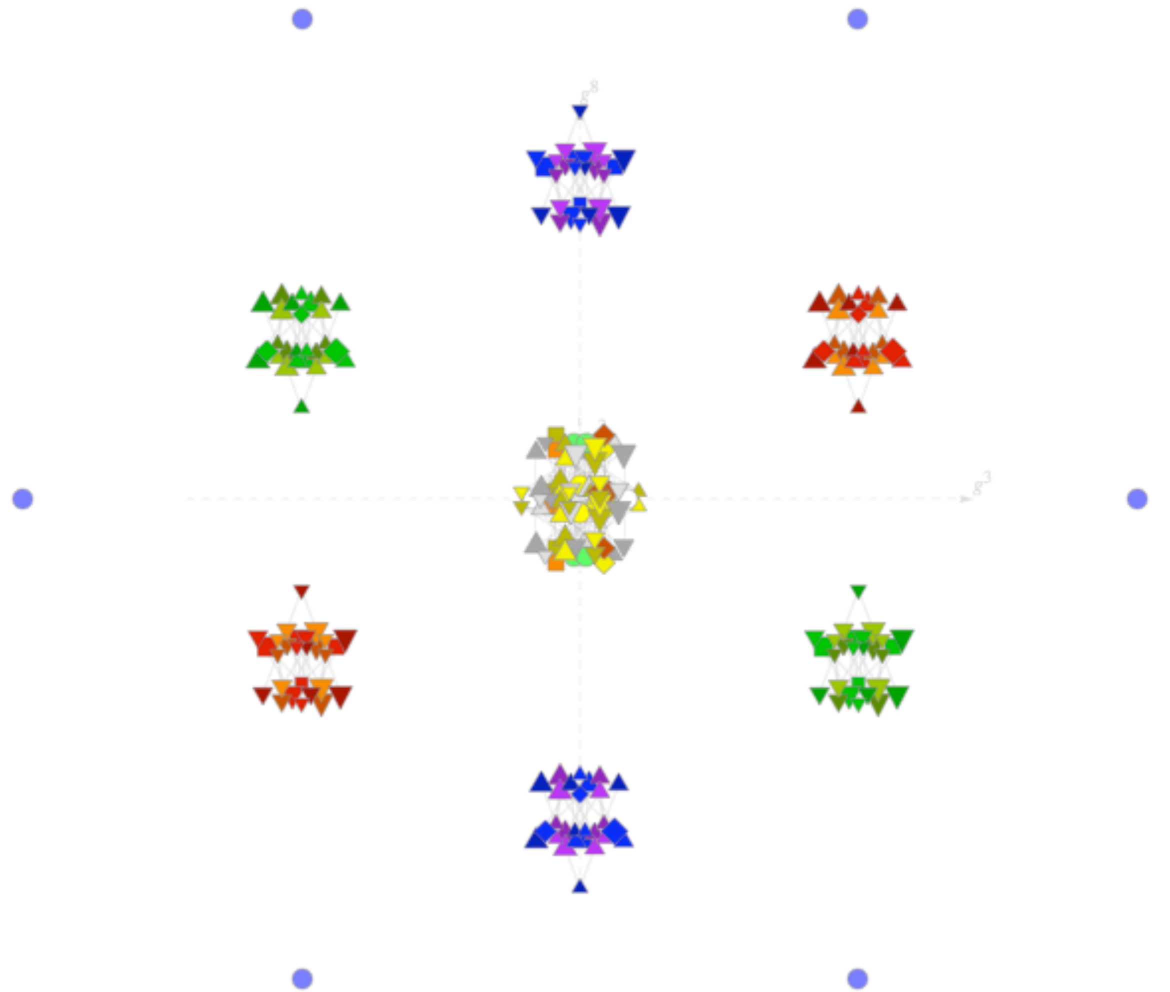
E8 root system ....x.x....x..x.



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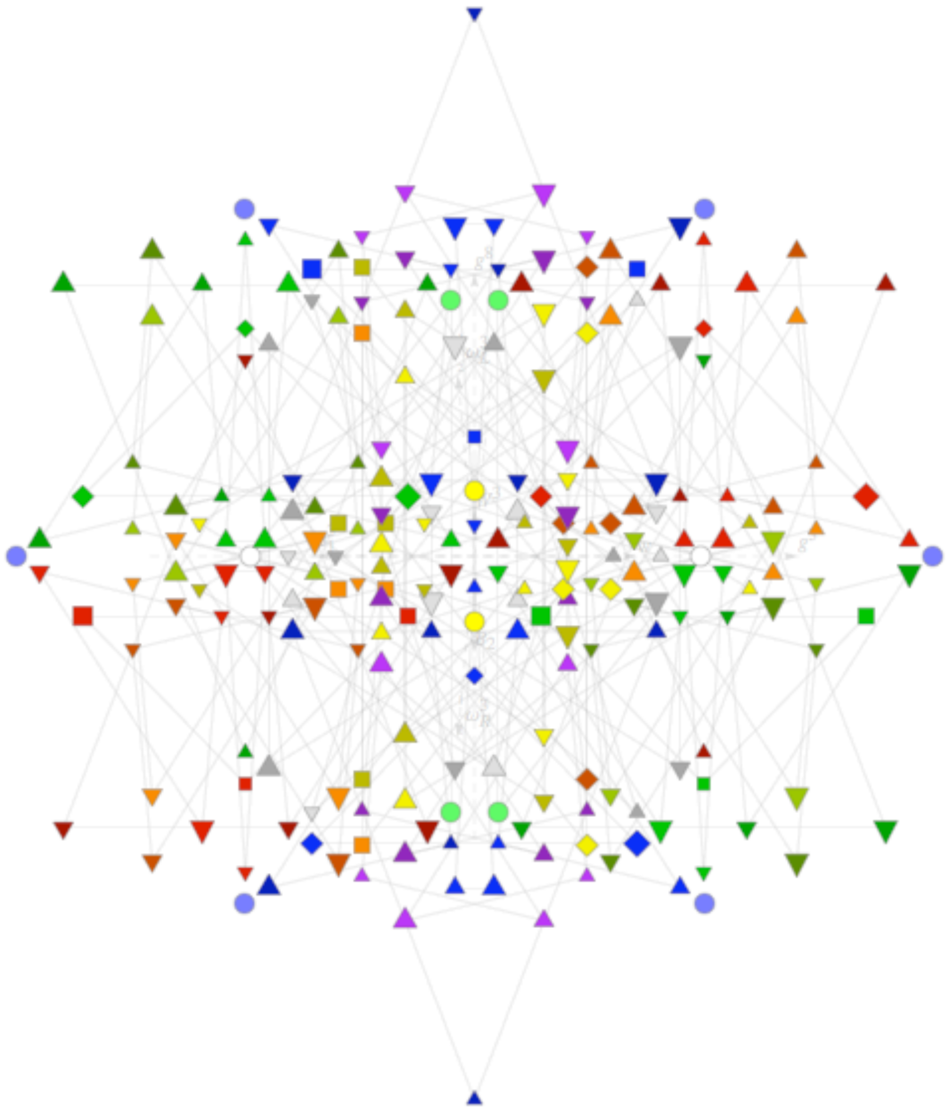
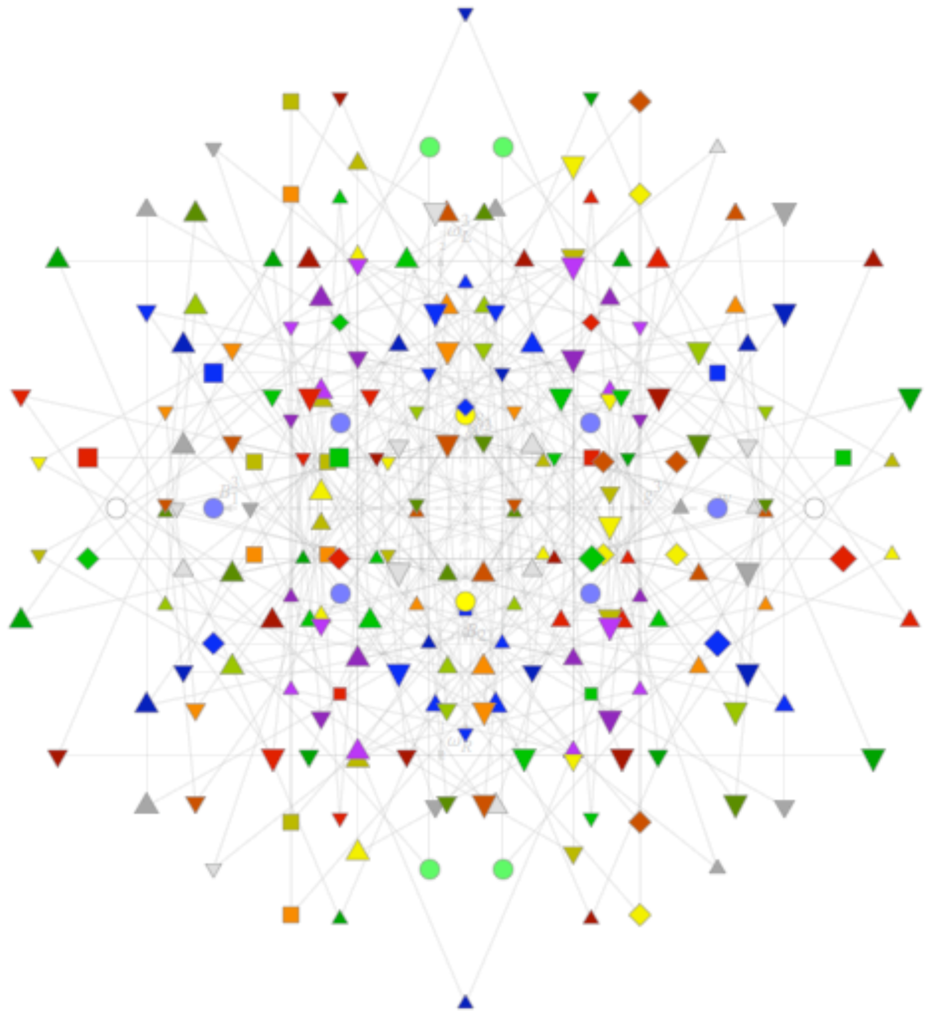
E8 root system ....x.x....x..x..



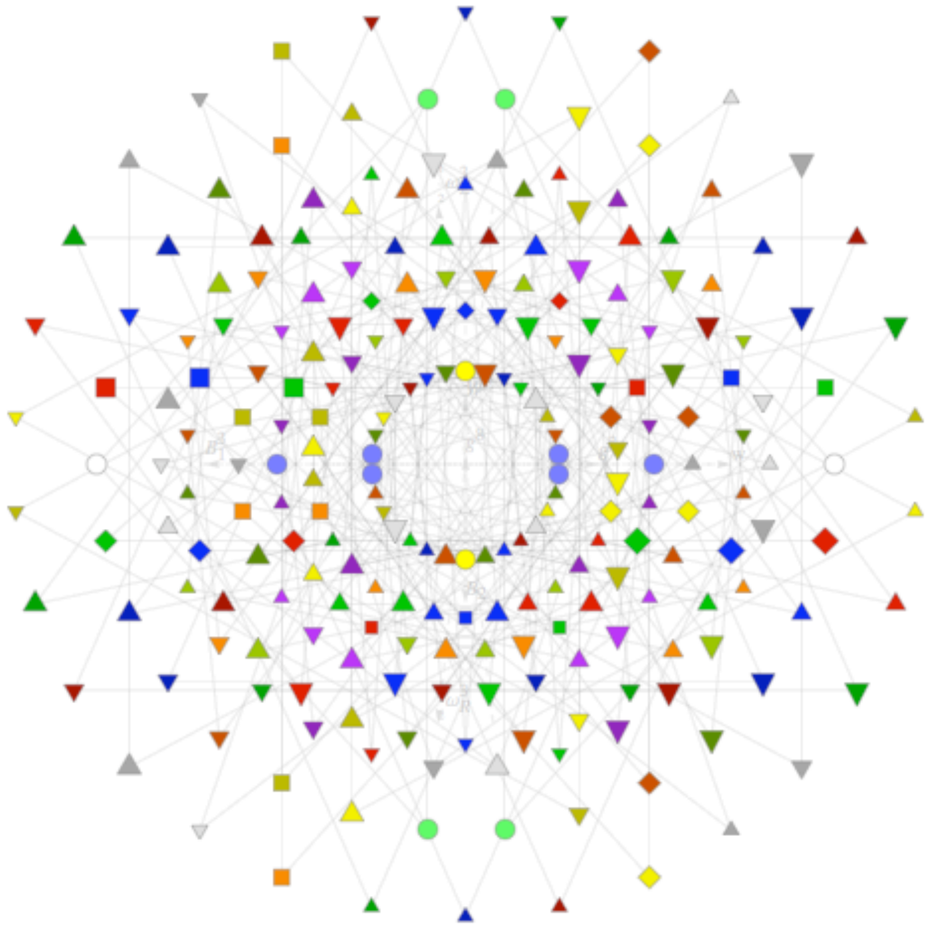
E.S.T.o.E.: Each E8 vertex corresponds to an elementary particle field.

E8 root system ....x.x....x..x...



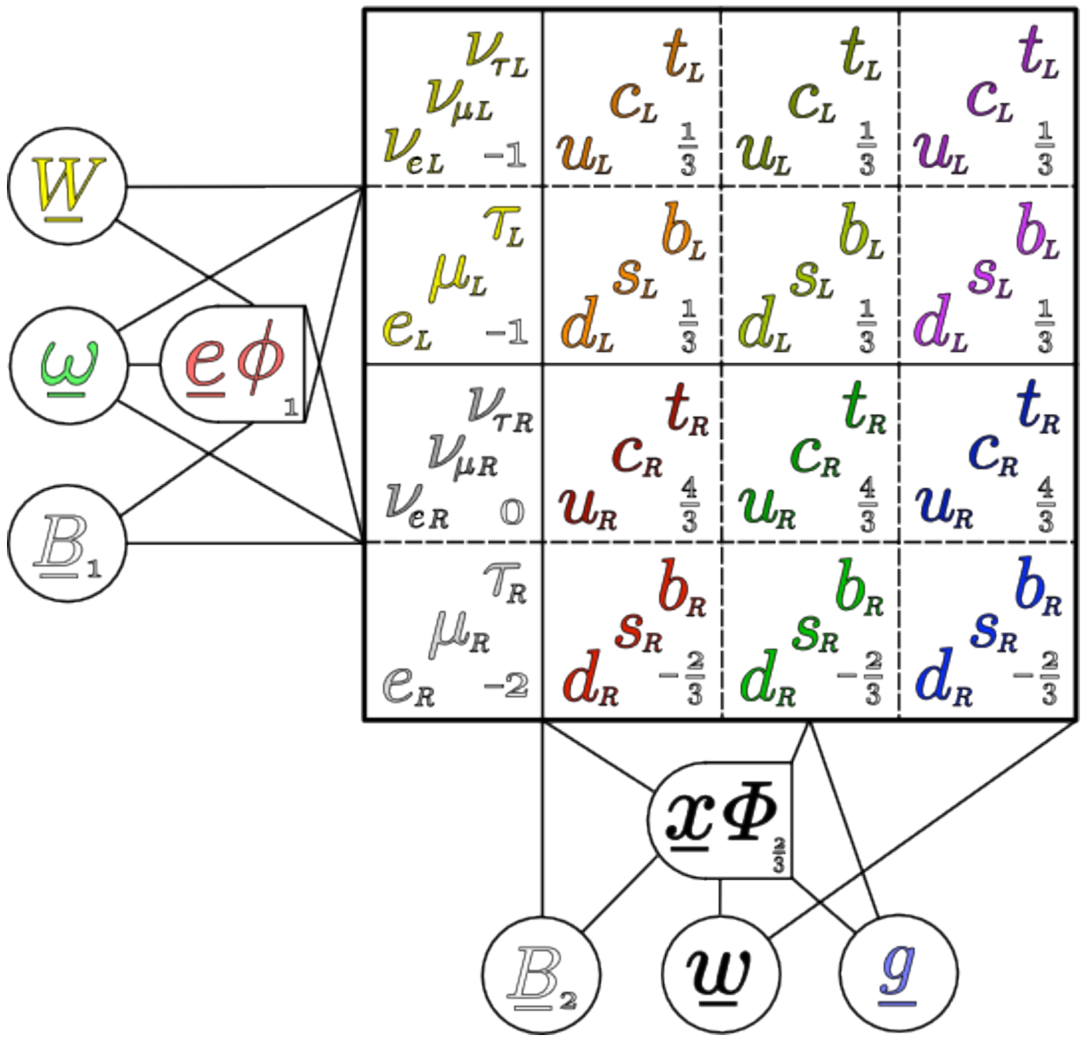
E.S.T.o.E.: Each E8 vertex corresponds to an elementary particle field.

E8 root system ....x.x....x..x...x



E.S.T.o.E.: Each E8 vertex corresponds to an elementary particle field.

E8 periodic table



"E8 is perhaps the most beautiful structure in all of mathematics, but it's very complex." — Hermann Nicolai

E8 connection

$$\underset{\cdot}{\underline{A}} = \underline{H}_1 + \underline{H}_2 + \underset{\cdot}{\Psi}_I + \underset{\cdot}{\Psi}_{II} + \underset{\cdot}{\Psi}_{III} \quad \in \quad \underset{\cdot}{\underline{e8}}$$

$$\begin{aligned}
\underline{H}_1 &= \tfrac{1}{2}\underline{\omega} + \tfrac{1}{4}\underline{e}\phi + \underline{W} + \underline{B}_1 && \in \quad \underline{so}(7,1) \\
&\underline{\omega} && \in \quad \underline{so}(3,1) \\
&\underline{e}\phi = (\underline{e}_1 + \underline{e}_2 + \underline{e}_3 + \underline{e}_4) \times (\phi_{+/0} + \phi_{-/1}) && \in \quad \underline{4} \times (2 + \bar{2}) \\
&\underline{W} + \underline{B}_1 && \in \quad \underline{su}(2) + \underline{su}(2) \\
\underline{H}_2 &= \underline{w} + \underline{B}_2 + \underline{x}\Phi + \underline{g} && \in \quad \underline{so}(8) \\
&\underline{w} + \underline{B}_2 && \in \quad \underline{u}(1) + \underline{u}(1) \\
&\underline{x}\Phi = (\underline{x}_1 + \underline{x}_2 + \underline{x}_3) \times (\Phi^{r/g/b} + \Phi^{\bar{r}/\bar{g}/\bar{b}}) && \in \quad \underline{3} \times (3 + \bar{3}) \\
&\underline{g} && \in \quad \underline{su}(3) \\
\underset{\cdot}{\Psi}_I &= \underset{\cdot}{\nu}_e + \underset{\cdot}{e} + \underset{\cdot}{u} + \underset{\cdot}{d} && \in \quad 8_{S+} \times 8_{S+} \\
\underset{\cdot}{\Psi}_{II} &= \underset{\cdot}{\nu}_\mu + \underset{\cdot}{\mu} + \underset{\cdot}{c} + \underset{\cdot}{s} && \in \quad 8_V \times 8_V \\
\underset{\cdot}{\Psi}_{III} &= \underset{\cdot}{\nu}_\tau + \underset{\cdot}{\tau} + \underset{\cdot}{t} + \underset{\cdot}{b} && \in \quad 8_{S-} \times 8_{S-}
\end{aligned}$$

E8 curvature

$$\underset{\cdot}{\underline{\underline{F}}} = \underline{d}\,\underset{\cdot}{\underline{A}} + \underset{\cdot}{\underline{A}}\,\underset{\cdot}{\underline{A}} = \underline{\underline{F}}_1 + \underline{\underline{F}}_2 + \underline{D}\Big(\underset{\cdot}{\Psi}_I + \underset{\cdot}{\Psi}_{II} + \underset{\cdot}{\Psi}_{III}\Big) \quad \in \quad \underset{\cdot}{\underline{\underline{e}}}8$$

$$\underline{\underline{F}}_1 = \tfrac{1}{2}\Big(\underline{\underline{R}} - \tfrac{1}{8}\underline{e}\,\underline{e}\,\phi^2\Big) + \tfrac{1}{4}\Big(\underline{\underline{T}}\phi - \underline{e}\,\underline{D}\phi\Big) + \Big(\underline{\underline{F}}_{B_1} + \underline{\underline{F}}_W\Big) \quad \in \quad s\underline{o}(7,1)$$

$$\underline{\underline{R}} = \underline{d}\,\underline{\omega} + \tfrac{1}{2}\underline{\omega}\,\underline{\omega} \quad \in \quad s\underline{o}(3,1)$$

$$\underline{\underline{T}}\phi - \underline{e}\,\underline{D}\phi = \Big(\underline{d}\,\underline{e} + \tfrac{1}{2}[\underline{\omega}, \underline{e}]\Big)\phi - \underline{e}\Big(\underline{d}\phi + [\underline{B}_1 + \underline{W}, \phi]\Big) \in \quad \underline{4} \times (2 + \bar{2})$$

$$\underline{\underline{F}}_{B_1} + \underline{\underline{F}}_W = (\underline{d}\,\underline{B}_1 + \underline{B}_1\underline{B}_1) + (\underline{d}\,\underline{W} + \underline{W}\,\underline{W}) \quad \in \quad s\underline{u}(2) + s\underline{u}(2)$$

$$\underline{\underline{F}}_2 = \Big(\underline{\underline{F}}_w + \underline{\underline{F}}_{B_2} + \underline{x}\Phi\underline{x}\Phi\Big) + \Big((\underline{D}\,\underline{x})\Phi - \underline{x}\,\underline{D}\Phi\Big) + \underline{\underline{F}}_g \quad \in \quad s\underline{o}(8)$$

$$\underline{\underline{F}}_w + \underline{\underline{F}}_{B_2} = \underline{d}\,\underline{w} + \underline{d}\,\underline{B}_2 \quad \in \quad \underline{u}(1) + \underline{u}(1)$$

$$(\underline{D}\,\underline{x})\Phi - \underline{x}\,\underline{D}\Phi = \Big(\underline{d}\,\underline{x} + [\underline{w} + \underline{B}_2, \underline{x}]\Big)\Phi - \underline{x}\Big(\underline{d}\Phi + [\underline{g}, \Phi]\Big) \in \quad \underline{3} \times (3 + \bar{3})$$

$$\underline{\underline{F}}_g = \underline{d}\,\underline{g} + \underline{g}\,\underline{g} \quad \in \quad s\underline{u}(3)$$

$$\underline{D}\underset{\cdot}{\Psi} = \Big(\underline{d} + \frac{1}{2}\underline{\omega} + \frac{1}{4}\underline{e}\phi\Big)\underset{\cdot}{\Psi} + \underline{W}\underset{\cdot}{\Psi}_L + \underline{B}_1\underset{\cdot}{\Psi}_R - \underset{\cdot}{\Psi}\Big(\underline{w} + \underline{B}_2 + \underline{x}\Phi\Big) - \underset{\cdot}{\Psi}_q\,\underline{g}$$

Action for everything

Modified BF action, using $\underset{=}{\dot{B}} = \underset{=}{B} + \underset{\equiv}{\dot{B}}$:

$$
\begin{aligned}
S &= \int \Big\langle \underset{=}{\dot{B}}\, \underset{=\cdot}{F} + \underset{\sim}{\Phi}(\underset{-}{H}_1, \underset{-}{H}_2, \underset{=}{B}) \Big\rangle \\
&= \int \Big\langle \underset{\equiv}{\dot{B}}\, \underset{-}{D} \underset{\cdot}{\Psi} + \underset{=}{B}\, \underset{=}{F} + \frac{\pi G}{4} \underset{=}{B}_G \underset{=}{B}_G \gamma + \underset{=}{B}' * \underset{=}{B}' \Big\rangle \\
&= \int \Big\langle \underset{\equiv}{\dot{B}}\, \underset{-}{D} \underset{\cdot}{\Psi} + \underset{\sim}{e} \frac{1}{16\pi G} \phi^2 \Big(R - \frac{3}{2}\phi^2\Big) + \frac{1}{4} \underset{=}{F}' * \underset{=}{F}' \Big\rangle
\end{aligned}
$$

Cosmological constant from the Higgs VEV: $\Lambda = \frac{3}{4}\phi^2$

Implies frame VEV is de Sitter: $\underset{=}{R} = \frac{\Lambda}{6} \underset{-}{e}\, \underset{-}{e}$ $\quad R = 4\Lambda$

Vacuum expectation value of the curvature vanishes: $\underset{=\cdot}{F} = 0$

Gravitational part of the action

$$S_G = \int \left\langle \underline{\underline{B}}_G \underline{\underline{F}}_G + \tfrac{\pi G}{4} \underline{\underline{B}}_G \underline{\underline{B}}_G \gamma \right\rangle \qquad \underline{\underline{F}}_G = \tfrac{1}{2}\left(\underline{\underline{R}} - \tfrac{1}{8}\underline{e}\,\underline{e}\,\phi^2\right) \in \underline{\underline{so}}(3,1)$$

$$\delta \underline{\underline{B}}_G \rightarrow \underline{\underline{B}}_G = \frac{1}{\pi G}\left(\underline{\underline{R}} - \tfrac{1}{8}\underline{e}\,\underline{e}\,\phi^2\right)\gamma \qquad \gamma = \gamma_1\gamma_2\gamma_3\gamma_4$$

$$S_G = \frac{1}{\pi G}\int \left\langle \underline{\underline{F}}_G \underline{\underline{F}}_G \gamma \right\rangle = \frac{1}{4\pi G}\int \left\langle \left(\underline{\underline{R}} - \tfrac{1}{8}\underline{e}\,\underline{e}\,\phi^2\right)\left(\underline{\underline{R}} - \tfrac{1}{8}\underline{e}\,\underline{e}\,\phi^2\right)\gamma \right\rangle$$

$$\left\langle \underline{\underline{R}}\,\underline{\underline{R}}\gamma \right\rangle = \underline{d}\left\langle \left(\underline{\omega}\,\underline{d}\,\underline{\omega} + \tfrac{1}{3}\underline{\omega}\,\underline{\omega}\,\underline{\omega}\right)\gamma \right\rangle \qquad \leftarrow \text{Chern-Simons}$$

$$\frac{1}{4!}\left\langle \underline{e}\,\underline{e}\,\underline{e}\,\underline{e}\,\gamma \right\rangle = -\underset{\sim}{e} \qquad \leftarrow \text{volume element}$$

$$\left\langle \underline{e}\,\underline{e}\,\underline{\underline{R}}\,\gamma \right\rangle = -\underset{\sim}{e}\, R \qquad \leftarrow \text{curvature scalar}$$

$$S_G = \frac{1}{16\pi G}\int \underset{\sim}{e}\,\phi^2\left(R - \tfrac{3}{2}\phi^2\right) \qquad \text{cosmological constant:} \quad \Lambda = \tfrac{3}{4}\phi^2$$

Fermionic part of the action

Choosing the anti-Grassmann 3-form to be $\underset{\equiv}{\dot{B}} = \underset{\sim}{e}\dot{\Psi}\overrightarrow{e}$ gives the massive Dirac action in curved spacetime:

$$S_f = \int \langle \underset{\equiv}{\dot{B}} \underset{=}{F}. \rangle = \int \langle \underset{\equiv}{\dot{B}} \underline{D} \underset{\cdot}{\Psi} \rangle$$

$$= \int \langle \underset{\sim}{e}\dot{\Psi}\overrightarrow{e} \big(\underline{d}\underset{\cdot}{\Psi} + \underline{H}_1 \underset{\cdot}{\Psi} - \underset{\cdot}{\Psi} \underline{H}_2 \big) \rangle$$

$$= \int \langle \underset{\sim}{e}\dot{\Psi}\overrightarrow{e} \big((\underline{d} + \tfrac{1}{2}\underline{\omega} + \tfrac{1}{4}\underline{e}\phi + \underline{W} + \underline{B}_1) \underset{\cdot}{\Psi} - \underset{\cdot}{\Psi}(\underline{w} + \underline{B}_2 + \underline{x}\Phi + \underline{g}) \big) \rangle$$

$$= \int \underset{\sim}{d^4x}\, |e| \, \langle \dot{\Psi}\gamma^\mu (e_\mu)^i \big(\partial_i \underset{\cdot}{\Psi} + \tfrac{1}{4}\omega_i{}^{\mu\nu}\gamma_{\mu\nu}\underset{\cdot}{\Psi} + W_i \underset{\cdot}{\Psi} + B_{1i}\underset{\cdot}{\Psi}$$

$$+ \underset{\cdot}{\Psi} w_i + \underset{\cdot}{\Psi} B_{2i} + \underset{\cdot}{\Psi} x_i \Phi + \underset{\cdot}{\Psi} g_i \big) + \dot{\Psi}\,\phi\,\underset{\cdot}{\Psi} \rangle$$

The $\dot{\Psi}\,\phi\,\underset{\cdot}{\Psi}$ is the standard Higgs mass term.

The $\dot{\Psi}\gamma^\mu \underset{\cdot}{\Psi} x_\mu \Phi$ term... I don't understand yet — promising for CKM.

E8 Theory summary

Everything in an $E8$ principal bundle connection,

$$\underset{\sim}{A} \in \underset{\sim}{e8}$$

Periodic table of interactions (Feynman vertices) from curvature,

$$\underset{\approx}{F} = \underset{\sim}{d} \underset{\sim}{A} + \frac{1}{2} \left[\underset{\sim}{A}, \underset{\sim}{A} \right]$$

described by the $E8$ root polytope. Three generations through triality,

$$T\,e = \mu \qquad T\,\mu = \tau \qquad T\,\tau = e$$

Pati-Salam $SU(2)_L \times SU(2)_R \times SU(4)$GUT and MM gravity together,

$$S = \int \left\langle \dot{\underset{\approx}{B}} \underset{\approx}{F} + \frac{\pi}{4} \underset{\approx}{B}_G \underset{\approx}{B}_G \gamma + \underset{\approx}{B}' * \underset{\approx}{B}' \right\rangle$$

No free parameters — masses from Higgs VEV's,

$$g_1 = \sqrt{\frac{3}{5}} \qquad g_2 = 1 \qquad g_3 = 1 \qquad \Lambda = \frac{3}{4}\phi^2 \qquad \phi_0, \phi_1, \Phi \ldots$$

Everything is pure geometry, and it's very beautiful.

E8 Theory discussion

- Quantization
  - Coupling constants run.
    - Large Λ compatible with UV fixed point.
  - Just a connection — amenable to LQG, spin foams, etc.
- Understand triality-generation relationship better
  - Possible collapse or mixing to graviweak $SL(2,\mathbb{C})$.
  - The role of $\underline{w} + \underline{x}\Phi$ and symmetry breaking.
  - Getting the CKMPMNS matrix would be nice.
- Why is the action what it is?
  - Pulling $\underline{e}$ out and putting it into $\underline{\underline{F}} * \underline{\underline{F}}$ and $\dot{\underline{\underline{\underline{B}}}}$ seems weird.
    - Why $\underline{e}\phi$ simple?
    - Four dimensional base manifold emergent?

What this theory will mean, if it all works:

- Combines standard model with gravity — with LQG, it's a T.o.E.
- Our universe is very pretty.

BRST extended connection

Start with $E8$ principal bundle connection and its curvature,

$$\underline{A} = \underline{H} + \underline{\Psi} \qquad\qquad \underline{\underline{F}} = (\underline{d}\,\underline{H} + \underline{H}\,\underline{H} + \underline{\Psi}\,\underline{\Psi}) + (\underline{d}\,\underline{\Psi} + \underline{H}\,\underline{\Psi} + \underline{\Psi}\,\underline{H})$$

Action such that $\underline{\Psi}$ part is pure gauge,

$$S = \int \left\langle \underline{\underline{B}}\,\underline{\underline{F}} + \tfrac{\pi G}{4}\underline{\underline{B}}_G \underline{\underline{B}}_G \gamma + \underline{\underline{B}}' {*} \underline{\underline{B}}' \right\rangle$$

BRST: Replace $\underline{\Psi}$ part with ghosts, $\underset{\cdot}{\Psi}$, in extended connection,

$$\underset{\cdot}{\underline{A}} = \underline{H} + \underset{\cdot}{\Psi} \qquad\qquad \underset{\cdot}{\underline{\underline{F}}} = \left(\underline{d}\,\underline{H} + \underline{H}\,\underline{H}\right) + \left(\underline{d}\,\underset{\cdot}{\Psi} + [\underline{H}, \underset{\cdot}{\Psi}]\right) = \underline{\underline{F}}_H + \underline{D}\,\underset{\cdot}{\Psi}$$

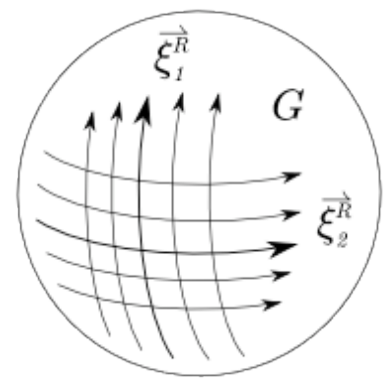
Effective action for gauge fields, ghosts, and anti-ghosts:

$$\begin{aligned} S &= \int \left\langle \dot{\underline{\underline{B}}}\,\underset{\cdot}{\underline{\underline{F}}} + \tfrac{\pi G}{4}\underline{\underline{B}}_G \underline{\underline{B}}_G \gamma + \underline{\underline{B}}' {*} \underline{\underline{B}}' \right\rangle \\ &= \int \left\langle \dot{\underline{\underline{\underline{B}}}}\,\underline{D}\,\underset{\cdot}{\Psi} + \underset{\sim}{e}\frac{1}{16\pi G}\phi^2\left(R - \frac{3}{2}\phi^2\right) + \frac{1}{4}\underline{\underline{F}}' {*} \underline{\underline{F}}' \right\rangle \end{aligned}$$

Geometry of Yang-Mills theory

Start with a **Lie group manifold** (*torsor*), G, coordinatized by y^p.
Two sets of invariant vector fields (*symmetries, **Killing vector** fields*):

$$\vec{\xi}^L_A(y)\,\underline{d}g = T_A\, g(y) \qquad \vec{\xi}^R_A(y)\,\underline{d}g = g(y)\, T_A$$



Lie derivative: $[\vec{\xi}^R_A, \vec{\xi}^R_B] = C_{AB}{}^C \vec{\xi}^R_C$

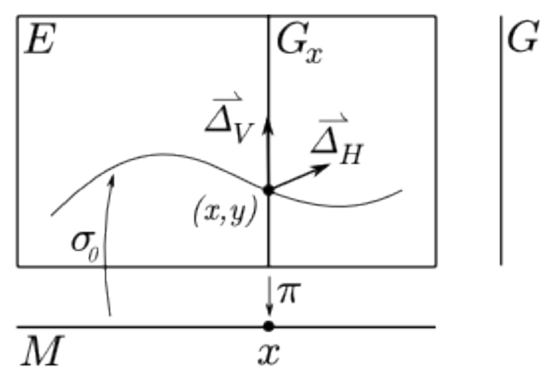
Lie bracket: $[T_A, T_B] = C_{AB}{}^C T_C$

Killing form (***Minkowski metric***): $g_{AB} = C_{AC}{}^D C_{BD}{}^C$

Maurer-Cartan form (***frame***): $\underline{\mathcal{I}} = \underline{d}y^p (\xi^R_p)^A T_A$

Entire space of a **principal bundle**: $E \sim M \times G$
Ehresmann principal bundle connection over patches of E:

$$\underline{\vec{\mathcal{E}}}(x,y) = \underline{d}x^i A_i{}^B(x)\,\vec{\xi}^L_B(y) + \underline{d}y^p \vec{\partial}_p$$



Gauge field **connection** over M:

$$\underline{A}(x) = \sigma_0^* \underline{\vec{\mathcal{E}}}\,\underline{\mathcal{I}} = \underline{d}x^i A_i{}^B(x)\, T_B$$

The Coleman-Mandula theorem

> Let G be a connected symmetry group of the S matrix, and let the following five conditions hold: (1) G contains a subgroup locally isomorphic to the Poincaré group. (2)... Then, we show that G is necessarily locally isomorphic to the direct product of an internal symmetry group and the Poincaré group.

E8 Theory does not allow a subgroup locally isomorphic to the Poincaré group. The S matrix only exists as an approximation, in which the theorem is satisfied.

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