Yukawa-Tomonaga Symposium

The future of Grand Unification

Howard Georgi
Harvard University

I am honored to be part of this extraordinary celebration - but I am little daunted that the organizers have asked me to talk about grand unified theories.

$$SU(2) \times U(1) \times SU(3) \longrightarrow SU(5)$$

$$\begin{array}{rcl} d_R = (1,3)_{-1/3} & \searrow & \\ & & 5_R \\ (e_R^+, \overline{\nu}_R) = (2,1)_{1/2} & \nearrow & \end{array}$$

$$\begin{array}{rcl} (u_L, d_L) = (2,3)_{1/6} & \searrow & \\ e_L^+ = (1,1)_1 & \longrightarrow & 10_L \\ \overline{u}_L = (1,\overline{3})_{-2/3} & \nearrow & \end{array}$$

Don't get me wrong. I think this is a beautiful idea. I think that the gorgeous fit of the standard model into $SU(5)$ must be an important and fundamental fact about the world.

$$SU(2) \times U(1) \times SU(3) \longrightarrow SU(5)$$

$$\left.\begin{array}{r} d_R = (1,3)_{-1/3} \\ (e_R^+, \overline{\nu}_R) = (2,1)_{1/2} \end{array}\right\} \longrightarrow 5_R$$

$$\left.\begin{array}{r} (u_L, d_L) = (2,3)_{1/6} \\ e_L^+ = (1,1)_1 \\ \overline{u}_L = (1,\overline{3})_{-2/3} \end{array}\right\} \longrightarrow 10_L$$

I was impressed and excited when I first found it a third of a century ago, and I am still impressed and excited about it today.

Yukawa-Tomonaga Symposium

The future (?) of Grand Unification

Howard Georgi
Harvard University

But I have written only one paper on grand unification since 1983! If there has been progress in grand unification since Savas Dimopoulos and I first constructed SUSY GUTs in 1981, I have not had much to do with it.

I continue to feel that grand unification and other such beautiful speculations will be much more worth speculating about after we learn from the LHC what breaks the electroweak symmetry.

But that won't be long now, and if we find that there is a fundamental looking Higgs boson with a low energy structure of super-partners, then perhaps it will be impossible to ignore the clues pointing to Grand Unified Theories.

Whatever happens at the LHC, it is certainly appropriate to talk about GUTs here at this symposium commemorating the birth of two of the founding fathers of quantum field theory.

Yukawa-Tomonaga Symposium

Grand Unification
A celebration of QFT

Howard Georgi
Harvard University

There is a real sense in which Grand Unified Theories are the ultimate application of renormalizable quantum field theory, and I am very glad to talk about this today as we celebrate two of its giants.

Grand Unified Theories

Stuart Raby
Department of Physics, The Ohio State University, 191 W. Woodruff Ave.
Columbus, OH 43210

Invited talk given at the 2nd World Summit on Physics Beyond the Standard Model
Galapagos Islands, Ecuador June 22-25, 2006

hep-ph/0608183 — I will steal liberally from this and other recent reviews

Fortunately for my review, there are a lot of really good people who continue to work on these ideas. I will steal liberally from them, particularly Stuart Raby, whose recent review hep-ph/0608183 is terrific.

Grand Unified Theories

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I should also say that I will stay in 4 dimensions. There has been a lot of work recently on GUTs and GUT symmetry breaking involving extra dimensions, including some by members of the audience, but my view of this is that unless you can find the GUT symmetry in the 4-d limit of your theory, it is not a GUT.

Grand Unified Theories

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This is really not such a strong constraint because most extra-dimensional features can be interpreted through deconstruction as 4-d models with expanded gauge symmetries.

the groups and representations

the predictions

the data

the history

My plan for the talk is to begin with a tour of the primary GUT groups.

the groups and representations — Lie Algebras and Yang-Mills theory

the predictions

the data

the history

I must confess that I like this stuff perhaps more than I should just for the beauty of the mathematics. I have loved symmetry and particularly Lie Algebras all my professional life. And really the love-affair started back in college when I first saw the operator treatment of angular momentum in quantum mechanics.

the groups and representations — Lie Algebras and Yang-Mills theory

the predictions — relations, proton decay, charge quantization, large scales

the data

the history

I will discuss the relations and the new physics that follow from grand unification.

the groups and representations — Lie Algebras and Yang-Mills theory

the predictions — relations, proton decay, charge quantization, large scales

the data — unification of gauge and Yukawa couplings — alas no proton decay

the history

I will then discuss how well the coupling relations work, and briefly discuss the so-far unsuccessful search for proton decay.

the groups and representations — Lie Algebras and Yang-Mills theory

the predictions — relations, proton decay, charge quantization, large scales

the data — unification of gauge and Yukawa couplings — alas no proton decay

the history — remembering a wonderful evening

Finally, I want to spend half my time on history — recalling the prehistory and discovery of GUTs, because I think it is such a fun example of a theoretical discovery. I hope that you will enjoy the story.

the groups
and representations

First the GUT groups —

$$SU(2) \times U(1) \times SU(3) \longrightarrow SU(5)$$

$$\left.\begin{array}{r} d_R = (1,3)_{-1/3} \\ (e_R^+, \overline{\nu}_R) = (2,1)_{1/2} \end{array}\right\} \longrightarrow 5_R$$

$$\left.\begin{array}{r} (u_L, d_L) = (2,3)_{1/6} \\ e_L^+ = (1,1)_1 \\ \overline{u}_L = (1,\overline{3})_{-2/3} \end{array}\right\} \longrightarrow 10_L$$

Let me begin with a quick and probably unnecessary reminder of how $SU(5)$ works.

	$SU(2) \times U(1) \times SU(3)$	$SU(5)$
$2+3=5$	$d_R = (1,3)_{-1/3}$	5
$\sum q = 0$	$(e^+_R, \overline{\nu}_R) = (2,1)_{1/2}$	
	$(u_L, d_L) = (2,3)_{1/6}$	$10 =$
	$e^+_L = (1,1)_1$	$[5 \times 5]$
	$\overline{u}_L = (1,\overline{3})_{-2/3}$	

It is really a beautiful structure. The 5_R is pretty obvious — just $2+3=5$ and finding a quark triplet and a lepton doublet in the standard model with the right charges so that the sum of the charges vanishes.

	$SU(2) \times U(1) \times SU(3)$	$SU(5)$
$2 + 3 = 5$	$d_R = (1, 3)_{-1/3}$	5
$\sum q = 0$	$(e_R^+, \overline{\nu}_R) = (2, 1)_{1/2}$	
$(1, 3)_{-1/3} \times (2, 1)_{1/2}$	$(u_L, d_L) = (2, 3)_{1/6}$	
$[(2, 1)_{1/2} \times (2, 1)_{1/2}]$	$e_L^+ = (1, 1)_1$	$10 =$ $[5 \times 5]$
$[(1, 3)_{-1/3} \times (1, 3)_{-1/3}]$	$\overline{u}_L = (1, \overline{3})_{-2/3}$	

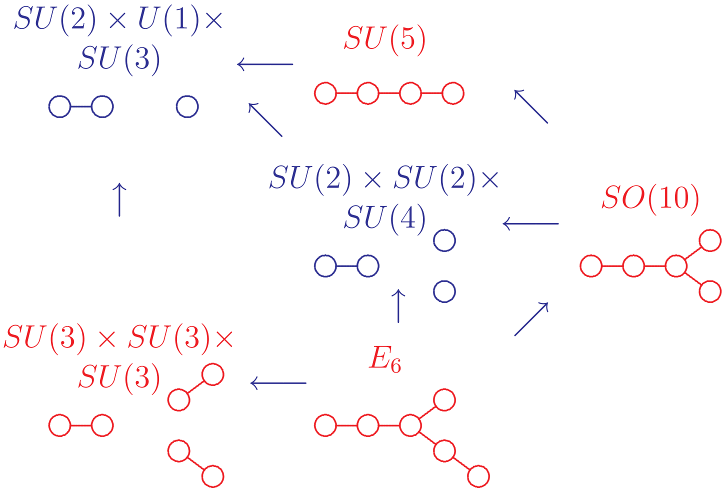
The magic is in the 10, which behaves like an antisymmetric product of two 5s. It is this antisymmetry that gets quarks and antiquarks into the same representation.

$$SU(2)\times U(1)\times SU(3) \longrightarrow SU(5)$$

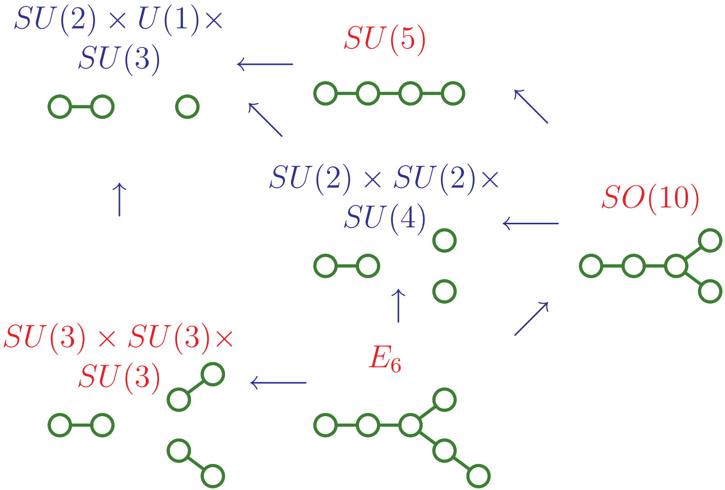
$$\begin{tikzcd}
\overline{d}_L = (1,\overline{3})_{1/3} \arrow[dr] & \\
 & \overline{5}_L \\
(\nu_L, e_L^-) = (2,1)_{-1/2} \arrow[ur] &
\end{tikzcd}$$

$$\begin{tikzcd}
(u_L, d_L) = (2,3)_{1/6} \arrow[dr] & \\
e_L^+ = (1,1)_1 \arrow[r] & 10_L \\
\overline{u}_L = (1,\overline{3})_{-2/3} \arrow[ur] &
\end{tikzcd}$$

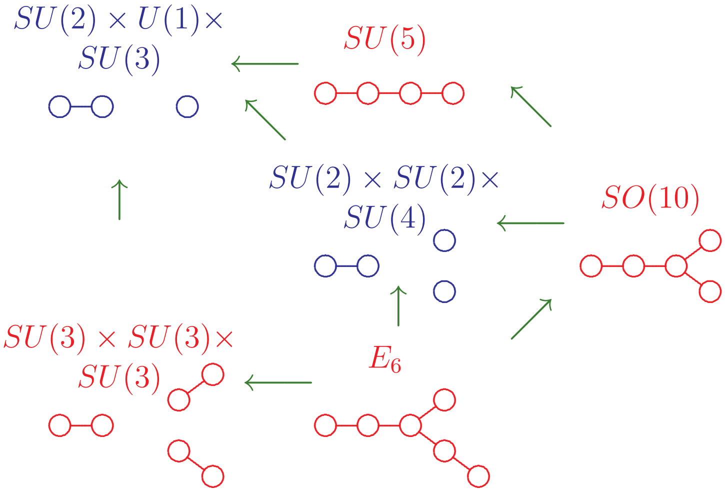
Just for future reference, let me say that we usually describe things entirely in terms of left-handed fields, in which case the $SU(5)$ representation is $10+\overline{5}$.



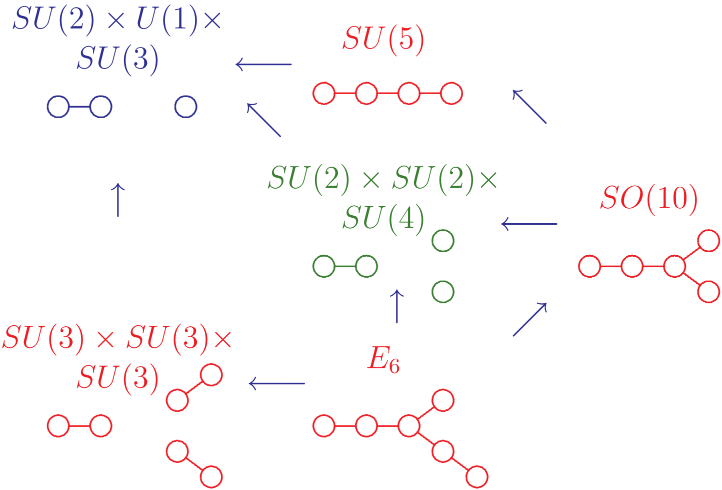
Here is a map of the principle GUT groups — at least those that incorporate the beautiful fit of the standard model that I referred to at the beginning. The ones in red are those that are unified in the sense that they may be described by just a single gauge coupling constant.



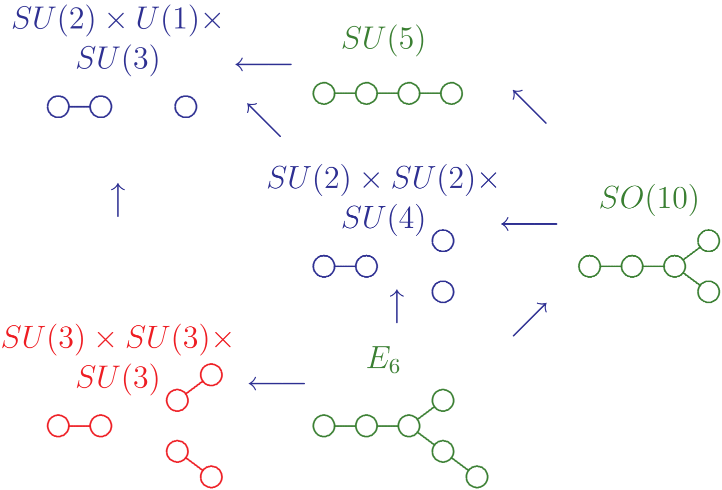
The little circles are the corresponding Dynkin diagrams.



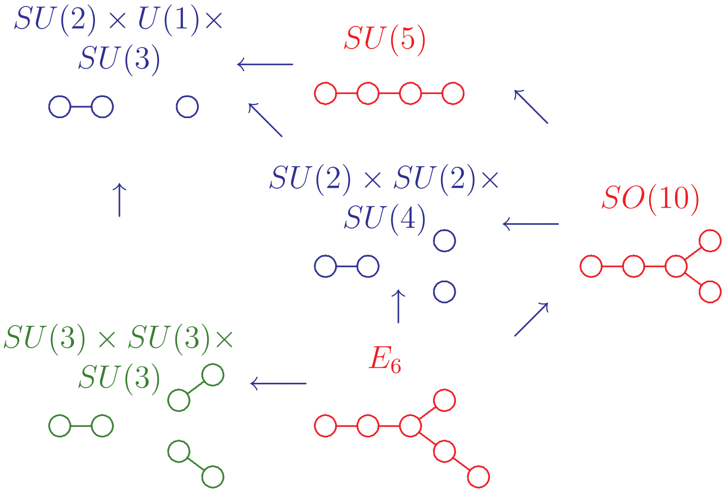
The arrows indicate the subgroup relationships — for example, $SU(5)$ is a subalgebra of $SO(10)$ which is a subalgebra of E_6, as is $SU(3) \times SU(3) \times SU(3)$.



The Pati-Salam group $SU(2) \times SU(2) \times SU(4)$ is not really unified, but it shares some interesting features with truly unified theories - such as charge quantization - and it was very important historically.



$SU(5)$, $SO(10)$ and $E(6)$ form a family of related models as you can see from the Dynkin diagrams. The list stops with E_6 as far as the 4-d field theory is concerned.



The $SU(3) \times SU(3) \times SU(3)$ is a subgroup of E_6 sometimes called "trinification." It requires some discrete symmetry to enforce the equality of the three $SU(3)$ couplings.

$$
\begin{tikzcd}
\begin{array}{c} SU(2)\times U(1)\times \\ SU(3) \\ (2,3)_{1/6}+(2,1)_{-1/2} \\ (1,\overline{3})_{-2/3}+(1,\overline{3})_{1/3} \\ +(1,1)_1 \end{array} & \begin{array}{c} SU(5) \\ 10+\overline{5} \end{array} \arrow[l] & \\
 & \begin{array}{c} SU(2)\times SU(2)\times \\ SU(4) \\ (2,1,4)+(1,2,\overline{4}) \end{array} \arrow[ul] & \begin{array}{c} SO(10) \\ 16 \end{array} \arrow[l] \arrow[ul, "\begin{array}{c} 16\rightarrow \\ 10+\overline{5}+1 \end{array}"'] \\
\begin{array}{c} SU(3)\times SU(3)\times \\ SU(3) \\ (3,\overline{3},1)+(1,3,\overline{3}) \\ +(\overline{3},1,3) \end{array} \arrow[uu] & \begin{array}{c} E_6 \\ 27 \end{array} \arrow[l] \arrow[u] \arrow[ur, "\begin{array}{c} 27\rightarrow \\ 16+10+1 \end{array}"'] &
\end{tikzcd}
$$

The representations containing a standard model family look like this. There is probably too much information here to digest unless you already understand it pretty well. Let me just emphasize a couple of important points.

$$SU(2) \times U(1) \times SU(3)$$

$$(2,3)_{1/6} + (2,1)_{-1/2}$$
$$(1,\overline{3})_{-2/3} + (1,\overline{3})_{1/3}$$
$$+(1,1)_1$$

$$\longleftarrow \quad \boxed{SU(5)} \quad \boxed{10 + \overline{5}}$$

$$\boxed{16 \rightarrow 10 + \overline{5} + 1}$$

$$SU(2) \times SU(2) \times SU(4)$$

$$(2,1,4) + (1,2,\overline{4})$$

$$\longleftarrow \quad \boxed{SO(10)} \quad \boxed{16}$$

$$SU(3) \times SU(3) \times SU(3)$$

$$(3,\overline{3},1) + (1,3,\overline{3})$$
$$+(\overline{3},1,3)$$

$$\longleftarrow \quad \boxed{E_6} \quad \boxed{27}$$

$$\boxed{27 \rightarrow 16 + 10 + 1}$$

As we move in the list of simple unifying groups from $SU(5)$ to $SO(10)$ to E_6, the representations get larger and the $SO(10)$ and E_6 representations contain the full standard model family in a single irreducible representation.

$$
\begin{tikzcd}
\begin{array}{c} SU(2)\times U(1)\times \\ SU(3) \\ (2,3)_{1/6}+(2,1)_{-1/2} \\ (1,\overline{3})_{-2/3}+(1,\overline{3})_{1/3} \\ +(1,1)_1 \end{array} & \begin{array}{c} \boxed{SU(5)} \\ \boxed{10+\overline{5}} \end{array} \arrow[l] & \boxed{\begin{array}{c} 16\rightarrow \\ 10+\overline{5}+1 \end{array}} \\
& \begin{array}{c} SU(2)\times SU(2)\times \\ SU(4) \\ (2,1,4)+(1,2,\overline{4}) \end{array} \arrow[ul] & \begin{array}{c} \boxed{SO(10)} \\ \boxed{16} \end{array} \arrow[l] \arrow[ul] \\
\begin{array}{c} SU(3)\times SU(3)\times \\ SU(3) \\ (3,\overline{3},1)+(1,3,\overline{3}) \\ +(\overline{3},1,3) \end{array} \arrow[uu] & \begin{array}{c} \boxed{E_6} \\ \boxed{27} \end{array} \arrow[l] \arrow[u] \arrow[ur] & \boxed{\begin{array}{c} 27\rightarrow \\ 16+10+1 \end{array}}
\end{tikzcd}
$$

However, this may be a mixed blessing because they also include additional fields not present in the standard model family — a singlet in $SO(10)$ and two singlets and an additional massive d quark and lepton doublet in E_6.

$$
\begin{tikzcd}
\begin{array}{c} SU(2)\times U(1)\times \\ SU(3) \\ (2,3)_{1/6}+(2,1)_{-1/2} \\ (1,\overline{3})_{-2/3}+(1,\overline{3})_{1/3} \\ +(1,1)_{1} \end{array} & \begin{array}{c} \boxed{SU(5)} \\ \boxed{10+\overline{5}} \end{array} \arrow[l] & \\
 & \begin{array}{c} SU(2)\times SU(2)\times \\ SU(4) \\ (2,1,4)+(1,2,\overline{4}) \end{array} \arrow[ul] & \begin{array}{c} \boxed{SO(10)} \\ \boxed{16} \end{array} \arrow[l] \arrow[ul, "\boxed{\begin{array}{c} 16\rightarrow \\ 10+\overline{5}+1 \end{array}}"'] \\
\begin{array}{c} SU(3)\times SU(3)\times \\ SU(3) \\ (3,\overline{3},1)+(1,3,\overline{3}) \\ +(\overline{3},1,3) \end{array} \arrow[uu] & \begin{array}{c} \boxed{E_6} \\ \boxed{27} \end{array} \arrow[l] \arrow[u] \arrow[ur, "\boxed{\begin{array}{c} 27\rightarrow \\ 16+10+1 \end{array}}"'] &
\end{tikzcd}
$$

There is a prejudice in favor of the extra singlet in $SO(10)$ because it provides a natural way to generate small neutrino masses via the see-saw mechanism. But who knows whether this makes any sense.

$$
\begin{tikzcd}
\begin{array}{c} SU(2)\times U(1)\times \\ SU(3) \\ (2,3)_{1/6}+(2,1)_{-1/2} \\ (1,\overline{3})_{-2/3}+(1,\overline{3})_{1/3} \\ +(1,1)_1 \end{array} & \begin{array}{c} SU(5) \\ 10+\overline{5} \end{array} \arrow[l] & \\
& \begin{array}{c} SU(2)\times SU(2)\times \\ SU(4) \\ (2,1,4)+(1,2,\overline{4}) \end{array} \arrow[ul] & \begin{array}{c} SO(10) \\ 16 \end{array} \arrow[l] \arrow[ul, "\begin{array}{c} 16\to \\ 10+\overline{5}+1 \end{array}"'] \\
\begin{array}{c} SU(3)\times SU(3)\times \\ SU(3) \\ (3,\overline{3},1)+(1,3,\overline{3}) \\ +(\overline{3},1,3) \end{array} \arrow[uu] & \begin{array}{c} E_6 \\ 27 \end{array} \arrow[l] \arrow[u] \arrow[ur, "\begin{array}{c} 27\to \\ 16+10+1 \end{array}"'] &
\end{tikzcd}
$$

One amusing feature of trinification is the Higgs could also be the $(3,\overline{3},1)+(1,3,\overline{3})+(\overline{3},1,3)$, so that one might need only one kind of representation. That is not true of any of the simple groups.

the predictions

You can probably tell that I find the mathematical structure of GUTs endlessly fascinating and I could go on talking about this all day. But this is a physics symposium so we should probably talk about the physics and not just the mathematics.

Charge quantization

A rather well-tested fact about the world is the neutrality of the atom - implying a relation between the charge of the quarks and the leptons. This is a natural consequence of GUTs that is not explained in the standard model.

Charge quantization

in non-GUT theories like the standard model, $U(1)$ charges not quantized — but any lone unbroken $U(1)$ in a GUT must be quantized.

The $U(1)$ charge in the standard model is not quantized. But in a GUT, any lone unbroken $U(1)$, like the $U(1)$ in the $SU(2) \times U(1) \times SU(3)$ standard model subgroup of a GUT group, must be quantized.

Charge quantization

in non-GUT theories like the standard model, $U(1)$ charges not quantized — but any lone unbroken $U(1)$ in a GUT must be quantized.

I have never found this beautiful fact about GUTs particularly obvious. You can see it by relying on the mathematics, using compactness but this is not particularly physical. Here is a more physical argument.

Charge quantization

in non-GUT theories like the standard model, $U(1)$ charges not quantized — but any lone unbroken $U(1)$ in a GUT must be quantized.

any GUT $U(1)$ linear combination of T_3s — Cartan-Dynkin

Because in a GUT group, there is no $U(1)$ that commutes with everything, any generator can be written as a sum of T_3s of some $SU(2)$ subgroups.

Charge quantization

in non-GUT theories like the standard model, $U(1)$ charges not quantized — but any lone unbroken $U(1)$ in a GUT must be quantized.

any GUT $U(1)$ linear combination of T_3s — Cartan-Dynkin

This is still not totally obvious to me, but it follows easily for the simple groups from the Cartan-Dynkin analysis that breaks up the generators into such T_3s and raising and lowering operators.

Charge quantization

in non-GUT theories like the standard model, $U(1)$ charges not quantized — but any lone unbroken $U(1)$ in a GUT must be quantized.

any GUT $U(1)$ linear combination of T_3s — Cartan-Dynkin

Each of the T_3s has quantized charges because of the structure of $SU(2)$.

Charge quantization

in non-GUT theories like the standard model, $U(1)$ charges not quantized — but any lone unbroken $U(1)$ in a GUT must be quantized.

any GUT $U(1)$ linear combination of T_3s — Cartan-Dynkin

but why not $\alpha T_3^1 + \beta T_3^2$ for α/β irrational??

But we must still explain why we cannot have a $U(1)$ that is some linear combination of two such T_3s with irrational coefficients, $\alpha T_3^1 + \beta T_3^2$.

Charge quantization

in non-GUT theories like the standard model, $U(1)$ charges not quantized — but any lone unbroken $U(1)$ in a GUT must be quantized.

any GUT $U(1)$ linear combination of T_3s — Cartan-Dynkin

but why not $\alpha T_3^1 + \beta T_3^2$ for α/β irrational??

Higgs mechanism — must be some element with non-zero T_3^1 and T_3^2 but zero $\alpha T_3^1 + \beta T_3^2$

The reason is the Higgs mechanism. Our hypothetical T_3^1 and T_3^2 are two separate $U(1)$s in the GUT theory. To end up with a lone $U(1)$ we would have to find a representation that breaks the two $U(1)$s down to a single $U(1)$.

Charge quantization

in non-GUT theories like the standard model, $U(1)$ charges not quantized — but any lone unbroken $U(1)$ in a GUT must be quantized.

any GUT $U(1)$ linear combination of T_3s — Cartan-Dynkin

but why not $\alpha T_3^1 + \beta T_3^2$ for α/β irrational??

Higgs mechanism — must be some element with non-zero T_3^1 and T_3^2 but zero $\alpha T_3^1 + \beta T_3^2$

But that requires a Higgs representation with an element that has non-zero charge under each of T_3^1 and T_3^2 but has zero charge under the linear combination $\alpha T_3^1 + \beta T_3^2$.

Charge quantization

in non-GUT theories like the standard model, $U(1)$ charges not quantized — but any lone unbroken $U(1)$ in a GUT must be quantized.

any GUT $U(1)$ linear combination of T_3s — Cartan-Dynkin

but why not $\alpha T_3^1 + \beta T_3^2$ for α/β irrational??

Higgs mechanism — must be some element with non-zero T_3^1 and T_3^2 but zero $\alpha T_3^1 + \beta T_3^2$

IMPOSSIBLE — thus charge is quantized

And that is impossible unless α and β are relatively rational. Thus charge is quantized.

coupling unification and running

$$T_a^2 = \begin{pmatrix} \frac{\sigma_a}{2} & 0 \\ 0 & 0 \end{pmatrix} \qquad T_a^3 = \begin{pmatrix} 0 & 0 \\ 0 & \frac{\lambda_a}{2} \end{pmatrix}$$

$$T^1 = \sqrt{\frac{3}{5}} \begin{pmatrix} I/2 & 0 \\ 0 & -I/3 \end{pmatrix} = \sqrt{\frac{3}{5}}(Q - T_3^2)$$

$$\Rightarrow g_3 = g_2 = \sqrt{\frac{5}{3}} g_1 \text{ at unification}$$

this plus logarithmic running

$\Rightarrow$ coupling unification at large scale

I am sure that everyone knows all about this and I don't think I can add much to what you already know. The GUT symmetry imposes constraints on the couplings at the scale at which it is broken.

coupling unification and running

running depends on the matter content of the low energy theory only weakly if the matter comes in full GUT multiplets

simplest non-SUSY models ruled out — but softly broken SUSY gauginos and different Higgs structure improve prospects — at least approximate unification at a few $\times 10^{16}$ GeV

fairly robust prediction because gauge coupling are so constrained by symmetries

At lower scales the couplings run logarithmically to the values we see at low energies. The running depends on the matter content of the low energy theory, but only weakly if the matter comes in full GUT multiplets.

coupling unification and running

running depends on the matter content of the low energy theory only weakly if the matter comes in full GUT multiplets

simplest non-SUSY models ruled out — but softly broken SUSY gauginos and different Higgs structure improve prospects — at least approximate unification at a few $\times 10^{16}$ GeV

fairly robust prediction because gauge coupling are so constrained by symmetries

The simplest non-SUSY models don't unify well without crazy assumptions, but softly broken SUSY improves the situation because the gauginos form incomplete GUT representations just like ordinary gauge bosons (and less importantly, because there must be two Higgs multiplets).

Yukawa coupling unification

HUGE CAVEAT! we really don't understand where the Yukawa couplings come from at all well

but if we assume that the couplings of the third family are dominantly to the simplest possible GUT Higgs multiplet we get something interesting

Yukawa couplings are much less constrained than gauge couplings, and we don't really know where they come from, but if we assume that the couplings of the third family are dominantly to the simplest possible GUT Higgs multiplet we get something interesting.

Yukawa coupling unification

HUGE CAVEAT! we really don't understand where the Yukawa couplings come from at all well

but if we assume that the couplings of the third family are dominantly to the simplest possible GUT Higgs multiplet we get something interesting

in $SU(5)$ — $f_\tau = f_b$ at unification scale

in SUSY $SO(10)$ — $f_\tau = f_b = f_t$ at unification scale

which implies that $\tan\beta$ - the ratio of the H_u to the H_d VEV - is very large — this gives a very predictive version of MSSM

In $SU(5)$, we get $f_\tau = f_b$ at the unification scale. In SUSY $SO(10)$, we get $f_\tau = f_b = f_t$ at the unification scale, which implies that $\tan\beta$ - the ratio of the H_u to the H_d VEV - is very large. This gives a very predictive version of MSSM.

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but if we assume that the couplings of the third family are dominantly to the simplest possible GUT Higgs multiplet we get something interesting

in $SU(5)$ — $f_\tau = f_b$ at unification scale

in SUSY $SO(10)$ — $f_\tau = f_b = f_t$ at unification scale which implies that $\tan\beta$ - the ratio of the H_u to the H_d VEV - is very large — this gives a very predictive version of MSSM

Note that $f_\tau = f_b = f_t$ really only makes sense in a two-Higgs theory like SUSY in which H_u and H_d survive into the low energy theory.

Yukawa coupling unification

coupling of 2nd family to 45 in $SU(5)$ (or 126 in $SO(10)$ gives $f_\mu = -3f_s$ at unification scale - not crazy - the factor of three is special because of the 3 colors

This kind of prediction often appears in attempt to combine $SO(10)$ unification with theories of flavor masses and mixing (see for example Blazek, Dermisek, Raby)

We might also get the so-called Georgi-Jarlskog relation - $f_\mu = -3f_s$ - but this is even more model dependent. Other relations are possible in flavor schemes, but have little to do with GUTs.

Yukawa coupling unification

$$g_3 = g_2 = \sqrt{\tfrac{5}{3}} g_1$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$f_\tau = f_b$$

increasing
model
dependence

$$\downarrow$$

$$f_\tau = f_b = f_t$$

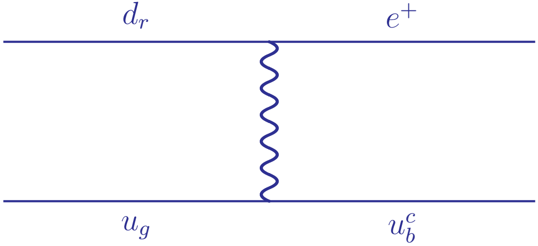
$$\downarrow \qquad\qquad\qquad \downarrow$$

$$f_\mu = -3 f_s$$

It should be obvious that as we go down this list of “relations” they get less and less robust.

Proton decay

generic GUTs have proton decay from gauge interactions



$\rightsquigarrow p \longrightarrow e^+\pi^0$

Generic GUTs have proton decay from gauge interactions. This is a big problem for the simplest non-SUSY $SU(5)$ model, but the GUT scale is pushed up if there are low-energy SUSY gauginos and it is not obviously ruled out.

Proton decay

in the standard model the lowest dimension b and l violating operators are 4-fermion - dimension 6 — $\propto 1/M_{GUT}^2$

much different in SUSY - sparticles carry b and l quantum numbers and their interactions can violate symmetries — dim 4 and 5 operators — dim 4 can (must) be forbidden by R-parity — dim 5 are generically produced by triplet in Higgs multiplet (which $\therefore$ must be heavy) — this means heavy flavors are important — $\rightsquigarrow$ $p \to \overline{\nu} K^+$

details are sensitive to details of SUSY breaking

In the standard model the lowest dimension b and l violating operators are 4-fermion operators with dimension 6 with coefficients $\propto 1/M_{GUT}^2$.

Proton decay

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details are sensitive to details of SUSY breaking

The situation is much different in SUSY because sparticles carry b and l quantum numbers and the sparticle interactions can violate symmetries, leading to dimension 4 and 5 operators that violate baryon and lepton number.

Proton decay

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much different in SUSY - sparticles carry b and l quantum numbers and their interactions can violate symmetries — dim 4 and 5 operators — dim 4 can (must) be forbidden by R-parity — dim 5 are generically produced by triplet in Higgs multiplet (which $\therefore$ must be heavy) — this means heavy flavors are important — $\rightsquigarrow$ $p \rightarrow \overline{\nu} K^+$

details are sensitive to details of SUSY breaking

The dimension 4 operators can be forbidden by R-parity and to get a consistent theory, they must be. But the dimension 5 operators are generically produced (for example) by the color triplet in the Higgs multiplets (which $\therefore$ must be heavy).

Proton decay

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much different in SUSY - sparticles carry b and l quantum numbers and their interactions can violate symmetries — dim 4 and 5 operators — dim 4 can (must) be forbidden by R-parity — dim 5 are generically produced by triplet in Higgs multiplet (which $\therefore$ must be heavy) — this means heavy flavors are important — $\rightsquigarrow p \to \overline{\nu} K^+$

details are sensitive to details of SUSY breaking

The fact that Higgs couplings are involved means that heavy flavors are important, so this increases the branching ratio for modes like $p \to \overline{\nu} K^+$.

Proton decay

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much different in SUSY - sparticles carry b and l quantum numbers and their interactions can violate symmetries — dim 4 and 5 operators — dim 4 can (must) be forbidden by R-parity — dim 5 are generically produced by triplet in Higgs multiplet (which $\therefore$ must be heavy) — this means heavy flavors are important — $\rightsquigarrow$ $p \to \overline{\nu} K^+$

details are sensitive to details of SUSY breaking

The details are sensitive to the details of SUSY breaking and SUSY breaking is NOT understood!

the

data

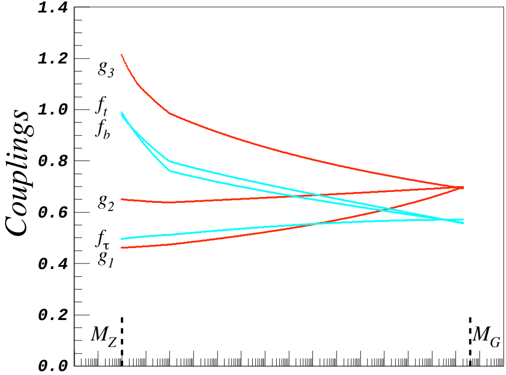
Coupling unification

There is a nice review of this from a few years ago by Daniel Auto, Howard Baer, Csaba Balázs, Alexander Belyaev, Javier Ferrandis, Xerxes Tata in hep-ph/0302155

They looked for models with $g_3 = g_2 = \sqrt{\frac{5}{3}} g_1$ and $f_\tau = f_b = f_t$ — many constraints (including cosmology because these models have a plausible LSP DM candidate)

They also discuss things that I am not even going to mention like unification of soft symmetry breaking parameter. I will show one picture completely out of context because it is so beautiful.

Coupling unification



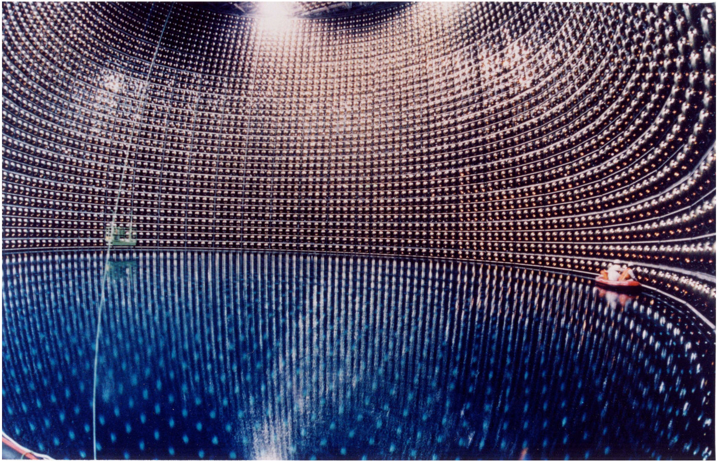
I can't pretend to have digested this paper completely, but it looks very exciting because the constraint of unification of Yukawa couplings is so tight that there will surely be tests at the LHC.

Proton decay

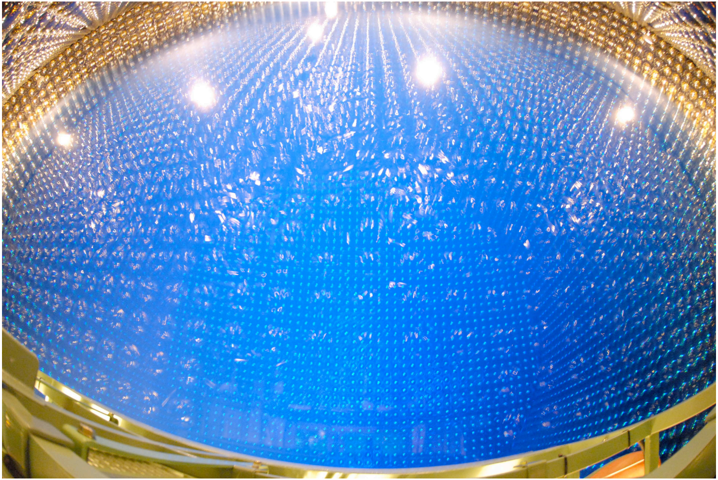
Here is an exciting statement about proton decay in SUSY GUTs by Stuart Raby in a very recent review article — hep-ph/0608183

Super-Kamiokande bounds on the proton lifetime severely constrain these dimension 6 and 5 operators with $\tau_{(p \to e^+ \pi^0)} > 5.0 \times 10^{33}$ yrs (79.3 ktyr exposure), $\tau_{(n \to e^+ \pi^-)} > 5 \times 10^{33}$ yrs (61 ktyr), and $\tau_{(p \to K^+ \bar{\nu})} > 2.3 \times 10^{33}$ yrs (92 ktyr), $\tau_{(n \to K^0 \bar{\nu})} > 1.3 \times 10^{32}$ yrs (92 ktyr) at (90% CL) based on the listed exposures [18]. These constraints are now sufficient to rule out minimal SUSY $SU(5)$ [19]. Non-minimal Higgs sectors in $SU(5)$ or $SO(10)$ theories still survive [9, 13]. The upper bound on the proton lifetime from these theories are approximately a factor of 5 above the experimental bounds. They are, however, being pushed to their theoretical limits. Hence if SUSY GUTs are correct, nucleon decay must be seen soon.

You probably can't read this, but what it says is that proton decay must show up soon. What can a theorist say except that as for the LHC, we are waiting as hard as we can!



I can't resist, at this point, showing these magnificent photos from the SuperK home page just because it is one of those things that gives me a sense of the majestic scope of particle physics.



I can’t resist, at this point, showing these magnificent photos from the SuperK home page just because it is one of those things that gives me a sense of the majestic scope of particle physics.

the
history

That concludes the serious part of my talk! I now want to tell you the story of the discovery of the $SO(10)$ and $SU(5)$ theories.

the history

It is hard to believe that it has been a third of a century since the evening I sat in a reclining chair in my living room and pieced together the $SO(10)$ and $SU(5)$ grand unified theories.

the history

I have talked about this a few times before and I apologize to the graybeards who have heard it already, but I thought some of the youngsters might find it amusing. I will do some of the group theoretical manipulations involved by the very pedestrian methods that we used at the time.

1967 — I start grad-school and ignore Weinberg's 67 paper (as did Weinberg)
1970 — GIM (charm predicted, but also pretty much ignored), anomalies
1971 — I start at a postdoc at Harvard and Ben Lee talks about t'Hooft's work on spontaneously broken gauge theories
1972 — $SO(3)$, Bj's formula and Weinberg's second model of leptons, partons
1973 — Pati-Salam, dimensional transmutation, asymptotic freedom, infrared slavery, GUTs

I will describe in detail what went on in Cambridge, Massachusetts, because that is what I know best, with only occasional reference to the rest of the world.

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In 67-71 I was a graduate student in sleepy New Haven, which was just as well. At Yale we didn't know enough to be depressed by the sorry state of quantum field theory.

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For me, the beginning was a talk at Harvard, early in the 71-72 academic year by Ben Lee, the great Korean physicist who went on to become Director of the theory group at Fermilab before his tragic death in an automobile accident.

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Ben had been trying to understand t'Hooft's papers on the renormalizability of spontaneously broken gauge field theories.

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Ben had done important work on the renormalization of field theories with spontaneously broken global symmetries, and was in a good position to make sense out of what t'Hooft had done. He reviewed t'Hooft's arguments for us, and emphasized the connection with Steve Weinberg's 67 paper.

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We had all seen this paper, of course, a few years before, but hadn't really known what to make of it. I remember my own reaction, as a graduate student at Yale to Steve's model of leptons. It didn't look renormalizable to me!

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Anyway, Ben clinched the deal by giving some specific examples, in a spontaneously broken $U(1)$ gauge theory, of what seemed to us at the time as miraculous cancellations required to allow renormalization.

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This got the attention of everyone at Harvard at the time. Ben Lee's talk was the first of an informal series of "gauge seminars" at Harvard that lasted into the 80s.

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We all dropped what we were doing and started working on various aspects of spontaneously broken gauge theories. Tom Appelquist, Helen Quinn and Joel Primack started work on renormalization in unitary gauge.

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Sidney Coleman and his students Eric Weinberg and David Politzer started the work that led to the famous Coleman-Weinberg paper and asymptotic freedom. Shelly Glashow and I started building models.

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At this point, GUTs were the farthest thing from our minds. In fact, we tried to think about strong interactions as little as possible. Our first motivation came from misleading data.

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In 1971, the neutral currents predicted by the $SU(2) \times U(1)$ model had not been seen. Quite the reverse, data on quasielastic neutrino proton scattering was apparently inconsistent with the model.

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Thus we set about to build renormalizable models without neutral currents. This was pretty easy. Before the Higgs mechanism, Shelly had investigated both the gauge structure of the $SU(2) \times U(1)$ model, and an $SO(3)$ model without neutral currents.

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It was a straightforward exercise to turn Shelly's old $SO(3)$ model into a renormalizable model with spontaneously broken gauge symmetry. The fermions were triplets and neutral singlets under $SO(3)$.

$$\begin{pmatrix} E^+ \\ \cos\xi\, E^0 + \sin\xi\, \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} E^+ \\ E^0 \\ e^- \end{pmatrix}_R$$

$$\begin{pmatrix} M^+ \\ \cos\xi\, M^0 + \sin\xi\, \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} M^+ \\ M^0 \\ \mu^- \end{pmatrix}_R$$

Lee, and independently Prentki and Zumino

$$\begin{pmatrix} E^+ \\ \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} M^+ \\ \nu_\mu \\ \mu^- \end{pmatrix}_L$$

The triplets looked like this. At about the same time, Lee, and independently Prentki and Zumino did something similar based on an $SO(3) \times U(1)$ gauge symmetry. In their model, only the left handed leptons are in triplets.

$$\begin{pmatrix} E^+ \\ \cos\xi\, E^0 + \sin\xi\, \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} E^+ \\ E^0 \\ e^- \end{pmatrix}_R$$

$$\begin{pmatrix} M^+ \\ \cos\xi\, M^0 + \sin\xi\, \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} M^+ \\ M^0 \\ \mu^- \end{pmatrix}_R$$

Lee, and independently Prentki and Zumino

$$\begin{pmatrix} E^+ \\ \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} M^+ \\ \nu_\mu \\ \mu^- \end{pmatrix}_L$$

Both these models were wrong, we now know. However, both broke theoretical ground by introducing new heavy fermions in order to complete gauge multiplets. It is worth saying that we did not imagine that these fermions would be very heavy.

$$\begin{pmatrix} E^+ \\ \cos\xi\, E^0 + \sin\xi\, \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} E^+ \\ E^0 \\ e^- \end{pmatrix}_R$$

$$\begin{pmatrix} M^+ \\ \cos\xi\, M^0 + \sin\xi\, \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} M^+ \\ M^0 \\ \mu^- \end{pmatrix}_R$$

Lee, and independently Prentki and Zumino

$$\begin{pmatrix} E^+ \\ \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} M^+ \\ \nu_\mu \\ \mu^- \end{pmatrix}_L$$

They just had to be heavy enough to have escaped detection so far. In addition, the $SO(3)$ model had two fascinating properties not shared by $SU(2) \times U(1)$ or $SO(3) \times U(1)$

$$
\begin{pmatrix} E^+ \\ \cos\xi\, E^0 + \sin\xi\, \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} E^+ \\ E^0 \\ e^- \end{pmatrix}_R
$$

$$
\begin{pmatrix} M^+ \\ \cos\xi\, M^0 + \sin\xi\, \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} M^+ \\ M^0 \\ \mu^- \end{pmatrix}_R
$$

Lee, and independently Prentki and Zumino

$$
\begin{pmatrix} E^+ \\ \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} M^+ \\ \nu_\mu \\ \mu^- \end{pmatrix}_L
$$

1 — because there was only one simple gauge group, there was a single gauge coupling — there was no analog of the arbitrary weak mixing angle θ_W (we called it the weak mixing angle rather than the Weinberg angle — after all it had first been written down by Glashow);

$$\begin{pmatrix} E^+ \\ \cos\xi\, E^0 + \sin\xi\, \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} E^+ \\ E^0 \\ e^- \end{pmatrix}_R$$

$$\begin{pmatrix} M^+ \\ \cos\xi\, M^0 + \sin\xi\, \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} M^+ \\ M^0 \\ \mu^- \end{pmatrix}_R$$

Lee, and independently Prentki and Zumino

$$\begin{pmatrix} E^+ \\ \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} M^+ \\ \nu_\mu \\ \mu^- \end{pmatrix}_L$$

2 — because the gauge group was semisimple, electric charge was quantized — later, of course, t'Hooft and Polyakov understood the connection between our version of charge quantization and Dirac's by finding magnetic monopoles in this model.

$$\begin{pmatrix} E^+ \\ \cos\xi\, E^0 + \sin\xi\, \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} E^+ \\ E^0 \\ e^- \end{pmatrix}_R$$

$$\begin{pmatrix} M^+ \\ \cos\xi\, M^0 + \sin\xi\, \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} M^+ \\ M^0 \\ \mu^- \end{pmatrix}_R$$

Lee, and independently Prentki and Zumino

$$\begin{pmatrix} E^+ \\ \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} M^+ \\ \nu_\mu \\ \mu^- \end{pmatrix}_L$$

Both of these features became important pieces of the puzzle that we tried to incorporate into future model building.

$$\begin{pmatrix} E^+ \\ \cos\xi\, E^0 + \sin\xi\, \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} E^+ \\ E^0 \\ e^- \end{pmatrix}_R$$

$$\begin{pmatrix} M^+ \\ \cos\xi\, M^0 + \sin\xi\, \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} M^+ \\ M^0 \\ \mu^- \end{pmatrix}_R$$

Lee, and independently Prentki and Zumino

$$\begin{pmatrix} E^+ \\ \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} M^+ \\ \nu_\mu \\ \mu^- \end{pmatrix}_L$$

The $SO(3)$ model, however, had a very ugly property — lack of universality. There was no good reason for the angle ξ to be the same in the e and μ multiplets. The Lee, Prentki, Zumino model solved this problem, at the cost of an extra coupling and no quantization of charge.

Bj's relation

$$\frac{m_e}{m_\mu} \approx \frac{3\alpha}{\pi} \ln 2$$

About this time, Bj confused matters enormously by writing down the suggestive formula $\frac{m_e}{m_\mu} \approx \frac{3\alpha}{\pi} \ln 2$.

Bj's relation

$$\frac{m_e}{m_\mu} \approx \frac{3\alpha}{\pi} \ln 2$$

He argued that this relation was correct at just the level that one would expect if there were radiative corrections — it is right up to order $\mathcal{O}(\alpha^2)$.

Bj's relation

$$\frac{m_e}{m_\mu} \approx \frac{3\alpha}{\pi} \ln 2$$

In hindsight, this seems completely ridiculous. But you have to remember, that at this time, in 1972, there was no τ lepton. There weren't really even any quarks. For most East coast physicists, quarks were still a shorthand for symmetry properties of the still mysterious strong interactions.

Bj's relation

$$\frac{m_e}{m_\mu} \approx \frac{3\alpha}{\pi} \ln 2$$

This situation would change like dreams over the next year, but in 1972, it seemed entirely reasonable to construct models in which the ratio, $\frac{m_\mu}{m_e}$, is calculable.

Bj's relation

$$\frac{m_e}{m_\mu} \approx \frac{3\alpha}{\pi} \ln 2$$

This problem was the whetstone on which we honed the tools that built grand unified theories.

This problem was the whetstone on which we honed the tools that built grand unified theories. At this time, Steve Weinberg was still at MIT, and he got interested in the same problem.

Bj's relation

$$\frac{m_e}{m_\mu} \approx \frac{3\alpha}{\pi} \ln 2$$

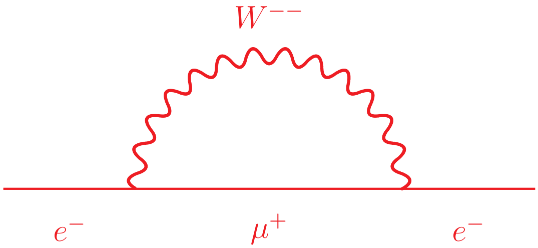
This problem was the whetstone on which we honed the tools that built grand unified theories.

It settled down to an amusing contest between Shelly and me a one end of Cambridge and Steve at the other, to see who could compute the electron-muon mass ratio. Although I think that Shelly and I eventually came closer to this goal, I must admit that Steve made the most lasting theoretical contribution.

$$\begin{pmatrix} \nu_e \\ e^- \\ \mu^+ \end{pmatrix}_L \qquad \begin{pmatrix} \nu_e \\ \mu^- \\ e^+ \end{pmatrix}_L$$

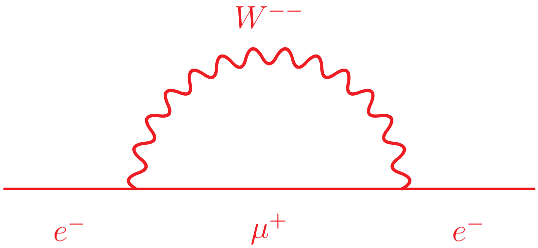
Steve wrote a paper which described a model of leptons (his second, and somewhat less well known than the first). It was based on an $SU(3) \times SU(3)$ gauge group, with the leptons transforming as triplets.

$$\begin{pmatrix} \nu_e \\ e^- \\ \mu^+ \end{pmatrix}_L \qquad \begin{pmatrix} \nu_e \\ \mu^- \\ e^+ \end{pmatrix}_L$$



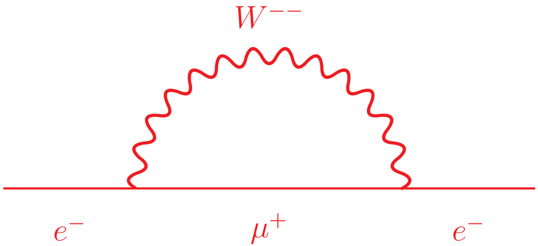
Steve reasoned that in this model, there would exist Feynman graphs like this one that could give a relation like Bj's for the electron mass as a radiative correction.

$$\begin{pmatrix} \nu_e \\ e^- \\ \mu^+ \end{pmatrix}_L \qquad \begin{pmatrix} \nu_e \\ \mu^- \\ e^+ \end{pmatrix}_L$$



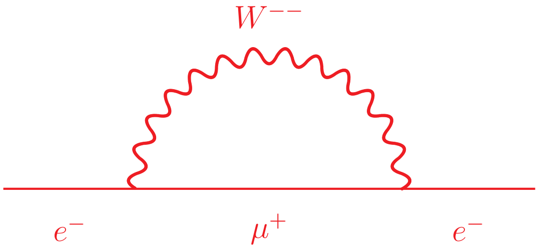
Unfortunately, Steve's model does not give anything like Bj's formula. In fact, in Steve's model, such diagrams do exist, but the electron mass is not actually calculable.

$$\begin{pmatrix} \nu_e \\ e^- \\ \mu^+ \end{pmatrix}_L \qquad \begin{pmatrix} \nu_e \\ \mu^- \\ e^+ \end{pmatrix}_L$$



Shelly and I showed this, and understood how to fix it, but the solution was not very interesting, because instead of the square-root of 2 in Bj's formula, we got a complicated ratio of heavy vector boson masses. If we knew all the masses, we would have a relation, but that didn't seem like much.

$$\begin{pmatrix} \nu_e \\ e^- \\ \mu^+ \end{pmatrix}_L \qquad \begin{pmatrix} \nu_e \\ \mu^- \\ e^+ \end{pmatrix}_L$$



The really interesting thing about Steve's model was not Bj's relation, but that the model is a kind of "proto-GUT".

$$\begin{pmatrix} \nu_e \\ e^- \\ \mu^+ \end{pmatrix}_L \qquad \begin{pmatrix} \nu_e \\ \mu^- \\ e^+ \end{pmatrix}_L$$

$$\langle \phi \rangle = \begin{pmatrix} u & 0 & 0 \\ 0 & u & 0 \\ 0 & 0 & -2u \end{pmatrix}$$

The point is that unlike $SU(2) \times U(1)$, but like all GUT models, this $SU(3)$ model has interactions that you don't see — in this case, the right handed weak interactions involving the top and bottom components of the triplets and weird doubly charged currents effects involving the bottom two.

$$\begin{pmatrix} \nu_e \\ e^- \\ \mu^+ \end{pmatrix}_L \qquad \begin{pmatrix} \nu_e \\ \mu^- \\ e^+ \end{pmatrix}_L$$

$$\langle \phi \rangle = \begin{pmatrix} u & 0 & 0 \\ 0 & u & 0 \\ 0 & 0 & -2u \end{pmatrix}$$

To get rid of them, Steve invoked "superstrong symmetry breaking". He imagined that there was an octet scalar field ϕ with a VEV u. If $u \gg v$, this breaks the $SU(3)$ gauge group down to $SU(2) \times U(1)$ and gives a large mass to all the unwanted gauge bosons.

$$\begin{pmatrix} \nu_e \\ e^- \\ \mu^+ \end{pmatrix}_L \qquad \begin{pmatrix} \nu_e \\ \mu^- \\ e^+ \end{pmatrix}_L$$

$$\langle \phi \rangle = \begin{pmatrix} u & 0 & 0 \\ 0 & u & 0 \\ 0 & 0 & -2u \end{pmatrix}$$

Unlike the heavy fermions in the $SO(3)$ models, these "superheavy" gauge bosons were constrained more by their virtual effects than by direct bounds on production. This was something new.

$$\begin{pmatrix} \nu_e \\ e^- \\ \mu^+ \end{pmatrix}_L \qquad \begin{pmatrix} \nu_e \\ \mu^- \\ e^+ \end{pmatrix}_L$$

$$\langle \phi \rangle = \begin{pmatrix} u & 0 & 0 \\ 0 & u & 0 \\ 0 & 0 & -2u \end{pmatrix}$$

At this point, it goes without saying, Steve was not thinking about what are now called GUT scale masses. A few or ten times the W mass would be plenty to suppress the unwanted interactions to an acceptable level.

$$\begin{pmatrix} \nu_e \\ e^- \\ \mu^+ \end{pmatrix}_L \qquad \begin{pmatrix} \nu_e \\ \mu^- \\ e^+ \end{pmatrix}_L$$

$$\langle \phi \rangle = \begin{pmatrix} u & 0 & 0 \\ 0 & u & 0 \\ 0 & 0 & -2u \end{pmatrix}$$

Nevertheless, at the time, this was a pretty radical concept. **You see, we all still thought of the W as very heavy!** To invent things that were heavier still was a bizarre act of genius.

$$\begin{pmatrix} \nu_e \\ e^- \\ \mu^+ \end{pmatrix}_L \qquad \begin{pmatrix} \nu_e \\ \mu^- \\ e^+ \end{pmatrix}_L$$

$$Q = T_3 + \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad T_3 = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\sin^2\theta_W = \frac{1/2}{1/2 + 3/2} = \frac{\operatorname{tr} T_3^2}{\operatorname{tr} Q^2} = \frac{1}{4}$$

This little proto-GUT had many of the interesting properties of GUTS. For example, the weak mixing angle was fixed.

$$\begin{pmatrix} \nu_e \\ e^- \\ \mu^+ \end{pmatrix}_L \qquad \begin{pmatrix} \nu_e \\ \mu^- \\ e^+ \end{pmatrix}_L$$

$$Q = T_3 + \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad T_3 = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\sin^2\theta_W = \frac{1/2}{1/2 + 3/2} = \frac{\operatorname{tr} T_3^2}{\operatorname{tr} Q^2} = \frac{1}{4}$$

The weak mixing angle (ignoring renormalization effect, which of course, nobody was thinking about at this time) is the fraction of the total charge (with $\operatorname{tr} T^2$ as the norm) in the T_3 sector.

$$\begin{pmatrix} \nu_e \\ e^- \\ \mu^+ \end{pmatrix}_L \qquad \begin{pmatrix} \nu_e \\ \mu^- \\ e^+ \end{pmatrix}_L$$

$$Q = T_3 + \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad T_3 = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\sin^2\theta_W = \frac{1/2}{1/2 + 3/2} = \frac{\mathrm{tr}\, T_3^2}{\mathrm{tr}\, Q^2} = \frac{1}{4}$$

This little model of Weinberg's, although not quite what he intended, had many nice features. It combined the best features of the $SO(3)$ and $SO(3) \times U(1)$ models.

$$\begin{pmatrix} \nu_e \\ e^- \\ \mu^+ \end{pmatrix}_L \qquad \begin{pmatrix} \nu_e \\ \mu^- \\ e^+ \end{pmatrix}_L$$

$$Q = T_3 + \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad T_3 = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\sin^2\theta_W = \frac{1/2}{1/2 + 3/2} = \frac{\operatorname{tr} T_3^2}{\operatorname{tr} Q^2} = \frac{1}{4}$$

The problem with it was that it could not accommodate the fractionally charged quarks. Because the charges of the 3 of $SU(3)$ contained only integral charges, all other representations had the same property. Charge was quantized in units of the proton charge.

$$\begin{pmatrix} \nu_e \\ e^- \\ \mu^+ \end{pmatrix}_L \qquad \begin{pmatrix} \nu_e \\ \mu^- \\ e^+ \end{pmatrix}_L$$

$$Q = T_3 + \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad T_3 = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\sin^2\theta_W = \frac{1/2}{1/2 + 3/2} = \frac{\operatorname{tr} T_3^2}{\operatorname{tr} Q^2} = \frac{1}{4}$$

At the time, this didn't seem so serious. But only a year after the model was actually published, quarks seemed much more real. The news that SLAC had seen fractionally charged quark partons was beginning to get to the US East Coast.

$$\begin{pmatrix} \nu_e \\ e^- \\ \mu^+ \end{pmatrix}_L \qquad \begin{pmatrix} \nu_e \\ \mu^- \\ e^+ \end{pmatrix}_L$$

$$Q = T_3 + \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad T_3 = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\sin^2\theta_W = \frac{1/2}{1/2 + 3/2} = \frac{\operatorname{tr} T_3^2}{\operatorname{tr} Q^2} = \frac{1}{4}$$

Shortly afterwards came the realization of dimensional transmutation, then asymptotic freedom, and soon after that, of the possibility of confinement.

$$\begin{pmatrix} \nu_e \\ e^- \\ \mu^+ \end{pmatrix}_L \qquad \begin{pmatrix} \nu_e \\ \mu^- \\ e^+ \end{pmatrix}_L$$

$$Q = T_3 + \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad T_3 = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\sin^2\theta_W = \frac{1/2}{1/2 + 3/2} = \frac{\operatorname{tr} T_3^2}{\operatorname{tr} Q^2} = \frac{1}{4}$$

From confusion, we were led quickly to the modern theory of the strong interactions in the space of only two years. It was clear that we had to understand how to incorporate the fractionally charged quarks.

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1970 — GIM (charm predicted, but also pretty much ignored), anomalies

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1972 — $SO(3)$, Bj's formula and Weinberg's second model of leptons, partons

1973 — Pati-Salam, dimensional transmutation, asymptotic freedom, infrared slavery, GUTs

One might ask why it took so long to take fractionally charged quarks seriously. Quarks had been used for years at Harvard as a short hand for the symmetry properties of the strong interactions.

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For example, in the Glashow, Iliopoulos, Maiani paper that established the rational for the charmed quark, the authors discussed how the quarks should be embedded in $SU(2) \times U(1)$.

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I think, however, that there was not much conviction that the quarks were real dynamical objects until a few years later. In fact, at the time, not even the GIM mechanism was sacred.

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We spent some of our time trying to build models in which we could avoid the need for a charmed quarks, until the discovery of the J/ψ convinced us that charm was real. But, the models were so contrived, that we were pretty convinced even before the actual discovery.

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The situation was actually delightfully confused up and even after the November revolution. The parton model picture seemed to work beautifully for the gross properties of deep inelastic lepton-hadron scattering.

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It showed fractionally charged quarks and neutral currents in accord with the $SU(2) \times U(1)$ model. But the same parton model predicted a constant R in e^+e^- annihilation, while that data actually seemed to be rising.

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There was an old data point from the Cambridge Electron Accelerator that was quite high compared to what one would expect from just u, d and s quarks, and preliminary data from SLAC seemed to confirm the rising value of R.

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This didn't seem so serious in 1972, because we didn't really understand why the parton model should work anyway. But after asymptotic freedom, when we learned how to actually calculate systematic corrections to the parton model, we began to believe it and recognize that there was a real problem.

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At some point in 1974, most of the young people at Harvard, Appelquist, DeRujula, me, Politzer and Quinn had gathered in Shelly's office and were discussing all this.

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Shelly realized that we were telling him that the problem was likely related to the existence of charm and said, "You mean that I am completely surrounded by people who believe that charm is actually being produced in e^+e^- annihilation?"

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Though it gets me far beyond the beginnings of GUTs, I can't help going on with the charm story a little bit, because there are some amusing tidbits there.

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A bit later, as the ψ was being discovered at SLAC, Burt Richter was at Harvard as a Loeb Lecturer, giving us lectures on his theory that the electron was a hadron some small fraction of the time.

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His view was that what we were seeing, in the apparently rising R, was the constant total cross section of the hadronic component of the electron.

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At a lunch with the department during his visit, Appelquist and Politzer suggested to Richter that he look for narrow peaks in the data. He did not appear to take this suggestion very seriously. The announcement of the discovery came a few weeks later.

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After the discovery(ies) of the J/ψ, and the success of the charmonium picture of the excited ψ states, we naturally assumed that everyone would be convinced that the charmed quark actually existed. Wrong!

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Quite the contrary, the difficulties of actually observing naked charm had convinced most of our colleagues at other institutions that the J/ψ must be something else.

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Coming from the sheltered haven of Harvard to a conference in Coral Gables, Florida, the year after the November revolution, I was shocked to discover that only a small group of theorists still believed in charm.

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Of course, the story finally ended happily. But we are now far ahead in time. Let me go back now to 1973, and continue with the GUT story.

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In late 72 and early 73, we were still not thinking about GUTs at all. We were trying to unify things without color.

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We may have thought about trying to include color sometimes, but we were always stopped by the vague feeling that the strong interactions are strong, and therefore shouldn't be put together with the electroweak gauge couplings.

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Anyway, it was still not obvious that we could not fit the fractionally charged quarks and the leptons into some simple group.

$$\vec{\sigma}\vec{\tau}\vec{\eta}\,, \quad \vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\eta}\,,$$

where $\vec{\sigma}$, $\vec{\tau}$, and $\vec{\eta}$ are commuting sets of Pauli matrices — the algebra closes under commutation

Model building with Shelly was a lot of fun. He would come in with bizarre questions every morning. For example, one day he came in with the algebra on the slide.

$$\vec{\sigma}\vec{\tau}\vec{\eta}\,, \quad \vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\eta}\,,$$

where $\vec{\sigma}$, $\vec{\tau}$, and $\vec{\eta}$ are commuting sets of Pauli matrices — the algebra closes under commutation — what is it?

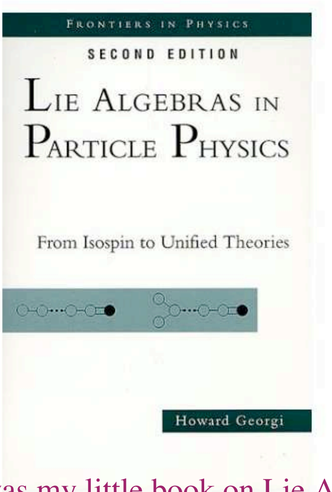
Evidently, the algebra closes under commutation. The question was this: What was the algebra? It has 36 generators, so Shelly suggested $SO(9)$. Of course, this is wrong. I would now answer immediately that this representation is pseudoreal, while $SO(9)$ has only real representations.

$$\vec{\sigma}\vec{\tau}\vec{\eta}\,, \quad \vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\eta}\,,$$

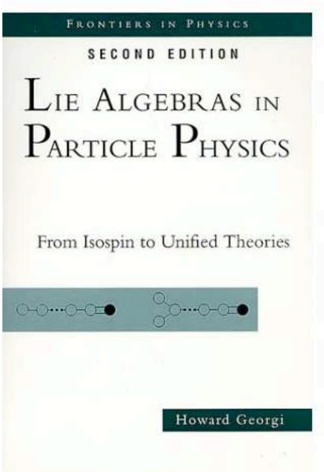
where $\vec{\sigma}$, $\vec{\tau}$, and $\vec{\eta}$ are commuting sets of Pauli matrices — the algebra closes under commutation — what is it?

$Sp(8)$ **NOT** $SO(9)$

It is actually $Sp(8)$, which has the same number of generators. But to defend myself from questions of this sort, I had to learn enough new group theoretical techniques to answer them quickly.



Eventually, the result was my little book on Lie Algebras.



The first half of 1973 was a period in which we simply tried lots of different things, building explicit gauge theory models out of all the groups we could find, just for practice. Nothing seemed to work as nicely for the quarks as Steve's little $SU(3)$ model did for the leptons.

$SO(5)$ 4 dimensional spinor is generated by $(\times 1/2)$

$$\vec{\sigma}\,, \quad \vec{\sigma}\tau_1\,, \quad \vec{\sigma}\tau_3\,, \quad \tau_2$$

$SO(6)$ 4 dimensional spinor is generated by $(\times 1/2)$

$$\vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\sigma}\vec{\tau}$$

$SO(7)$ 8 dimensional spinor is generated by $(\times 1/2)$

$$\vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\sigma}\eta_1\,, \quad \vec{\tau}\eta_2\,, \quad \vec{\sigma}\vec{\tau}\eta_3$$

However, the practice was useful. I got good at writing down representations of all sorts of groups, like the spinor representation of $SO(5)$, $SO(6)$ and $SO(7)$ shown in the slide.

$SO(5)$ 4 dimensional spinor is generated by $(\times 1/2)$

$$\vec{\sigma}\,, \quad \vec{\sigma}\tau_1\,, \quad \vec{\sigma}\tau_3\,, \quad \tau_2$$

$SO(6)$ 4 dimensional spinor is generated by $(\times 1/2)$

$$\vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\sigma}\vec{\tau}$$

$SO(7)$ 8 dimensional spinor is generated by $(\times 1/2)$

$$\vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\sigma}\eta_1\,, \quad \vec{\tau}\eta_2\,, \quad \vec{\sigma}\vec{\tau}\eta_3$$

The very naive, constructive way that I thought about these spinor representation of the orthogonal groups was to write down one set of Pauli matrices for each independent $SO(3)$ subgroup I could fit in.

$SO(5)$ 4 dimensional spinor is generated by $(\times 1/2)$

$$\vec{\sigma}\,, \quad \vec{\sigma}\tau_1\,, \quad \vec{\sigma}\tau_3\,, \quad \tau_2$$

$SO(6)$ 4 dimensional spinor is generated by $(\times 1/2)$

$$\vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\sigma}\vec{\tau}$$

$SO(7)$ 8 dimensional spinor is generated by $(\times 1/2)$

$$\vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\sigma}\eta_1\,, \quad \vec{\tau}\eta_2\,, \quad \vec{\sigma}\vec{\tau}\eta_3$$

For example, one for $SO(5)$, rotating the (123) indices of the five dimensional space of the vector representation, and two for $SO(6)$ and $SO(7)$, one for the (123) indices and one for the (456).

$SO(5)$ 4 dimensional spinor is generated by $(\times 1/2)$

$$\vec{\sigma}\,, \quad \vec{\sigma}\tau_1\,, \quad \vec{\sigma}\tau_3\,, \quad \tau_2$$

$SO(6)$ 4 dimensional spinor is generated by $(\times 1/2)$

$$\vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\sigma}\vec{\tau}$$

$SO(7)$ 8 dimensional spinor is generated by $(\times 1/2)$

$$\vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\sigma}\eta_1\,, \quad \vec{\tau}\eta_2\,, \quad \vec{\sigma}\vec{\tau}\eta_3$$

Then if there were any other generators that commuted with these, like M_{45} in $SO(5)$ I put them in as additional commuting Pauli matrices.

$SO(5)$ 4 dimensional spinor is generated by $(\times 1/2)$

$$\vec{\sigma}\,, \quad \vec{\sigma}\tau_1\,, \quad \vec{\sigma}\tau_3\,, \quad \tau_2$$

$SO(6)$ 4 dimensional spinor is generated by $(\times 1/2)$

$$\vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\sigma}\vec{\tau}$$

$SO(7)$ 8 dimensional spinor is generated by $(\times 1/2)$

$$\vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\sigma}\eta_1\,, \quad \vec{\tau}\eta_2\,, \quad \vec{\sigma}\vec{\tau}\eta_3$$

That is where the τ_2 comes from in $SO(5)$. Then the other generators can be easily constructed, according to the indices. For example, M_{j4} and M_{j5} in $SO(5)$ for $j = 1$ to 3, must be proportional to $\vec{\sigma}$ and must anticommute with τ_2 — it's just fun.

standard model
LH fermions
in complex
representation
$q_L \neq -q_L (= q_R)$

One of the obvious but important features of the standard model is that the left handed fermions do not have same charges as the right handed fermions. In group theory language, this means that the left handed representation is complex.

standard model
LH fermions
in complex
representation
$q_L \neq -q_L (= q_R)$

GUT fermion representation must be complex

If a GUT theory contains the standard model with only normal left handed fermion families, the representation of the left handed fermions in the GUT must also be complex. Heavy Dirac fermions have $q_L = -q_L$, so they don't affect this argument.

standard model
LH fermions
in complex
representation
$q_L \neq -q_L (= q_R)$

$\rightsquigarrow$

possibility
of anomalies
breaking
gauge symmetry

gauge anomaly
cancellation

$\Rightarrow$

$$A = \sum q_L^3 = 0$$
for any q

GUT fermion representation must be complex

Bouchiat, Iliopoulos and Meyer (BIM) and Gross and Jackiw recognized that the existence of the triangle anomaly put interesting constraints on spontaneously broken gauge models.

standard model
LH fermions
in complex
representation
$q_L \neq -q_L (= q_R)$

$\rightsquigarrow$

possibility
of anomalies
breaking
gauge symmetry

gauge anomaly
cancellation

$\Rightarrow$

$A = \sum q_L^3 = 0$
for any q

GUT fermion representation must be complex

A cancels for three colors in the standard model

BIM noticed that 3 colors was the right number for anomaly cancellation in the electroweak model. Shelly and I had some fun working out the group theory of anomaly cancellation.

standard model
LH fermions
in complex
representation
$q_L \neq -q_L (= q_R)$

$\rightsquigarrow$

possibility
of anomalies
breaking
gauge symmetry

Only $SU(N)$ groups have anomalies without $U(1)$s

GUT fermion representation must be complex

A cancels for three colors in the standard model

It is very simple when you are interested in models without $U(1)$s, as we were. We soon discovered that only the $SU(N)$ groups for $N \geq 3$ could support anomalies without $U(1)$s.

standard model
LH fermions
in complex
representation
$q_L \neq -q_L (= q_R)$

$\rightsquigarrow$

possibility
of anomalies
breaking
gauge symmetry

Only $SU(N)$ groups have anomalies without $U(1)$s
$SO(4N+2)$ and E_6 have complex representations

GUT fermion representation must be complex
A cancels for three colors in the standard model

It probably should have piqued our curiosity at the time, but didn't, that the $SO(4N+2)$ groups have complex spinor representations, but are anomaly free, for $N > 1$. I think that, at the time, $SO(10)$ (and certainly E_6) was just too complicated to think about.

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We were by no means the only people building models at this time. I think that the most interesting model building going on elsewhere was done in Maryland by Jogesh Pati and Abdus Salam. They discovered a very beautiful thing — lepton number as a fourth color, the Pati-Salam $SU(4)$.

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1971 — I start at a postdoc at Harvard and Ben Lee talks about t'Hooft's work on spontaneously broken gauge theories
1972 — $SO(3)$, Bj's formula and Weinberg's second model of leptons, partons
1973 — Pati-Salam, dimensional transmutation, asymptotic freedom, infrared slavery, GUTs

I'll talk about $SU(4)$ in detail later. They were first people to write down a model with charge quantization that incorporated the fractionally charged quarks, their beautiful $SU(2) \times SU(2) \times SU(4)$ model.

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In fact, ironically, they were the first people to write down the full gauge structure of the standard model, which is contained within $SU(2) \times SU(2) \times SU(4)$.

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I say that this is ironic, because having written down the right gauge structure, they proceeded to do something absolutely disgusting to it — they spontaneously broke the color $SU(3)$ and the electroweak $U(1)$ down to a subgroup that left the quarks with integral, Han-Nambu charges.

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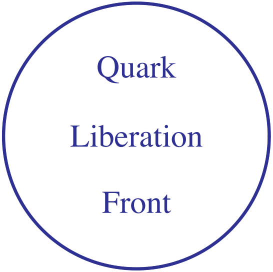
I think that Salam had some philosophical problem with fractionally charged quarks. Anyway, this model was a disaster. It was not consistent with the picture of fractionally charged quarks emerging from deep inelastic lepton-hadron scattering experiments.



Nevertheless, they stuck to it long after people almost everywhere else had gotten used to QCD and confinement. Salam used to wear Quark Liberation Front buttons —

Quark

Liberation

Front

It is worth noting that Pati and Salam also talk about proton decay, but they were actually talking about the decay of their silly, integrally charged quarks. Their model had no proton decay if color was not broken.

Quark

Liberation

Front

Their insistence on breaking the color $SU(3)$ symmetry was particularly unfortunate because it kept many people from appreciating the beauty of Pati-Salam $SU(4)$. While Shelly and I knew about it, we didn't take the idea as seriously as we should have. We kept on looking.

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The key came when we finally understood that it might not be necessary to break the color $SU(3)$ symmetry at all, that the massless gluons might be confined and not show up as massless states. I think that we learned this from Steve Weinberg who had probably discussed it with David Gross.

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Shelly suggested that this might allow us to unify the quarks with the leptons and not worry about the small size of the electroweak couplings.

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He reasoned that the strength of the strong interactions was coming from confinement, rather than a large gauge coupling. This was not quite the whole story, but it was enough to get us started. We tried a few things for an afternoon, and nothing worked. I went home.

Pati-Salam $SU(2) \times SU(2) \times SU(4)$
LH fermions in $(2,1,4)$ — RH in $(1,2,4)$

$$\vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\sigma}\vec{\tau}\,.$$

contains the following matrix

$$T_4 = -\frac{\sigma_3 + \tau_3 + \sigma_3\tau_3}{6} = \begin{pmatrix} -\frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} \end{pmatrix}$$

$$S = T_{R3} + T_4$$

After dinner, I decided to look at the Pati-Salam model assuming infrared slavery. I went through the exercise of seeing how it worked **with the color $SU(3)$ unbroken which was easy because of my study of spinors.**

Pati-Salam $SU(2) \times SU(2) \times SU(4)$
LH fermions in $(2, 1, 4)$ — RH in $(1, 2, 4)$

$$\vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\sigma}\vec{\tau}\,.$$

contains the following matrix

$$T_4 = -\frac{\sigma_3 + \tau_3 + \sigma_3\tau_3}{6} = \begin{pmatrix} -\frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} \end{pmatrix}$$

$$S = T_{R3} + T_4$$

The matrix T_4 commutes with the generators of the $SU(3)$ subgroup that acts on the last three indices. This is the $U(1)$ of the $SU(3) \times U(1)$ subgroup of $SU(4)$.

Pati-Salam $SU(2) \times SU(2) \times SU(4)$
LH fermions in $(2, 1, 4)$ — RH in $(1, 2, 4)$

$$\vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\sigma}\vec{\tau}\,.$$

contains the following matrix

$$T_4 = -\frac{\sigma_3 + \tau_3 + \sigma_3\tau_3}{6} = \begin{pmatrix} -\frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} \end{pmatrix}$$

$$S = T_{R3} + T_4$$

In the Pati-Salam $SU(2)\times SU(2)\times SU(4)$ model, the left-handed quarks and leptons are in a (2,1,4), transforming under the $SU(2)_L$ and the $SU(4)$.

Pati-Salam $SU(2) \times SU(2) \times SU(4)$
LH fermions in $(2, 1, 4)$ — RH in $(1, 2, 4)$

$$\vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\sigma}\vec{\tau}\,.$$

contains the following matrix

$$T_4 = -\frac{\sigma_3 + \tau_3 + \sigma_3\tau_3}{6} = \begin{pmatrix} -\frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} \end{pmatrix}$$

$$S = T_{R3} + T_4$$

The right handed quarks and leptons, together with a right-handed neutrino, are in a (1,2,4), transforming under the $SU(2)_R$ and the $SU(4)$. The $U(1)$ of the standard model then just involves the right handed $SU(2)$ and T_4, so that $S = T_{R3} + T_4$.

Pati-Salam $SU(2) \times SU(2) \times SU(4)$
LH fermions in $(2, 1, 4)$ — RH in $(1, 2, 4)$

$$\vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\sigma}\vec{\tau}\,.$$

contains the following matrix

$$T_4 = -\frac{\sigma_3 + \tau_3 + \sigma_3\tau_3}{6} = \begin{pmatrix} -\frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} \end{pmatrix}$$

$$S = T_{R3} + T_4$$

Of course, there was still the right handed neutrino to get rid of. I realized that this would be easy if there were a neutral singlet lepton coupled to the $(1,2,4)_R$ by a (1,2,4) of scalars.

Pati-Salam $SU(2) \times SU(2) \times SU(4)$
LH fermions in $(2, 1, 4)$ — RH in $(1, 2, 4)$

$$\vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\sigma}\vec{\tau}\,.$$

contains the following matrix

$$T_4 = -\frac{\sigma_3 + \tau_3 + \sigma_3\tau_3}{6} = \begin{pmatrix} -\frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} \end{pmatrix}$$

$$S = T_{R3} + T_4$$

This was not the now popular see-saw mechanism. It preserves lepton number. But this mechanism was very easy to think about so it was very useful to me that evening.

Pati-Salam $SU(2)\times SU(2)\times SU(4)$
LH fermions in $(2,1,4)$ — RH in $(1,2,4)$

$$\vec{\sigma}\,,\quad \vec{\tau}\,,\quad \vec{\sigma}\vec{\tau}\,.$$

contains the following matrix

$$T_4=-\frac{\sigma_3+\tau_3+\sigma_3\tau_3}{6}=\begin{pmatrix}-\frac{1}{2} & 0 & 0 & 0\\ 0 & \frac{1}{6} & 0 & 0\\ 0 & 0 & \frac{1}{6} & 0\\ 0 & 0 & 0 & \frac{1}{6}\end{pmatrix}$$

$$S=T_{R3}+T_4$$

When the (1,2,4) develops a vacuum expectation value in the right handed neutrino direction, it breaks the $SU(2)\times SU(2)\times SU(4)$ down to the $SU(2)\times U(1)\times SU(3)$ of the standard model.

Pati-Salam $SU(2) \times SU(2) \times SU(4)$
LH fermions in $(2, 1, 4)$ — RH in $(1, 2, 4)$

$$\vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\sigma}\vec{\tau}\,.$$

contains the following matrix

$$T_4 = -\frac{\sigma_3 + \tau_3 + \sigma_3\tau_3}{6} = \begin{pmatrix} -\frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{6} \end{pmatrix}$$

$$S = T_{R3} + T_4$$

Finally, I knew from playing with L-R symmetric models in general that I could give mass to the quarks and leptons with scalars transforming like (2,2,1).

Pati-Salam $SU(2) \times SU(2) \times SU(4)$
LH fermions in $(2, 1, 4)$ — RH in $(1, 2, 4)$

$$SU(2) \times SU(2) = SO(4) \qquad SU(4) = SO(6)$$

$$4 + 6 = 10$$

$$SU(2) \times SU(2) \times SU(4) = SO(4) \times SO(6) \subset SO(10)$$

Now I knew from my adventures in group theory that the algebra of $SU(2) \times SU(2)$ is the same as $SO(4)$ and $SU(4)$ is $SO(6)$. So the Pati-Salam $SU(2) \times SU(2) \times SU(4)$, now that I understood it, immediately suggested $SO(10)$

Pati-Salam $SU(2) \times SU(2) \times SU(4)$
LH fermions in $(2, 1, 4)$ — RH in $(1, 2, 4)$

$$SU(2) \times SU(2) = SO(4) \qquad SU(4) = SO(6)$$

$$4 + 6 = 10$$

$$SU(2) \times SU(2) \times SU(4) = SO(4) \times SO(6) \subset SO(10)$$

$SO(10)$ has complex spinors

The nice thing about $SO(10)$ was I didn't have to guess what representation to look at. From my work on anomalies, I knew that the complex spinor representation was the obvious choice even though I didn't see in detail how it was going to work.

$$
\begin{array}{l}
SO(10): \\
\qquad\quad \vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\eta}\,, \\
\quad \vec{\sigma}\,\alpha_1\,, \quad \vec{\tau}\,\alpha_2,, \quad \vec{\eta}\,\alpha_3\,, \\
\vec{\sigma}\,\vec{\tau}\,\alpha_3\,, \quad \vec{\tau}\,\vec{\eta}\,\alpha_1\,, \quad \vec{\eta}\,\vec{\sigma}\,\alpha_2\,. \\
\\
SU(2)_L: \quad \vec{\eta}\,(1+\alpha_3)/2 \\
SU(2)_R: \quad \vec{\eta}\,(1-\alpha_3)/2 \\
SU(4): \quad \vec{\sigma}\,,\ \vec{\tau}\,,\ \vec{\sigma}\,\vec{\tau}\,\alpha_3
\end{array}
\qquad
\begin{pmatrix}
\nu \\ u_r \\ u_g \\ u_b \\ e^- \\ d_r \\ d_g \\ d_b \\ d_b^c \\ d_g^c \\ d_r^c \\ e^+ \\ u_b^c \\ u_g^c \\ u_r^c \\ \nu^c
\end{pmatrix}
$$

I wrote down the following representation of $SO(10)$ for the 16_L. This was tremendously exciting. I understood now that the reason that we had been having such difficulty in constructing interesting models is that we had not thought of putting quarks and antiquarks together in the same multiplets.

$$SO(10):$$

$$\vec\sigma\,,\quad \vec\tau\,,\quad \vec\eta\,,$$

$$\vec\sigma\,\alpha_1\,,\quad \vec\tau\,\alpha_2,\,,\quad \vec\eta\,\alpha_3\,,$$

$$\vec\sigma\,\vec\tau\,\alpha_3\,,\quad \vec\tau\,\vec\eta\,\alpha_1\,,\quad \vec\eta\,\vec\sigma\,\alpha_2\,.$$

$$SU(2)_L: \quad \vec\eta\,(1+\alpha_3)/2$$

$$SU(2)_R: \quad \vec\eta\,(1-\alpha_3)/2$$

$$SU(4): \quad \vec\sigma\,,\ \vec\tau\,,\ \vec\sigma\,\vec\tau\,\alpha_3$$

$$\begin{pmatrix} \nu \\ u_r \\ u_g \\ u_b \\ e^- \\ d_r \\ d_g \\ d_b \\ d^c_b \\ d^c_g \\ d^c_r \\ e^+ \\ u^c_b \\ u^c_g \\ u^c_r \\ \nu^c \end{pmatrix}$$

In fact, **I** had not **thought of it** this time. That is what I like about this story. The group theory had done it for me! I have no idea how long it would have taken to try this if the spinor representation of $SO(10)$ had not been such an obvious thing to look at.

$$
\begin{array}{l}
SO(10): \\
\qquad \vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\eta}\,, \\
\vec{\sigma}\,\alpha_1\,, \quad \vec{\tau}\,\alpha_2,, \quad \vec{\eta}\,\alpha_3\,, \\
\vec{\sigma}\,\vec{\tau}\,\alpha_3\,, \quad \vec{\tau}\,\vec{\eta}\,\alpha_1\,, \quad \vec{\eta}\,\vec{\sigma}\,\alpha_2\,.
\end{array}
$$

$$
\begin{array}{ll}
SU(2)_L: & \vec{\eta}\,(1+\alpha_3)/2 \\
SU(2)_R: & \vec{\eta}\,(1-\alpha_3)/2 \\
SU(4): & \vec{\sigma}\,,\ \vec{\tau}\,,\ \vec{\sigma}\,\vec{\tau}\,\alpha_3
\end{array}
$$

$$
\begin{pmatrix} \nu \\ u_r \\ u_g \\ u_b \\ e^- \\ d_r \\ d_g \\ d_b \\ d_b^c \\ d_g^c \\ d_r^c \\ e^+ \\ u_b^c \\ u_g^c \\ u_r^c \\ \nu^c \end{pmatrix}
$$

I immediately realized that the strategy used in the Pati-Salam model for getting rid of the right handed neutrino would work in this model. If a neutral singlet lepton coupled to the 16 of fermions with a 16 of scalars, then the 16 of scalars could get a vacuum expectation value in the right handed neutrino direction.

$$
\begin{array}{c}
SO(10): \\
\vec\sigma\,, \quad \vec\tau\,, \quad \vec\eta\,, \\
\vec\sigma\,\alpha_1\,, \quad \vec\tau\,\alpha_2,, \quad \vec\eta\,\alpha_3\,, \\
\vec\sigma\,\vec\tau\,\alpha_3\,, \quad \vec\tau\,\vec\eta\,\alpha_1\,, \quad \vec\eta\,\vec\sigma\,\alpha_2\,.
\end{array}
$$

$$
\begin{array}{ll}
SU(2)_L: & \vec\eta\,(1+\alpha_3)/2 \\
SU(2)_R: & \vec\eta\,(1-\alpha_3)/2 \\
SU(4): & \vec\sigma\,,\ \vec\tau\,,\ \vec\sigma\,\vec\tau\,\alpha_3
\end{array}
$$

$$
\begin{pmatrix} \nu \\ u_r \\ u_g \\ u_b \\ e^- \\ d_r \\ d_g \\ d_b \\ d^c_b \\ d^c_g \\ d^c_r \\ e^+ \\ u^c_b \\ u^c_g \\ u^c_r \\ \nu^c \end{pmatrix}
$$

I also found superstrong symmetry breaking that would break the symmetry down to the standard model, by a combination of the 16 of scalars and the adjoint, 45 with a VEV in the T_4 direction in the $SU(4)$ subgroup. And, I understood how to give mass to the quarks and leptons, using a 10 of scalars. The 10 contained a (2,2,1).

$$
\begin{array}{l}
SO(10): \\
\qquad\qquad \vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\eta}\,, \\
\qquad \vec{\sigma}\,\alpha_1\,, \quad \vec{\tau}\,\alpha_2,\,, \quad \vec{\eta}\,\alpha_3\,, \\
\vec{\sigma}\,\vec{\tau}\,\alpha_3\,, \quad \vec{\tau}\,\vec{\eta}\,\alpha_1\,, \quad \vec{\eta}\,\vec{\sigma}\,\alpha_2\,.
\end{array}
\qquad
\begin{pmatrix} \nu \\ u_r \\ u_g \\ u_b \\ e^- \\ d_r \\ d_g \\ d_b \\ d_b^c \\ d_g^c \\ d_r^c \\ e^+ \\ u_b^c \\ u_g^c \\ u_r^c \\ \nu^c \end{pmatrix}
$$

$$
\begin{array}{ll}
SU(2)_L: & \vec{\eta}\,(1+\alpha_3)/2 \\
SU(2)_R: & \vec{\eta}\,(1-\alpha_3)/2 \\
SU(4): & \vec{\sigma}\,,\ \vec{\tau}\,,\ \vec{\sigma}\,\vec{\tau}\,\alpha_3
\end{array}
$$

I was having a really good time exploring the $SO(10)$ model, but something was nagging at me. In the Pati-Salam model, getting rid of the right handed neutrino broke the symmetry down to the $SU(2) \times U(1) \times SU(3)$ of the standard model.

$$SO(10):$$
$$\vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\eta}\,,$$
$$\vec{\sigma}\,\alpha_1\,, \quad \vec{\tau}\,\alpha_2,\,, \quad \vec{\eta}\,\alpha_3\,,$$
$$\vec{\sigma}\,\vec{\tau}\,\alpha_3\,, \quad \vec{\tau}\,\vec{\eta}\,\alpha_1\,, \quad \vec{\eta}\,\vec{\sigma}\,\alpha_2\,.$$

$$SU(2)_L: \quad \vec{\eta}\,(1+\alpha_3)/2$$
$$SU(2)_R: \quad \vec{\eta}\,(1-\alpha_3)/2$$
$$SU(4): \quad \vec{\sigma}\,,\ \vec{\tau}\,,\ \vec{\sigma}\,\vec{\tau}\,\alpha_3$$

$$\begin{pmatrix} \nu \\ u_r \\ u_g \\ u_b \\ e^- \\ d_r \\ d_g \\ d_b \\ d_b^c \\ d_g^c \\ d_r^c \\ e^+ \\ u_b^c \\ u_g^c \\ u_r^c \\ \nu^c \end{pmatrix}$$

What did the VEV of the 16 do here? I should have known how the 16 of $SO(10)$ decomposed under $SU(5)$, but I didn't have that analysis at my fingertips at the time.

$$SO(10):$$

$$\vec\sigma\,, \quad \vec\tau\,, \quad \vec\eta\,,$$

$$\vec\sigma\,\alpha_1\,, \quad \vec\tau\,\alpha_2,\,, \quad \vec\eta\,\alpha_3\,,$$

$$\vec\sigma\,\vec\tau\,\alpha_3\,, \quad \vec\tau\,\vec\eta\,\alpha_1\,, \quad \vec\eta\,\vec\sigma\,\alpha_2\,.$$

$$SU(2)_L: \quad \vec\eta\,(1+\alpha_3)/2$$

$$SU(2)_R: \quad \vec\eta\,(1-\alpha_3)/2$$

$$SU(4): \quad \vec\sigma\,,\ \vec\tau\,,\ \vec\sigma\,\vec\tau\,\alpha_3$$

$$\begin{pmatrix} \nu \\ u_r \\ u_g \\ u_b \\ e^- \\ d_r \\ d_g \\ d_b \\ d^c_b \\ d^c_g \\ d^c_r \\ e^+ \\ u^c_b \\ u^c_g \\ u^c_r \\ \nu^c \end{pmatrix}$$

I certainly did know how to find the regular $SU(5)$ subgroup of $SO(10)$ just by removing a root from the Dynkin diagram, but for some reason, I didn't do that right away (it was very late).

$$
\begin{array}{c}
SO(10): \\
\vec{\sigma}\,, \quad \vec{\tau}\,, \quad \vec{\eta}\,, \\
\vec{\sigma}\,\alpha_1\,, \quad \vec{\tau}\,\alpha_2,, \quad \vec{\eta}\,\alpha_3\,, \\
\vec{\sigma}\,\vec{\tau}\,\alpha_3\,, \quad \vec{\tau}\,\vec{\eta}\,\alpha_1\,, \quad \vec{\eta}\,\vec{\sigma}\,\alpha_2\,.
\end{array}
\qquad
\begin{pmatrix} \nu \\ u_r \\ u_g \\ u_b \\ e^- \\ d_r \\ d_g \\ d_b \\ d_b^c \\ d_g^c \\ d_r^c \\ e^+ \\ u_b^c \\ u_g^c \\ u_r^c \\ \nu^c \end{pmatrix}
$$

$$
\begin{array}{ll}
SU(2)_L: & \vec{\eta}\,(1+\alpha_3)/2 \\
SU(2)_R: & \vec{\eta}\,(1-\alpha_3)/2 \\
SU(4): & \vec{\sigma}\,, \ \vec{\tau}\,, \ \vec{\sigma}\,\vec{\tau}\,\alpha_3
\end{array}
$$

I plodded along and constructed the hermitian linear combinations of the $SO(10)$ generators that annihilated the right handed neutrino component of the 16. This is a disgusting analysis that doesn't teach you very much. But at least, at the end, I could count 24 of them.

$$24 = 5^2 - 1 \qquad\qquad 2+3=5$$

$$(1,3)_{-1/3} + (2,1)_{1/2}$$

under the $SU(2) \times U(1) \times SU(3)$ subgroup — I could now do the by now familiar calculation:

$$\begin{aligned} 10 &= \Big[5 \times 5\Big] \\ &= \Big[\Big((1,3)_{-1/3} + (2,1)_{1/2}\Big) \times \Big((1,3)_{-1/3} + (2,1)_{1/2}\Big)\Big] \\ &= (1,\overline{3})_{-2/3} + (2,3)_{1/6} + (1,1)_1 \end{aligned}$$

the 16 of $SO(10)$ is $10 + \overline{5} + 1$ of $SU(5)$

Finally, when the number of generators totaled 24, the light dawned. 24=5^2−1 is the number of generators of $SU(5)$! $SU(5)$ contains $SU(2) \times U(1) \times SU(3)$ because 2+3=5! At that point, I finally did the Dynkin analysis.

$$24 = 5^2 - 1 \qquad 2 + 3 = 5$$

$$(1,3)_{-1/3} + (2,1)_{1/2}$$

under the $SU(2) \times U(1) \times SU(3)$ subgroup — I could now do the by now familiar calculation:

$$\begin{aligned} 10 &= \Big[5 \times 5\Big] \\ &= \Big[\Big((1,3)_{-1/3} + (2,1)_{1/2}\Big) \times \Big((1,3)_{-1/3} + (2,1)_{1/2}\Big)\Big] \\ &= (1,\overline{3})_{-2/3} + (2,3)_{1/6} + (1,1)_1 \end{aligned}$$

the 16 of $SO(10)$ is $10 + \overline{5} + 1$ of $SU(5)$

From there on, the path was straight and simple. It was obvious that the 5 of $SU(5)$ had to transform like $(1,3)_{-1/3} + (2,1)_{1/2}$ under the $SU(2) \times U(1) \times SU(3)$ subgroup. I could now do the calculation of the antisymmetric combinaiton of two 5s to find the 10.

$$24 = 5^2 - 1 \qquad\qquad 2 + 3 = 5$$

$$(1,3)_{-1/3} + (2,1)_{1/2}$$

under the $SU(2) \times U(1) \times SU(3)$ subgroup — I could now do the by now familiar calculation:

$$10 = \left[5 \times 5\right]$$
$$= \left[\Big((1,3)_{-1/3} + (2,1)_{1/2}\Big) \times \Big((1,3)_{-1/3} + (2,1)_{1/2}\Big)\right]$$
$$= (1,\overline{3})_{-2/3} + (2,3)_{1/6} + (1,1)_1$$

the 16 of $SO(10)$ is $10 + \overline{5} + 1$ of $SU(5)$

Obvioiusly the 16 was $10 + \overline{5} + 1$. It was easy to see that the required superstrong symmetry breaking to get from $SU(5)$ to $SU(2) \times U(1) \times SU(3)$ could be done with a 24 (adjoint) of scalars, with a VEV in the $U(1)$ direction. It was also easy to work out what the $U(1)$ was, and compute $\sin^2\theta = 3/8$.

$$24 = 5^2 - 1 \qquad\qquad 2 + 3 = 5$$

$$(1,3)_{-1/3} + (2,1)_{1/2}$$

under the $SU(2) \times U(1) \times SU(3)$ subgroup — I could now do the by now familiar calculation:

$$10 = \Big[5 \times 5\Big]$$
$$= \Big[\Big((1,3)_{-1/3} + (2,1)_{1/2}\Big) \times \Big((1,3)_{-1/3} + (2,1)_{1/2}\Big)\Big]$$
$$= (1,\overline{3})_{-2/3} + (2,3)_{1/6} + (1,1)_1$$

the 16 of $SO(10)$ is $10 + \overline{5} + 1$ of $SU(5)$

I also noticed that if I gave quarks and leptons masses with a 5 of scalars, the masses of the charge $-1/3$ quarks would be the same as that of the corresponding charged leptons.

$$SU(2) \times U(1) \times SU(3) \longrightarrow SU(5)$$

$$\left.\begin{array}{r}
d_R = (1,3)_{-1/3} \\
(e_R^+, \overline{\nu}_R) = (2,1)_{1/2}
\end{array}\right\} \longrightarrow 5_R$$

$$\left.\begin{array}{r}
(u_L, d_L) = (2,3)_{1/6} \\
e_L^+ = (1,1)_1 \\
\overline{u}_L = (1,\overline{3})_{-2/3}
\end{array}\right\} \longrightarrow 10_L$$

By this time, I was pretty excited. I found $SU(5)$ much more appealing than $SO(10)$. Partly, this was because I didn't like the right handed neutrino in $SO(10)$. Party, it was because I liked the way it fit together. But mostly, I think, it was because $SU(5)$ was so simple that I could understand it even late at night.

$$SU(2) \times U(1) \times SU(3) \longrightarrow SU(5)$$

$$\left.\begin{array}{r} d_R = (1,3)_{-1/3} \\ (e_R^+, \overline{\nu}_R) = (2,1)_{1/2} \end{array}\right\} \longrightarrow 5_R$$

$$\left.\begin{array}{r} (u_L, d_L) = (2,3)_{1/6} \\ e_L^+ = (1,1)_1 \\ \overline{u}_L = (1,\overline{3})_{-2/3} \end{array}\right\} \longrightarrow 10_L$$

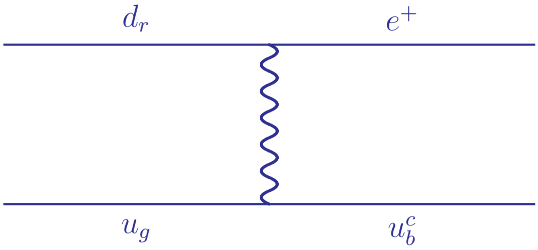
I understood finally that the $SU(2) \times U(1) \times SU(3)$ model had been trying to tell me this all along. 2+3=5. I just wouldn't listen, because I didn't want to put quarks and antiquarks together into the same multiplet.

$$SU(2) \times U(1) \times SU(3) \longrightarrow SU(5)$$

$$\left.\begin{array}{l} d_R = (1,3)_{-1/3} \\ (e_R^+, \overline{\nu}_R) = (2,1)_{1/2} \end{array}\right\} \longrightarrow 5_R$$

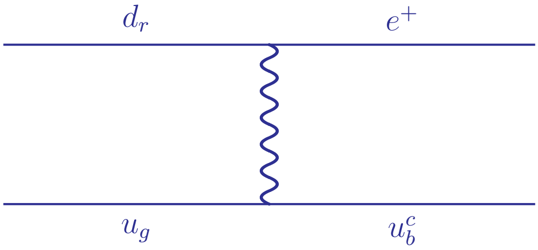
$$\left.\begin{array}{l} (u_L, d_L) = (2,3)_{1/6} \\ e_L^+ = (1,1)_1 \\ \overline{u}_L = (1,\overline{3})_{-2/3} \end{array}\right\} \longrightarrow 10_L$$

But that thought triggered a worry. Maybe there was a good reason to avoid such things. I now looked at what the extra gauge bosons did. I had avoided this in $SO(10)$ because it looked hard. But in $SU(5)$ it was easy.



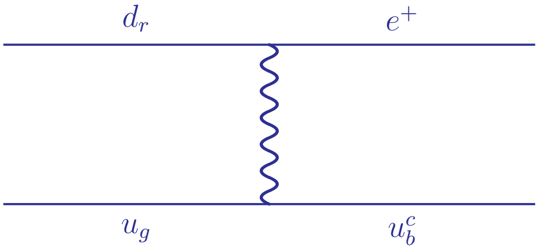
$$P \to e^+\pi^0$$

I drew the relevant diagram. It was clear that this would cause proton decay, $P \to e^+\pi^0$.



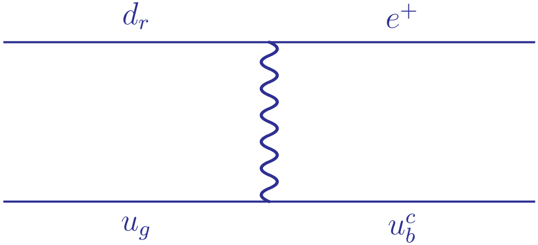
$$P \to e^+\pi^0$$

I was crushed. I knew that the proton was stable. I couldn't think of what to do about it though. The model was so unique. I went to bed. Years later, Bj told me that his experience in reading our paper on $SU(5)$, was similar to mine in constructing the model.



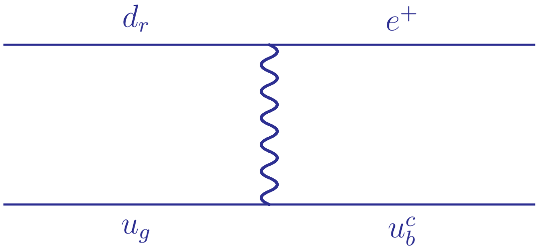
$$P \to e^+ \pi^0$$

Reading along, he got more and more impressed by the beauty and uniqueness of the model and was convinced it was right until he got to proton decay, at which point he thought it was crazy.



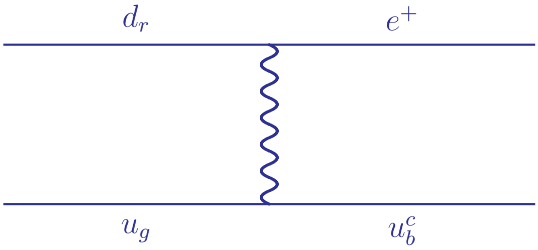
$$P \to e^+ \pi^0$$

Shelly, on the other hand, when I told him about it the next morning, was more excited about proton decay than about anything else. He was right, of course. This was the way to try to look for the new interactions. We went to the library to find the bounds on proton decay.



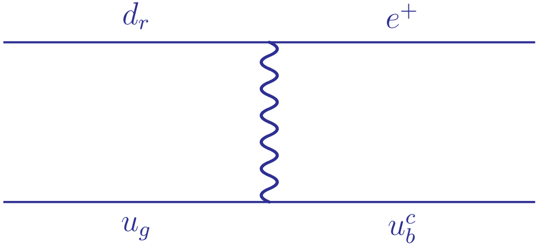
$$P \rightarrow e^+ \pi^0$$

We gulped a little when we found that we would have to make the mass of the superheavy gauge bosons greater than 10^14 GeV to be consistent with the data at the time.



$$P \to e^+\pi^0$$

But we wrote the paper and went on to some of the other exciting things that were going on at the time (like charm and QCD).



$$P \rightarrow e^+\pi^0$$

That - as they say - is history. It was a fun time, and I feel very blessed to have had the opportunity to participate in the kind of theoretical discoveries that were so familiar to Yukawa and Tomonaga.

From http://th.physik.uni-frankfurt.de/ jr/physpictheo.html

Happy Birthday! Thanks for inviting me to celebrate with all of you.