SLAC-PUB-13079
January, 2008

Supersymmetry in Elementary Particle Physics

Michael E. Peskin[1]

Stanford Linear Accelerator Center, Stanford University
2575 Sand Hill Road, Menlo Park, California 94025 USA

ABSTRACT

These lectures give a general introduction to supersymmetry, emphasizing its application to models of elementary particle physics at the 100 GeV energy scale. I discuss the following topics: the construction of supersymmetric Lagrangians with scalars, fermions, and gauge bosons, the structure and mass spectrum of the Minimal Supersymmetric Standard Model (MSSM), the measurement of the parameters of the MSSM at high-energy colliders, and the solutions that the MSSM gives to the problems of electroweak symmetry breaking and dark matter.

lectures presented at the 2006 TASI Summer School
Boulder, Colorado, June 4–30, 2006

[1]Work supported by the US Department of Energy, contract DE–AC02–76SF00515.

Contents

1 Introduction

1.1 Overview

It is an exciting time now in high-energy physics. For many years, ever since the Standard Model was established in the late 1970's, the next logical question in the search for the basic laws of physics has been that of the mechanism by which the weak interaction gauge symmetry is spontaneously broken. This seemed at the time the one important gap that kept the Standard Model from being a complete theory of the strong, weak,and electromagnetic interactions [1,2,3]. Thirty years later, after many precision experiments at high-energy e^+e^- and hadron colliders, this is still our situation. In the meantime, another important puzzle has been recognized, the fact that 80% of the mass in the universe is composed of 'dark matter', a particle species not included in the Standard Model. Both problems are likely to be solved by new fundamental interactions operating in the energy range of a few hundred GeV. Up to now, there is no evidence from particle physics for such new interactions. But, in the next few years, this situation should change dramatically. Beginning in 2008, the CERN Large Hadron Collider (LHC) should give us access to physics at energies well above 1 TeV and thus should probe the energy region responsible for electroweak symmetry breaking. Over a longer term, we can look forward to precision experiments in e^+e^- annihilation in this same energy region at the proposed International Linear Collider (ILC).

Given this expectation, it is important for all students of elementary particle physics to form concrete ideas of what new phenomena we might find as we explore this new energy region. Of course, we have no way of knowing exactly what we will find there. But this makes it all the more important to study the alternative theories that have been put forward and to understand their problems and virtues.

Many different models of new physics relevant to electroweak symmetry breaking are being discussed at this TASI school. Among these, supersymmetry has pride of place. Supersymmetry (or SUSY) provides an explicit realization of all of the aspects of new physics expected in the hundred GeV energy region. Because SUSY requires only weak interactions to build a realistic theory, it is possible in a model with SUSY to carry out explicit calculations and find the answers that the model gives to all relevant phenomenological questions.

In these lectures, I will give an introduction to supersymmetry as a context for building models of new physics associated with electroweak symmetry breaking. Here is an outline of the material: In Section 2, I will develop appropriate notation and then construct supersymmetric Lagrangians for scalar, spinor, and vector fields. In Section 3, I will define the canonical phenomenological model of supersymmetry, the Minimal Supersymmetric Standard Model (MSSM). I will discuss the quantum numbers of new particles in the MSSM and the connection of the MSSM to the idea of grand unification.

The remaining sections of these lectures will map out the phenomenology of the new particles and interactions expected in models of supersymmetry. I caution you that I will draw only those parts of the map that cover the simplest and most well-studied class of models. Supersymmetry has an enormous parameter space which contains many different scenarios for particle physics, more than I have room to cover here. I will at least try to indicate the possible branches in the path and give references that will help you follow some of the alternative routes.

With this restriction, the remaining sections will proceed as follows: In Section 4, I will compute the mass spectrum of the MSSM from its parameters. I will also discuss the parameters of the MSSM that characterize supersymmetry breaking. In Section 5, I will describe how the MSSM parameters will be measured at the LHC and the ILC. Finally, Section 6 will discuss the answers

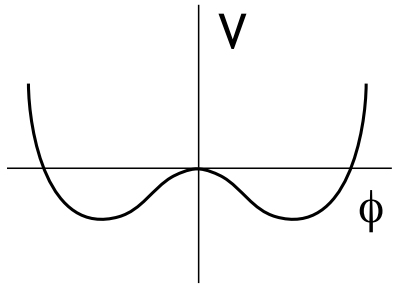

Figure 1: The Standard Model Higgs potential (1).

that supersymmetry gives to the two major questions posed at the beginning of this discussion, the origin of electroweak symmetry breaking, and the origin of cosmic dark matter.

Although I hope that these lectures will be useful to students in studying supersymmetry, there are many other excellent treatments of the subject available. A highly recommended introduction to SUSY is the 'Supersymmetry Primer' by Steve Martin [6]. An excellent presentation of the formalism of supersymmetry is given in the texbook of Wess and Bagger [7]. Supersymmetry has been reviewed at previous TASI schools by Bagger [8], Lykken [9], and Kane [10], among others. Very recently, three textbooks of phenomenological supersymmetry have appeared, by Drees, Godbole, and Roy [11], Binetruy [12], and Baer and Tata [13]. A fourth textbook, by Dreiner, Haber, and Martin [14], is expected soon.

It would be wonderful if all of these articles and books used the same conventions, but that is too much to expect. In these lectures, I will use my own, somewhat ideosyncratic conventions. These are explained in Section 2.1. Where possible, within the philosophy of that section, I have chosen conventions that agree with those of Martin's primer [6].

1.2 Motivation and Structure of Supersymmetry

If we propose supersymmetry as a model of electroweak symmetry breaking, we might begin by asking: What is the problem of electroweak symmetry breaking, and what are the alternatives for solving it?

Electroweak symmetry is spontaneously broken in the minimal form of the Standard Model, which I will refer to as the MSM. However, the explanation that the MSM gives for this phenomenon is not satisfactory. The sole source of symmetry breaking is a single elementary Higgs boson field. All mass of quarks, leptons, and gauge bosons arise from the couplings of those particles to the Higgs field.

To generate symmetry breaking, we postulate a potential for the Higgs field

$$V = \mu^2|\varphi|^2 + \lambda|\varphi|^4 \ , \tag{1}$$

shown in Fig. 1. The assumption that $\mu^2 < 0$ is the complete explanation for electroweak symmetry breaking in the MSM. Since μ is a renormalizable coupling of this theory, the value of μ cannot be computed from first principles, and even its sign cannot be predicted.

In fact, this explanation has an even worse feature. The parameter μ^2 receives large additive radiative corrections from loop diagrams. For example, the two diagrams shown in Fig. 2 are

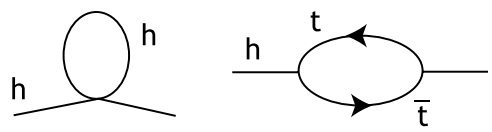

Figure 2: Two Standard Model diagrams that give divergent corrections to the Higgs mass parameter μ^2.

ultraviolet divergent. Supplying a momentum cutoff Λ, the two diagrams contribute

$$\mu^2 = \mu^2_{\text{bare}} + \frac{\lambda}{8\pi^2}\Lambda^2 - \frac{3y_t^2}{8\pi^2}\Lambda^2 + \cdots \tag{2}$$

If we view the MSM as an effective theory, Λ should be taken to be the largest momentum scale at which this theory is still valid. The presence of large additive corrections implies that the criterion $\mu^2 < 0$ is not a simple condition on the underlying parameters of the effective theory. The radiative corrections can easily change the sign of μ^2. Further, if we insist that the MSM has a large range of validity, the corrections become much larger than the desired result. To obtain the Higgs field vacuum expectation value required for the weak interactions, $|\mu|$ should be about 100 GeV. If we insist at the same time that the MSM is valid up to the Planck scale, $\Lambda \sim 10^{19}$ GeV, the formula (2) requires a cancellation between the bare value of μ and the radiative corrections in the first 36 decimal places. This problem has its own name, the 'gauge hierarchy problem'. But, to my mind, the absence of a logical explantion for electroweak symmetry breaking in the MSM is already problem enough.

How could we solve this problem? There are two different strategies. One is to look for new strong-couplings dynamics at an energy scale of 1 TeV or below. Then the Higgs field could be composite and its potential could be the result, for example, of pair condensation of fermion constituents. Higgs actually proposed that his field was a phenomenological description of a fermion pair condensation mechanism similar to that in superconductivity [4]. Sometime later, Susskind [2] and Weinberg [3] proposed an explicit model of electroweak symmetry breaking by new strong interactions, called 'technicolor'.

Today, this approach is disfavored. Technicolor typically leads to flavor-changing neutral currents at an observable level, and also typically conflicts with the accurate agreement of precision electroweak theory with experiment. Specific models do evade these difficulties, but they are highly constrained [5].

The alternative is to postulate that the electroweak symmetry is broken by a weakly-coupled Higgs field, but that this field is part of a model in which the Higgs potential is computable. In particular, the Higgs mass term $\mu^2|\varphi|^2$ should be generated by well-defined physics within the model. A prerequisite for this is that the μ^2 term not receive additive radiative corrections. This requires that, at high energy, the appearance of a nonzero μ^2 in the Lagrangian should be forbidden by a symmetry of the theory.

There are three ways to arrange a symmetry that forbids the term $\mu^2|\varphi|^2$. We can postulate a symmetry that shifts φ

$$\delta\varphi = \epsilon v \ . \tag{3}$$

We can postulate a symmetry that connects φ to a gauge field, whose mass can then forbidden by gauge symmetry

$$\delta\varphi = \epsilon \cdot A \ . \tag{4}$$

We can postulate a symmetry that connects φ to a fermion field, whose mass can then be forbidden by a chiral symmetry.

$$\delta\varphi = \epsilon \cdot \psi \ . \tag{5}$$

The options (3) and (4) lead, respectively, to 'little Higgs' models [15,16,17] and to models with extra space dimensions [18,19]. The third option leads to supersymmetry. This is the route we will now follow.

The symmetry (5) looks quite innocent, but it is not. In quantum theory, a symmetry that links a boson with a fermion is generated by a conserved charge Q_α that carries spin-1/2

$$[Q_\alpha, \varphi] = \psi_\alpha \ , \qquad [Q_\alpha, H] = 0 \ . \tag{6}$$

Such a Q_α implies the existence of a conserved 4-vector charge R_m defined by

$$\{Q_\alpha, Q^\dagger_\beta\} = 2\gamma^m_{\alpha\beta} R_m \tag{7}$$

(It may not be obvious to you that there is no Lorentz scalar component in this anticommutator, but I will show this in Section 2.1.) The charge R_m is conserved, because both Q and $Q^\dagger$ commute with H. It is nonzero, as we can see by taking the expectation value of (7) in any state and setting $\alpha = \beta$

$$\begin{aligned} \langle A| \{Q_\alpha, Q^\dagger_\alpha\} |A\rangle &= \langle A| Q_\alpha Q^\dagger_\alpha |A\rangle + \langle A| Q_\alpha Q^\dagger_\alpha |A\rangle \\ &= \| Q_\alpha |A\rangle \|^2 + \| Q^\dagger_\alpha |A\rangle \|^2 \ . \end{aligned} \tag{8}$$

This expression is non-negative; it can be zero only if Q_α and $Q^\dagger_\alpha$ annihilate every state in the theory.

However, in a relativistic quantum field theory, we do not have the freedom to introduce arbitrary charges that have nontrivial Lorentz transformation properties. Conservation of energy-momentum and angular momentum are already very constraining. For example, in two-body scattering, the scattering amplitude for fixed center of mass energy can only be a function of one variable, the center of mass scattering angle θ. If one adds a second conserved 4-vector charge, almost all values of θ will also be forbidden. Coleman and Mandula proved a stronger version of this statement: In a theory with an addtional conserved 4-vector charge, there can be no scattering at all, and so the theory is trivial [20].

If we would like to have (5) as an exact symmetry, then, the only possibility is to set $R_m = P_m$. That is, the square of the fermionic charge Q_α must be *the total energy-momentum of everything.* We started out trying to build a theory in which the fermionic charge acted only on the Higgs field. But now, it seems, the fermionic charge must act on every field in the theory. Everything—quarks, leptons, gauge bosons, even gravitons—must have partners under the symmetry generated by Q_α. Q_α is fermionic and carries spin $\frac{1}{2}$. Then every particle in the theory must have a partner with the opposite statistics and spin differing by $\frac{1}{2}$ unit.

The idea that the transformation (5) leads to a profound generalization of space-time symmetry was discovered independently several times in the early 1970's [22,23]. The 1974 paper by Wess and Zumino [24] which gave simple linear realizations of this algebra on multiplets of fields launched the detailed exploration of this symmetry and its application to particle physics.

The pursuit of (5) then necessarily leads us to introduce a very large number of new particles. This seems quite daunting. It might be a reason to choose one of the other paths, except that these also lead to new physics models of similarly high complexity. I encourage you to carry on with this line of analysis a bit longer. It will lead to a beautiful structure with many interesting implications for the theory of Nature.

2 Formalism of Supersymmetry

2.1 Fermions in 4 Dimensions

To work out the full consequences of (5), we will need to write this equation more precisely. To do this, we need to set up a formalism that describes relativistic fermions in four dimensions in the most general way. There is no general agreement on the best conventions to use, but every discussion of supersymmetry leans heavily on the particular choices made. I will give my choice of conventions in this section.

There are two basic spin-$\frac{1}{2}$ representations of the Lorentz group. Each is two-dimensional. The transformation laws are those of left- and right-handed Weyl (2-component) fermions,

$$\begin{aligned} \psi_L &\to (1 - i\vec{\alpha}\cdot\vec{\sigma}/2 - \vec{\beta}\cdot\vec{\sigma}/2)\,\psi_L \\ \psi_R &\to (1 - i\vec{\alpha}\cdot\vec{\sigma}/2 + \vec{\beta}\cdot\vec{\sigma}/2)\,\psi_R \ , \end{aligned} \tag{9}$$

where $\vec{\alpha}$ is an infinitesimal rotation angle and $\vec{\beta}$ is an infinitesimal boost. The four-component spinor built from these ingredients, $\Psi = (\psi_L, \psi_R)$, is a Dirac fermion.

Define the matrix

$$c = -i\sigma^2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \ . \tag{10}$$

This useful matrix satisfies $c^2 = -1$, $c^T = -c$. The combination

$$\psi_{1L}^T c \psi_{2L} = -\epsilon_{\alpha\beta}\psi_{1L\alpha}\psi_{2L\beta} \tag{11}$$

is the basic Lorentz invariant product of spinors. Many treatments of supersymmetry, for example, that in Wess and Bagger's book [7], represent c implicitly by raising and lowering of spinor indices. I will stick to this more prosaic approach.

Using the identity $\vec{\sigma}c = -c(\vec{\sigma})^T$, it is easy to show that the quantity $(-c\psi_L^*)$ transforms like ψ_R. So if we wish, we can replace every ψ_R by a ψ_L and write all fermions in the theory as left-handed Weyl fermions. With this notation, for example, we would call e_L^- and e_L^+ fermions and e_R^- and e_R^+ antifermions. This convention does not respect parity, but parity is not a symmetry of the Standard Model. The convention of representing all fermions in terms of left-handed Weyl fermions turns out to be very useful for not only for supersymmetry but also for other theories of physics beyond the Standard Model.

Applying this convention, a Dirac fermion takes the form

$$\Psi = \begin{pmatrix} \psi_{1L} \\ -c\psi_{2L}^* \end{pmatrix} \tag{12}$$

Write the Dirac matrices in terms of 2×2 matrices as

$$\gamma^m = \begin{pmatrix} 0 & \sigma^m \\ \overline{\sigma}^m & 0 \end{pmatrix} \tag{13}$$

with

$$\sigma^m = (1, \vec{\sigma})^m \qquad \overline{\sigma}^m = (1, -\vec{\sigma})^m \qquad c\sigma^m = (\overline{\sigma}^m)^T c \tag{14}$$

Then the Dirac Lagrangian can be rewritten in the form

$$\begin{aligned} Ł &= \overline{\Psi} i\gamma\cdot\partial\Psi - M\overline{\Psi}\Psi \\ &= \psi_{1L}^\dagger i\overline{\sigma}\cdot\partial\psi_{1L} + \psi_{2L}^\dagger i\overline{\sigma}\cdot\partial\psi_{2L} \\ &\qquad -(m\psi_{1L}^T c\psi_{2L} - m^*\psi_{1L}^\dagger c\psi_{2L}^*) \ . \end{aligned} \tag{15}$$

For the bilinears in the last line, we can use fermion anticommutation and the antisymmetry of c to show

$$\psi_{1L}^T c\psi_{2L} = +\psi_{2L}^T c\psi_{1L} \ . \tag{16}$$

and, similarly,

$$(\psi_{1L}^T c\psi_{2L})^\dagger = \psi_{2L}^\dagger(-c)\psi_{1L}^* = -\psi_{1L}^\dagger c\psi_{2L}^* \ . \tag{17}$$

The mass term looks odd, because it is fermion number violating. However, the definition of fermion number is that given in the previous paragraph. The fields ψ_{1L} and ψ_{2L} annihilate, respectively, e_L^- and e_L^+. So this mass term generates the conversion of e_L^- to e_R^-, which is precisely what we would expect a mass term to do.

If we write all fermions as left-handed Weyl fermions, the possibilities for fermion actions are highly restricted. The most general Lorentz-invariant free field Lagrangian takes the form

$$Ł = \psi_k^\dagger i\overline{\sigma}\cdot\partial\psi_k - \frac{1}{2}(m_{jk}\psi_j^T c\psi_k - m_{jk}^*\psi_j^\dagger c\psi_k^*) \ . \tag{18}$$

where j, k index the fermion fields. Here and in the rest of these lectures, I drop the subscript L. The matrix m_{jk} is a complex symmetric matrix. For a Dirac fermion,

$$m_{jk} = \begin{pmatrix} 0 & m \\ m & 0 \end{pmatrix}_{jk} \tag{19}$$

as we have seen in (15). This matrix respects the charge

$$Q\psi_1 = +\psi_1 \ , \qquad Q\psi_2 = -\psi_2 \ , \tag{20}$$

which is equivalent to the original Dirac fermion number. A Majorana fermion is described in the same formalism by the mass matrix

$$m_{jk} = m\delta_{jk} \ . \tag{21}$$

The most general fermion mass is a mixture of Dirac and Majorana terms. We will meet such fermion masses in our study of supersymmetry. These more general mass matrices also occur in other new physics models and in models of the masses of neutrinos.

The SUSY charges are four-dimensional fermions. The minimum set of SUSY charges thus includes one Weyl fermion Q_α and its Hermitian conjugate $Q_\alpha^\dagger$. We can now analyze the anticommutator $\{Q_\alpha, Q_\beta^\dagger\}$. Since the indices belong to different Lorentz representations, this object does not contain a scalar. The indices transform as do the spinor indices of σ^m, and so we can rewrite (7) with $R^m = P^m$ as

$$\{Q_\alpha, Q_\beta^\dagger\} = 2\sigma_{\alpha\beta}^m P_m \ . \tag{22}$$

It is possible to construct quantum field theories with larger supersymmetry algebras. These must include (22), and so the general form is [21]

$$\{Q_\alpha^i, Q_\beta^{\dagger j}\} = 2\sigma_{\alpha\beta}^m P_m \delta^{ij} \ , \tag{23}$$

for $i, j = 1 \ldots N$. This relation can be supplemented by a nontrivial anticommutator

$$\{Q_\alpha^i, Q_\beta^j\} = 2\epsilon_{\alpha\beta}\mathcal{Q}^{ij} \tag{24}$$

where the *central charge* $\mathcal{Q}^{ij}$ is antisymmetric in $[ij]$. Theories with $N > 4$ necessarily contain particles of spin greater than 1. Yang-Mills theory with $N = 4$ supersymmetry is an especially

beautiful model with exact scale invariance and many other attractive formal properties [25]. In these lectures, however, I will restrict myself to the minimal case of $N = 1$ supersymmetry.

I will discuss supersymmetry transformations using the operation on fields

$$\delta_\xi \Phi = [\, \xi^T c Q + Q^\dagger c \xi^*, \Phi] \ . \tag{25}$$

Note that the operator δ_ξ contains pairs of anticommuting objects and so obeys commutation rather than anticommutation relations. The operator P_m acts on fields as the generator of translations, $P_m = i\partial_m$. Using this, we can rewrite (22) as

$$[\delta_\xi, \delta_\eta] = 2i \left(\xi^\dagger \overline{\sigma}^m \eta - \eta^\dagger \overline{\sigma}^m \xi \right) \ \partial_m \tag{26}$$

I will take this equation as the basic (anti)-commutation relation of supersymmetry. In the next two sections, I will construct some representations of this commutation relation on multiplets of fields.

2.2 Supersymmetric Lagrangians with Scalars and Fermions

The simplest representation of the supersymmetry algebra (26) directly generalizes the transformation (5) from which we derived the idea of supersymmetry. The full set of fields required includes a complex-valued boson field ϕ and a Weyl fermion field ψ. These fields create and destroy a scalar particle and its antiparticle, a left-handed massless fermion, and its right-handed antiparticle. Note that the particle content has an equal number of fermions and bosons. This particle content is called a *chiral supermultiplet*.

I will now write out the transformation laws for the fields corresponding to a chiral supermultiplet. It is convenient to add a second complex-valued boson field F that will have no associated particles. Such a field is called an *auxiliary field*. We can then write the transformations that generalize (5) as

$$\begin{aligned} \delta_\xi \phi &= \sqrt{2} \xi^T c \psi \\ \delta_\xi \psi &= \sqrt{2} i \sigma^n c \xi^* \partial_n \phi + \sqrt{2} F \, \xi \\ \delta_\xi F &= -\sqrt{2} i \xi^\dagger \overline{\sigma}^m \partial_m \psi \ . \end{aligned} \tag{27}$$

The conjugates of these transformations are

$$\begin{aligned} \delta_\xi \phi^* &= -\sqrt{2} \psi^\dagger c \xi^* \\ \delta_\xi \psi^\dagger &= \sqrt{2} i \xi^T c \sigma^n \partial_n \phi^* + \sqrt{2} \xi^\dagger F^* \\ \delta_\xi F^* &= \sqrt{2} i \partial_m \psi^\dagger \overline{\sigma}^m \xi \ . \end{aligned} \tag{28}$$

These latter transformations define the *antichiral supermultiplet*. I claim that the transformations (27) and (28), first, satisfy the fundamental commutation relation (26) and, second, leave a suitable Lagrangian invariant. Both properties are necessary, and both must be checked, in order for a set of transformations to generate a symmetry group of a field theory.

The transformation laws (27) seem complicated. You might wonder if there is a formalism that generates these relations automatically and manipulates them more easily than working with the three distinct component fields (ϕ, ψ, F). In the next section, I will introduce a formalism called *superspace* that makes it almost automatic to work with the chiral supermultiplet. However, the superspace description of the multiplet containing gauge fields is more complicated, and the difficulty of working with superspace becomes exponentially greater in theories that include gravity, higher dimensions, or $N > 1$ supersymmetry. At some stage, one must go back to components. I strongly

recommend that you gain experience by working through the component field calculations described in these notes in full detail, however many large pieces of paper that might require.

To verify each of the two claims I have made for (27) requires a little calculation. Here is the check of the commutation relation applied to the field ϕ:

$$\begin{aligned}
[\delta_\xi, \delta_\eta]\phi &= \delta_\xi(\sqrt{2}\eta^T c\psi) - (\xi \leftrightarrow \eta) \\
&= \sqrt{2}\eta^T c(\sqrt{2}i\sigma^n c\xi^* \partial_n \phi) - (\xi \leftrightarrow \eta) \\
&= -2i\eta^T (\overline{\sigma}^n)^T \xi^* \partial_n \phi - (\xi \leftrightarrow \eta) \\
&= 2i[\xi^\dagger \overline{\sigma}^n \eta - \eta^\dagger \overline{\sigma}^n \xi]\partial_n \phi
\end{aligned} \tag{29}$$

The check of the commutation relation applied to F is equally straightforward. The check on ψ is a bit lengthier. It requires a *Fierz identity*, that is, a spinor index rearrangment identity. Specifically, we need

$$\eta_\alpha \xi^\dagger_\beta = -\frac{1}{2}(\xi^\dagger \overline{\sigma}_m \eta)\sigma^m_{\alpha\beta} \ , \tag{30}$$

which you can derive by writing out the four components explicitly. After some algebra that involves the use of this identity, you can see that the SUSY commutation relation applied to ψ also takes the correct form.

Next, I claim that the Lagrangian

$$Ł = \partial^m \phi^* \partial_m \phi + \psi^\dagger i\overline{\sigma} \cdot \partial \psi + F^* F \tag{31}$$

is invariant to the transformation (27). I will assume that the Lagrangian (31) is integrated $\int d^4x$ and use integration by parts freely. Then

$$\begin{aligned}
\delta_\xi Ł &= \partial^m \phi^* \partial_m(\sqrt{2}\xi^T c\psi) + (-\sqrt{2}\partial^m \psi^\dagger c\xi^*)\partial\phi \\
&\quad + \psi^\dagger i\overline{\sigma} \cdot \partial[\sqrt{2}i\sigma^n c\xi^* \partial_m \phi + \sqrt{2}\xi F] \\
&\quad + [\sqrt{2}i\partial_n \phi^* \xi^T c\sigma^n + \sqrt{2}\xi^\dagger F^*] i\overline{\sigma} \cdot \partial\psi \\
&\quad + F^*[-\sqrt{2}i\xi^\dagger \overline{\sigma}^m \partial_m \psi] + [\sqrt{2}i\partial_m \psi^\dagger \overline{\sigma}^m \xi]F \\
&= -\phi^* \sqrt{2}\xi^T c\partial^2 \psi + \sqrt{2}\partial_n \phi^* \xi^T c\sigma^n \overline{\sigma}^m \partial_n \partial_m \psi \\
&\quad + \sqrt{2}\psi^\dagger c\xi^* \partial^2 \phi - \sqrt{2}\psi^\dagger \overline{\sigma}^m \sigma^n c\xi^* \partial_m \partial_n \phi \\
&\quad + \sqrt{2}i\psi^\dagger \overline{\sigma}^m \partial_m F\xi + \sqrt{2}i\partial_m \psi^\dagger \overline{\sigma}^m \xi F \\
&\quad - \sqrt{2}i\xi^\dagger F^* \overline{\sigma}^m \partial_m \psi + \sqrt{2}iF^* \xi^\dagger \overline{\sigma}^m \partial_m \psi \\
&= 0 \ .
\end{aligned} \tag{32}$$

In the final expression, the four lines cancel line by line. In the first two lines, the cancellation is made by using the identity $(\overline{\sigma} \cdot \partial)(\sigma \cdot \partial) = \partial^2$.

So far, our supersymmetry Lagrangian is just a massless free field theory. However, it is possible to add rather general interactions that respect the symmetry. Let $W(\phi)$ be an analytic function of ϕ, that is, a function that depends on ϕ but not on ϕ^*. Let

$$Ł_W = F\frac{\partial W}{\partial \phi} - \frac{1}{2}\psi^T c\psi \frac{\partial^2 W}{\partial \phi^2} \tag{33}$$

I claim that $Ł_W$ is invariant to (27). Then we can add $(Ł_W + Ł_W^\dagger)$ to the free field Lagrangian to introduce interactions into the theory. The function W is called the *superpotential*.

We can readily check that $Ł_W$ is indeed invariant:

$$\begin{aligned}\delta_\xi Ł_W = F\frac{\partial^2 W}{\partial\phi^2}(\sqrt{2}\xi^T c\psi) - \sqrt{2}F\xi^T c\psi\frac{\partial^2 W}{\partial\phi^2} \\ -\sqrt{2}i\xi^\dagger\overline{\sigma}^m\partial_m\psi\frac{\partial W}{\partial\phi} - \psi^T c\sqrt{2}i\sigma^n c\xi^*\partial_n\phi\frac{\partial^2 W}{\partial\phi^2} \\ -\psi^T c\psi\frac{\partial^3 W}{\partial\phi^3}\sqrt{2}\xi^T c\psi\ .\end{aligned} \tag{34}$$

The second line rearranges to

$$-\sqrt{2}i\xi^\dagger\overline{\sigma}\left(\partial_n\psi\frac{\partial W}{\partial\phi} + \psi\partial_n\phi\frac{\partial^2 W}{\partial\phi^2}\right)\ , \tag{35}$$

which is a total derivative. The third line is proportional to $\psi_\alpha\psi_\beta\psi_\gamma$, which vanishes by fermion antisymmetry since the spinor indices take only two values. Thus it is true that

$$\delta_\xi Ł_W = 0\ . \tag{36}$$

The proofs of invariance that I have just given generalize straightforwardly to systems of several chiral supermultiplets. The requirement on the superpotential is that it should be an analytic function of the complex scalar fields ϕ_k. Then the following Lagrangian is supersymmetric:

$$Ł = \partial^m\phi_k^*\partial_m\phi_k + \psi_k^\dagger i\overline{\sigma}\cdot\partial\psi_k + F_k^*F_k + Ł_W + Ł_W^\dagger\ , \tag{37}$$

where

$$Ł_W = F_k\frac{\partial W}{\partial\phi_k} - \frac{1}{2}\psi_j^T c\psi_k\frac{\partial^2 W}{\partial\phi_j\partial\phi_k}\ . \tag{38}$$

In this Lagrangian, the fields F_k are Lagrange multipliers. They obey the constraint equations

$$F_k^* = -\frac{\partial W}{\partial\phi_k}\ . \tag{39}$$

Using these equations to eliminate the F_k, we find an interacting theory with the fields ϕ_k and ψ_k, a Yukawa coupling term proportional to the second derivative of W, as given in (38), and the potential energy

$$V_F = \sum_k\left|\frac{\partial W}{\partial\phi_k}\right|^2\ . \tag{40}$$

I will refer to V_F as the *F-term potential*. Later we will meet a second contribution V_D, the *D-term potential*. These two terms, both obtained by integrating out auxiliary fields, make up the classical potential energy of a general supersymmetric field theory of scalar, fermion, and gauge fields.

The simplest example of the F-term potential appears in the theory with one chiral supermultiplet and the superpotential $W = \frac{1}{2}m\phi^2$. The constraint equation for F is [27]

$$F^* = -m\phi\ . \tag{41}$$

After eliminating F, we find the Lagrangian

$$Ł = \partial^n\phi^*\partial_n\phi - |m|^2\phi^*\phi + \psi^\dagger i\overline{\sigma}\cdot\partial\psi - \frac{1}{2}(m\psi^T c\psi - m^*\psi^\dagger c\psi^*) \tag{42}$$

This is a theory of two free scalar bosons of mass $|m|$ and a free Majorana fermion with the same mass $|m|$. The Majorana fermion has two spin states, so the number of boson and fermion physical states is equal, as required.

The form of the expression (40) implies that $V_F \geq 0$, and that $V_F = 0$ only if all $F_k = 0$. This constraint on the potential energy follows from a deeper consideration about supersymmetry. Go back to the anticommutation relation (22), evaluate it for $\alpha = \beta$, and take the vacuum expectation value. This gives

$$\langle 0| \{Q_\alpha, Q_\alpha^\dagger\} |0\rangle = \langle 0| (H - P^3) |0\rangle = \langle 0| H |0\rangle \ , \tag{43}$$

since the vacuum expectation value of P^3 vanishes by rotational invariance. Below (7), I argued that the left-hand side of this equation is greater than or equal to zero. It is equal to zero if and only if

$$Q_\alpha |0\rangle = Q_\alpha^\dagger |0\rangle = 0 \tag{44}$$

The formulae (44) give the criterion than the vacuum is invariant under supersymmetry. If this relation is not obeyed, supersymmetry is spontaneously broken. Taking the vacuum expectation value of the transformation law for the chiral representation, we find

$$\begin{aligned} \langle 0| [\xi^T c Q + Q^\dagger c \xi^*, \psi_k] |0\rangle &= \langle 0| \sqrt{2} i \sigma^n \xi^* \partial_n \phi_k + \xi F_k |0\rangle \\ &= \xi \langle 0| F_k |0\rangle \ . \end{aligned} \tag{45}$$

In the last line I have used the fact that the vacuum expectation value of $\phi(x)$ is translation invariant, so its derivative vanishes. The left-hand side of (45) vanishes if the vacuum state is invariant under supersymmetry.

The results of the previous paragraph can be summarized in the following way: If supersymmetry is a manifest symmetry of a quantum field theory,

$$\langle 0| H |0\rangle = 0 \ , \text{ and } \langle 0| F_k |0\rangle = 0 \tag{46}$$

for every F field of a chiral multiplet. In complete generality,

$$\langle 0| H |0\rangle \geq 0 \ . \tag{47}$$

The case where $\langle H \rangle$ is positive and nonzero corresponds to spontaneously broken supersymmetry. If the theory has a state satisfying (44), this is necesssarily the state in the theory with lowest energy. Thus, supersymmetry can be spontaneously broken only if a supersymmetric vacuum state does not exist*

For the moment, we will work with theories that preserve supersymmetry. I will give examples of theories with spontaneous supersymmetry breaking in Section 3.5.

The results we have just derived are exact consequences of the commutation relations of supersymmetry. It must then be true that the vacuum energy of a supersymmetric theory must vanish in perturbation theory. This is already nontrivial for the free theory (42). But it is correct. The positive zero point energy of the boson field exactly cancels the negative zero point energy of the fermion field. With some effort, one can show the cancellation also for the leading-order diagrams in an interacting theory. Zumino proved that this cancellation is completely general [29].

I would like to show you another type of cancellation that is also seen in perturbation theory in models with chiral fields. Consider the model with one chiral field and superpotential

$$W = \frac{\lambda}{3} \phi^3 \ . \tag{48}$$

*It is possible that a supersymmetric vacuum state might exist but that a higher-energy vacuum state might be metastable. A model built on this metastable state would show spontaneous breaking of supersymmetry [26].

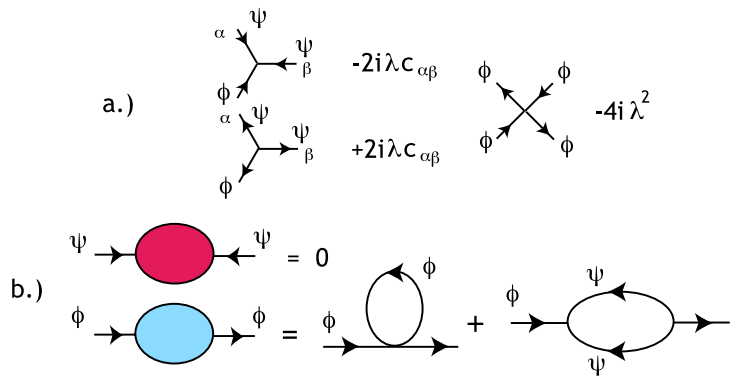

Figure 3: Perturbation theory for the supersymmetric model (49): (a) vertices of the model; (b) corrections to the fermion and scalar masses.

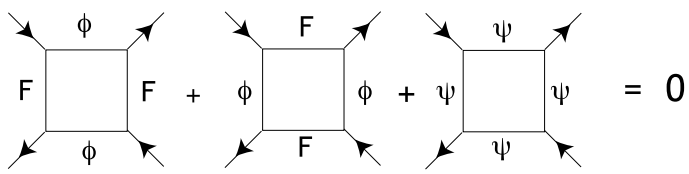

Figure 4: Scheme of cancellations of one-loop corrections to the F-term potential.

After eliminating F, the Lagrangian becomes

$$\text{Ł} = \partial\phi^*\partial_m\phi + \psi^\dagger i\overline{\sigma}\cdot\partial\psi - \lambda(\phi\psi^T c\psi - \phi^*\psi^\dagger c\psi^*) - \lambda^2|\phi|^4 \ . \tag{49}$$

The vertices of this theory are shown in Fig. 3(a).

From our experience in (2), we might expect to find an addditive radiative correction to the scalar mass. The corrections to the fermion and scalar mass terms are given by the diagrams in Fig. 3(b). Actually, there are no diagrams that correct the fermion mass; you can check that there it is not possible to match the arrows appropriately. For the scalar mass correction, the two diagrams shown contribute

$$-4i\lambda^2\int\frac{d^4p}{(2\pi)^4}\frac{i}{p^2} + \frac{1}{2}(-2i\lambda)(+2i\lambda)\int\frac{d^4p}{(2\pi)^4}\text{tr}\left[\frac{i\sigma\cdot p}{p^2}c\frac{i\sigma^T\cdot(-p)}{p^2}c\right] \tag{50}$$

Using $\sigma\cdot p\overline{\sigma}\cdot p = p^2$ in the second term and then taking the trace, we see that these two contributions cancel precisely. In this way, supersymmetry really does control radiative corrections to the Higgs mass, following the logic that we presented in Section 1.2.

In fact, it can be shown quite generally that not only the mass term but the whole superpotential W receives no additive radiative corrections in any order of perturbation theory [30]. For example, the one-loop corrections to quartic terms in the Lagrangian cancel in a simple way that is indicated in Fig. 4. The field strength renormalization of chiral fields can be nonzero, so the form of W can be changed by radiative corrections by the rescaling of fields. Examples are known in which W receives additive radiative corrections from nonperturbative effects [31].

2.3 Superspace

Because the commutation relations of supersymmetry include the generators of translations, supersymmetry is a space-time symmetry. It is an attractive idea that supersymmetry is the natural set of translations on a generalized space-time with commuting and anticommuting coordinates. In this section, I will introduce the appropriate generalization of space-time and use it to re-derive some of the results of Section 2.2.

Consider, then, a space with four ordinary space-time coordinates x^μ and four anticommuting coordinates $(\theta_\alpha, \overline{\theta}_\alpha)$. I will take the coordinates θ_α to transform as 2-component Weyl spinors; the $\overline{\theta}_\alpha$ are the complex conjugates of the θ_α. This is *superspace*. A *superfield* is a function of these superspace coordinates: $\Phi(x, \theta, \overline{\theta})$.

It is tempting to define supersymmetry transformations as translations $\theta \to \theta + \xi$. However, this does not work. These transformations commute, $[\delta_\xi, \delta_\eta] = 0$, and we have seen in Section 1.2 that this implies that the S-matrix of the resulting field theory must be trivial. To construct a set of transformations with the correct commutation relations, we must write

$$\delta_\xi \Phi = \mathcal{Q}_\xi \Phi \ , \tag{51}$$

where

$$\mathcal{Q}_\xi = \left(-\frac{\partial}{\partial\theta} - i\overline{\theta}\overline{\sigma}^m \partial_m \right) \xi + \xi^\dagger \left(\frac{\partial}{\partial\overline{\theta}} + i\overline{\sigma}^m \theta \partial_m \right) \ . \tag{52}$$

This is a translation of the fermionic coordinates $(\theta, \overline{\theta})$ plus a translation of the ordinary space-time coordinates proportional to θ, $\overline{\theta}$. It is straightforward to show that these operators satisfy

$$[\mathcal{Q}_\xi, \mathcal{Q}_\eta] = -2i \left(\xi^\dagger \overline{\sigma}^m \eta - \eta^\dagger \overline{\sigma}^m \xi \right) \ \partial_m \ . \tag{53}$$

Despite the fact that this equation has an extra minus sign on the right-hand side with respect to (26), it is the relation that we want. (The difference is similar to that between active and passive transformations.) Combined with the decomposition of the superfield that I will introduce below, this relation will allow us to derive the chiral supermultiplet transformation laws (27).

Toward this goal, we need one more ingredient. Define the superspace derivatives

$$D_\alpha = \frac{\partial}{\partial\theta_\alpha} - i(\overline{\theta}\sigma^m)_\alpha \partial_m \qquad \overline{D}_\alpha = -\frac{\partial}{\partial\overline{\theta}_\alpha} + i(\sigma^m\theta)_\alpha \partial_m \ , \tag{54}$$

such that $(D_\alpha \Phi)^\dagger = \overline{D}_\alpha \Phi^\dagger$. These operators commute with $\mathcal{Q}_\xi$:

$$[D_\alpha, \mathcal{Q}_\xi] = 0 \qquad [\overline{D}_\alpha, \mathcal{Q}_\xi] = 0 \ . \tag{55}$$

Thus, we can constrain Φ by the equation

$$D_\alpha \Phi = 0 \qquad \text{or} \qquad \overline{D}_\alpha \Phi = 0 \ , \tag{56}$$

and these constraints are consistent with supersymmetry. What we have just shown is that the general superfield $\Phi(x, \theta, \overline{\theta})$ is a *reducible* representation of supersymmetry. It can be decomposed into a direct sum of three smaller representations, one constrained by the first of the relations (56), one constrained by the second of these relations, and the third containing whatever is left over in Φ when these pieces are removed.

Let's begin with the constraint $\overline{D}_\alpha \Phi = 0$. The solution of this equation can be written

$$\Phi(x, \theta, \overline{\theta}) = \Phi(x + i\overline{\theta}\overline{\sigma}^m \theta, \theta) \ , \tag{57}$$

that is, this solution is parametrized by a general function of x and θ. Since θ is a two-component anticommuting object, this general function of x and θ can be represented as

$$\Phi(x,\theta) = \phi(x) + \sqrt{2}\theta^T c\psi(x) + \theta^T c\theta F(x) \ . \tag{58}$$

The field content of this expression is exactly that of the chiral supermultiplet. The supersymmetry transformation of this field should be

$$\delta_\xi \Phi = \mathcal{Q}_\xi \Phi(x + i\overline{\theta}\overline{\sigma}^m\theta, \theta) \ . \tag{59}$$

It is straightforward to compute the right-hand side of (59) in terms of θ, $\overline{\theta}$, and the component fields of (58). The coefficients of powers of θ are precisely the supersymmetry variations given in (27). Thus a superfield satisfying

$$\overline{D}_\alpha \Phi = 0 \tag{60}$$

is equivalent to a chiral supermultiplet, and the transformation (59) gives the supersymmetry transformation of this multiplet. A superfield satisfying (60) is called a *chiral superfield*. Similarly, a superfield satisfying

$$D_\alpha \Phi = 0 \tag{61}$$

is called an *antichiral superfield*. This superfield has a component field decomposition (ϕ^*, ψ^*, F^*), on which $\mathcal{Q}_\xi$ induces the transformation (28). I will describe the remaining content of the general superfield Φ in Section 2.5.

A Lagrangian on Minkowski space is integrated over d^4x. A superspace Lagrangian should be also be integrated over the θ coordinates. Integration over fermionic coordinates is defined to be proportional to the coefficient of the highest power of θ. I will define integration over superspace coordinates by the formulae

$$\int d^2\theta \, 1 = \int d^2\theta \, \theta_\alpha = 0 \qquad \int d^2\theta (\theta^T c\theta) = 1 \tag{62}$$

and their conjugates. To use these formulae, expand the superfields in powers of θ and pick out the terms proportional to $(\theta^T c\theta)$. Then, if Φ is a chiral superfield constrained by (60) and $W(\Phi)$ is an analytic function of Φ,

$$\begin{aligned} &\int d^2\theta \ \Phi(x,\theta) = F(x) \\ &\int d^2\theta \ W(\Phi) = F(x)\frac{\partial W}{\partial \phi} - \frac{1}{2}\psi^T c\psi \frac{\partial^2 W}{\partial^2 \phi} \ , \end{aligned} \tag{63}$$

where, in the second line, W on the right-hand side is evaluated with $\Phi = \phi(x)$. With somewhat more effort, one can show

$$\int d^2\theta \int d^2\overline{\theta} \ \Phi^\dagger \Phi = \partial^m \phi^* \partial_m \phi + \psi^\dagger i\overline{\sigma} \cdot \partial\psi + F^* F \ . \tag{64}$$

These formulae produce the invariant Lagrangians of chiral supermultiplets from a superspace point of view. The most general Lagrangian of chiral superfields Φ_k takes the form

$$Ł = \int d^4\theta \, K(\Phi, \Phi^\dagger) + \int d^2\theta \, W(\Phi) + \int d^2\overline{\theta} \, (W(\Phi))^\dagger \ , \tag{65}$$

where $W(\Phi)$ is an analytic function of complex superfields and $K(\Phi, \Phi^\dagger)$ is a general real-valued function of the superfields. The Lagrangian (37) is generated from this expression by taking

$K(\Phi, \Phi^\dagger) = \Phi_k^\dagger \Phi_k$. The most general *renormalizable* Lagrangian of chiral supermultiplets is obtained by taking K to be of this simple form and taking W to be a polynomial of degree at most 3.

Because the integral $d^2\theta$ exposes the Lagrange multiplier F in (58), I will refer to a term with this superspace integral as an *F-term*. For similar reasons that will become concrete in the next section, I will call a term with a $d^4\theta$ integral a *D-term*.

In the remainder of these lectures, I will restrict myself to discussing renormalizable supersymmetric theories. But, still, it is interesting to ask what theories we obtain when we take more general forms for K. The Lagrangian for ϕ turns out to be a nonlinear sigma model for which the target space is a complex manifold with the metric [32]

$$g_{m\overline{n}} = \frac{\partial^2}{\partial \Phi^m \partial \Phi^{\dagger \overline{n}}} K(\Phi, \Phi^\dagger) \tag{66}$$

A complex manifold whose metric is derived from a potential in this way is called a *Kähler manifold*. The function K is the *Kähler potential*. It is remarkable that, wherever in ordinary quantum field theory we find a general structure from real analysis, the supersymmetric version of the theory has a corresponding complex analytic structure.

Now that we have a Lagrangian in superspace, it is possible to derive Feynman rules and compute Feynman diagrams in superspace. I do not have space here to discuss this formalism; it is discussed, for example, in [7] and [30]. I would like to state one important consequence of this formalism. It turns out that, barring some special circumstances related to perturbation theory anomalies, these Feynman diagrams always generate corrections to the effective Lagrangian that are D-terms,

$$\int d^4\theta \, X(\Phi, \Phi^\dagger) \ . \tag{67}$$

The perturbation theory does not produce terms that are integrals $\int d^2\theta$. This leads to an elegant proof of the result cited at the end of the previous section that the superpotential is not renormalized at any order in perturbation theory [30].

2.4 Supersymmetric Lagrangians with Vector Fields

To construct a supersymmetric model that can include the Standard Model, we need to be able to write supersymmetric Lagrangians that include Yang-Mills vector fields. In this section, I will discuss how to do that.

To prepare for this discussion, let me present my notation for gauge fields in a general quantum field theory. The couplings of gauge bosons to matter are based on the covariant derivative, which I will write as

$$\mathcal{D}_m \phi = (\partial_m - igA_m^a t_R^a)\phi \tag{68}$$

for a field ϕ that belongs to the representation R of the gauge group G. In this formula, t_R^a are the representation matrices of the generators of G in the representation R. These obey

$$[t_R^a, t_R^b] = if^{abc} t_R^c \tag{69}$$

The coefficients f^{abc} are the *structure constants* of G. They are independent of R; essentially, their values define the multiplication laws of G. They can be taken to be totally antisymmetric.

The generators of G transform under G according to a representation called the *adjoint representation*. I will denote this representation by $R = G$. Its representation matrices are

$$(t_G^a)_{bc} = if^{bac} \tag{70}$$

These matrices satisfy (69) by virtue of the Jacobi identity. The covariant derivative on a field in the adjoint representation takes the form

$$\mathcal{D}_m \Phi^a = \partial_m \Phi^a + g f^{abc} A^b_m \Phi^c \tag{71}$$

The field strengths F^a_{mn} are defined from the covariant derivative (in any representation) by

$$[\mathcal{D}_m, \mathcal{D}_n] = -igF^a_{mn} t^a_R \ . \tag{72}$$

This gives the familiar expression

$$F^a_{mn} = \partial_m A^a_n - \partial_n A^a_m + g f^{abc} A^b_m A^c_n \ . \tag{73}$$

Now we would like to construct a supersymmetry multiplet that contains the gauge field A^a_m. The fermion in the multiplet should differ in spin by $\frac{1}{2}$ unit. To write a renormalizable theory, we must take this to be a spin-$\frac{1}{2}$ Weyl fermion. I will then define the *vector supermultiplet*

$$(A^a_m, \lambda^a_\alpha, D^a) \tag{74}$$

including the gauge field, a Weyl fermion in the adjoint representation of the gauge group, and an auxililary real scalar field, also in the adjoint representation, that will have no independent particle content. The particle content of this multiplet is one massless vector boson, with two transverse polarization states, and one massless fermion and antifermion, for each generator of the gauge group. The fermion is often called a *gaugino*. The number of physical states is again equal between bosons and fermions.

The supersymmetry transformations for this multiplet are

$$\begin{aligned}
\delta_\xi A^{am} &= [\xi^\dagger \overline{\sigma}^m \lambda^a + \lambda^{\dagger a} \overline{\sigma}^m \xi] \\
\delta_\xi \lambda^a &= [i\sigma^{mn} F^a_{mn} + D^a]\xi \\
\delta_\xi \lambda^{\dagger a} &= \xi^\dagger [i\overline{\sigma}^{mn} F^a_{mn} + D^a] \\
\delta_\xi D^a &= -i[\xi^\dagger \overline{\sigma}^m \mathcal{D}_m \lambda^a - \mathcal{D}_m \lambda^{\dagger a} \overline{\sigma}^m \xi]
\end{aligned} \tag{75}$$

where

$$\sigma^{mn} = \frac{1}{4}(\sigma^m \overline{\sigma}^n - \sigma^n \overline{\sigma}^m) \ . \tag{76}$$

I encourage you to verify that these tranformations obey the algebra

$$[\delta_\xi, \delta_\eta] = 2i \left(\xi^\dagger \overline{\sigma}^m \eta - \eta^\dagger \overline{\sigma}^m \xi\right) \partial_m + \delta_\alpha \ , \tag{77}$$

where δ_α is a gauge tranformation with the gauge parameter

$$\alpha = -2i(\xi^\dagger \overline{\sigma}^m \eta - \eta^\dagger \overline{\sigma}^m \xi) A^a_m \ . \tag{78}$$

Acting on λ^a, the extra term δ_α in (77) can be combined with the translation to produce the commutation relation

$$[\delta_\xi, \delta_\eta]\, \lambda^a = 2i \left(\xi^\dagger \overline{\sigma}^m \eta - \eta^\dagger \overline{\sigma}^m \xi\right) (\mathcal{D}_m \lambda)^a \ . \tag{79}$$

This rearrangement applies also for the auxiliary field D^a and for any matter field that tranforms linearly under G. The gauge field A^{am} does not satisfy this last criterion; instead, we find

$$\begin{aligned}
[\delta_\xi, \delta_\eta] A^a_m &= 2i(\xi^\dagger \overline{\sigma}^n \eta - \eta^\dagger \overline{\sigma}^n \xi)(\partial_n A^a_m - \mathcal{D}_m A_n) \\
&= 2i(\xi^\dagger \overline{\sigma}^n \eta - \eta^\dagger \overline{\sigma}^n \xi)\, F^a_{nm}
\end{aligned} \tag{80}$$

The proof that (75) satisfies the supersymmetry algebra is more tedious than for (29), but it is not actually difficult. For the transformation of λ^a we need both the Fierz identity (30) and the relation

$$\eta_\alpha \xi_\beta - (\xi \leftrightarrow \eta) = -(\xi^T c \sigma_{pq} \eta)(\sigma^{pq} c)_{\alpha\beta} \ . \tag{81}$$

The matrices $\sigma^{pq}c$ and $c\overline{\sigma}^{pq}$ are symmetric in their spinor indices.

Again, the transformation laws leave a simple Lagrangian invariant. For the vector supermultiplet, this Lagrangian is that of the renormalizable Yang-Mills theory including the gaugino:

$$Ł_F = -\frac{1}{4}(F^a_{mn})^2 + \lambda^{\dagger a} i\overline{\sigma} \cdot \mathcal{D}\lambda^a + \frac{1}{2}(D^a)^2 \tag{82}$$

The kinetic term for D^a contains no derivatives, so this field will be a Lagrange multiplier.

The vector supermultiplet can be coupled to matter particles in chiral supermultiplets. To do this, we must first modify the transformation laws of the chiral supermultiplet so that the commutators of supersymmetry transformations obey (77) or (79). The modified transformation laws are:

$$\begin{aligned} \delta_\xi \phi &= \sqrt{2}\xi^T c\psi \\ \delta_\xi \psi &= \sqrt{2} i\sigma^n c\xi^* \mathcal{D}_n \phi + \sqrt{2} F\xi \\ \delta_\xi F &= -\sqrt{2} i\xi^\dagger \overline{\sigma}^m \mathcal{D}_m \psi - 2g\xi^\dagger c\lambda^{a*} t^a \phi \end{aligned} \tag{83}$$

In this formula, the chiral fields ϕ, ψ, F must belong to the same representation of G, with t^a a representation matrix in that representation. From the transformation laws, we can construct the Lagrangian. Start from (31), replace the derivatives by covariant derivatives, add terms to the Lagrangian involving the λ^a to cancel the supersymmetry variation of these terms, and then add terms involving D^a to cancel the remaining supersymmetry variation of the λ^a terms. The result is

$$\begin{aligned} Ł_D = \mathcal{D}^m \phi^* \mathcal{D}_m \phi + \psi^\dagger i\overline{\sigma} \cdot \mathcal{D}\psi + F^* F \\ -\sqrt{2} g(\phi^* \lambda^{aT} t^a c\psi - \psi^\dagger c\lambda^{a*} t^a \phi) + gD^a \phi^a t^a \phi \ . \end{aligned} \tag{84}$$

The proof that this Lagrangian is supersymmetric, $\delta_\xi Ł = 0$, is completely straightforward, but it requires a very large sheet of paper.

The gauge invariance of the theory requires the superpotential Lagrangian $Ł_W$ to be invariant under G as a global symmetry. Under this condition, $Ł_W$, which contains no derivatives, is invariant under (83) without modification. The combination of $Ł_F$, $Ł_D$, and $Ł_W$, with W a polynomial of degree at most 3, gives the most general renormalizable supersymmetric gauge theory.

As we did with the F field of the chiral multiplet, it is interesting to eliminate the Lagrange multiplier D^a. For the Lagrangian which is the sum of (82) and (84), the equation of motion for D^a is

$$D^a = -g\phi^* t^a \phi \ . \tag{85}$$

Eliminating D^a gives a second potential energy term proportional to $(D^a)^2$. This is the *D-term potential* promised below (40). I will write the result for a theory with several chiral multiplets:

$$V_D = \frac{1}{2} g^2 \left(\sum_k \phi_k^* t^a \phi_k \right)^2 \ . \tag{86}$$

As with the F-term potential, $V_D \geq 0$ and vanishes if and only if $D^a = 0$. It can be shown by an argument similar to (45) that

$$\langle 0 | \, D^a \, | 0 \rangle = 0 \tag{87}$$

unless supersymmetry is spontaneously broken.

It makes a nice illustration of this formalism to show how the Higgs mechanism works in supersymmetry. For definiteness, consider a supersymmetric gauge theory with the gauge group $U(1)$.

Introduce chiral supermultiplets ϕ_+, ϕ_-, and X, with charges $+1$, -1, and 0, respectively, and the superpotential

$$W = \lambda(\phi_+\phi_- - v^2)X \ . \tag{88}$$

The $F = 0$ equations are

$$F_X^* = (\phi_+\phi_- - v^2) = 0 \qquad F_\pm^* = \phi_\pm X = 0 \ . \tag{89}$$

To solve these equations, set

$$X = 0 \qquad \phi_+ = v/y \qquad \phi_- = vy \ , \tag{90}$$

where y is a complex-valued parameter. The $D = 0$ equation is

$$\phi_+^\dagger\phi_+ - \phi_-^\dagger\phi_- = 0 \ . \tag{91}$$

This implies $|y| = 1$. So y is a pure phase and can be removed by a $U(1)$ gauge transformation.

Now look at the pieces of the Lagrangian that give mass to gauge bosons, fermions, and scalars. The gauge field receives mass from the Higgs mechanism. To compute the mass, we can look at the scalar kinetic terms

$$\phi_+^\dagger(-\mathcal{D}^2)\phi_+ + \phi_-^\dagger(-\mathcal{D}^2)\phi_- = \cdots + \phi_+^\dagger(g^2A^2)\phi_+ + \phi_-^\dagger(g^2A^2)\phi_- \ . \tag{92}$$

Putting in the vacuum expectation values $\phi_+ = \phi_- = v$, we find

$$m^2 = 4g^2v^2 \tag{93}$$

for the vector fields. The mode of the scalar field

$$\delta\phi_+ = \eta/\sqrt{2} \qquad \delta\phi_- = -\eta/\sqrt{2} \ , \tag{94}$$

with η real, receives a mass from the D-term potential energy

$$\frac{g^2}{2}(\phi_+^\dagger\phi_+ - \phi_-^\dagger\phi_-)^2 \tag{95}$$

Expanding to quadratic order in η, we see that η also receives the mass $m^2 = 4g^2v^2$. The corresponding mode for η imaginary is the infinitesimal version of the phase rotation of y that we have already gauged away below (91). The mode of the fermion fields

$$\delta\psi_+ = \chi/\sqrt{2} \qquad \delta\psi_- = -\chi/\sqrt{2} \tag{96}$$

mixes with the gaugino through the term

$$-\sqrt{2}g(\phi_+^\dagger\lambda^T c\psi_+ - \phi_-^\dagger\lambda^T c\psi_-) + h.c. \tag{97}$$

Putting in the vacuum expectation values $\phi_+ = \phi_- = v$, we find a Dirac mass with the value

$$m = 2gv \tag{98}$$

In all, we find a massive vector boson, a massive real scalar, and a massive Dirac fermion, all with the mass $m = 2gv$. The system has four physical bosons and four physical fermions, all with the same mass, as supersymmetry requires.

2.5 The Vector Supermultiplet in Superspace

The vector supermultiplet has a quite simple representation in superspace. This multiplet turns out to be the answer to the question that we posed in our discussion of superspace in the previous section: When the chiral and antichiral components of a general superfield are removed, what is left over? To analyze this issue, I will write a Lagrangian containing a local symmetry that allows us to gauge away the chiral and antichiral components of this superfield. Let $V(x,\theta,\overline{\theta})$ be a real-valued superfield, acted on by a local gauge transformation in superspace

$$\delta V = -\frac{i}{g}(\Lambda - \Lambda^\dagger) \tag{99}$$

where Λ is a chiral superfield and $\Lambda^\dagger$ is its conjugate. Since Λ satisfies (60), its expansion in powers of θ contains

$$\Lambda(x,\theta,\overline{\theta}) = \Lambda(x + i\overline{\theta}\overline{\sigma}\theta, \theta) = \alpha(x) + \cdots + i\overline{\theta}\overline{\sigma}^m\theta\partial_m\alpha(x) + \cdots \tag{100}$$

The general superfield V contains a term[†]

$$V(x,\theta,\overline{\theta}) = \cdots + 2\overline{\theta}\overline{\sigma}^m\theta\, A_m(x) + \cdots \tag{101}$$

So the superfield V contains a space-time vector field $A_m(x)$, and under (99), A_m transforms as

$$\delta A_m = \frac{1}{g}\partial_m(\mathrm{Re}\,\alpha)\ . \tag{102}$$

This is just what we would like for an Abelian gauge field. So we should accept (99) as the generalization of the Abelian gauge transformation to superspace.

The real-valued superfield transforming under (99) is called a *vector superfield*. To understand its structure, use the gauge transformation to remove all components with powers of θ or $\overline{\theta}$ only. This choice is called *Wess-Zumino gauge* [33]. What remains after this gauge choice is

$$V(x,\theta,\overline{\theta}) = 2\overline{\theta}\overline{\sigma}^m\theta\, A_m(x) + 2\overline{\theta}^2\theta^T c\lambda - 2\theta^2\overline{\theta}^T c\lambda^* + \theta^2\overline{\theta}^2 D\ . \tag{103}$$

This expression has exactly the field content of the Abelian vector supermultiplet (A_m, λ, D).

This gauge multiplet can be coupled to matter described by chiral superfields. For the moment, I will continue to discuss the Abelian gauge theory. For a chiral superfield Φ with charge Q, the gauge transformation

$$\delta\Phi = iQ\Lambda\Phi \tag{104}$$

contains a standard Abelian gauge transformation with gauge parameter $\mathrm{Re}\,\alpha(x)$ and also preserves the chiral nature of Φ. Then the superspace Lagrangian

$$\int d^2\theta d^2\overline{\theta}\ \Phi^\dagger e^{gQV}\Phi \tag{105}$$

is gauge-invariant. Using the representation (103) and the rules (62), it is straightforward to carry out the integrals explicitly and show that (105) reduces to (84), with $t^a = Q$ for this Abelian theory.

We still need to construct the pure gauge part of the Lagrangian. To do this, first note that, because a quantity antisymmetrized on three Weyl fermion indices vanishes,

$$\overline{D}_\alpha\overline{D}^2 X = 0 \tag{106}$$

[†]The factor 2 in this equation is convenient but disagrees with some standard treatments, *e.g.*, [7].

for any superfield X. Thus, acting with $\overline{D}^2$ makes any superfield a chiral superfield. The following is a chiral superfield that also has the property that its leading component is the gaugino field $\lambda(x)$:

$$W_\alpha = -\frac{1}{8}\overline{D}^2 (Dc)_\alpha V \ . \tag{107}$$

Indeed, working this out in full detail, we find that $W_\alpha = W_\alpha(x + i\overline{\theta}\sigma\theta, \theta)$, with

$$W_\alpha(x,\theta) = \lambda_\alpha + \left[(i\sigma^{mn}F_{mn} + D)\theta\right]_\alpha + \theta^T c\theta \left[\partial_m \lambda^* i\overline{\sigma}^m c\right]_\alpha \ . \tag{108}$$

The chiral superfield W_α is the superspace analogue of the electromagnetic field strength. The Lagrangian

$$\int d^2\theta \, \frac{1}{2} W^T c W \tag{109}$$

reduces precisely to the Abelian version of (82). It is odd that the kinetic term for gauge fields is an F-term rather than a D-term. It turns out that this term can be renormalized by loop corrections as a consequence of the trace anomaly [34]. However, the restricted form of the correction has implications, both some simple ones that I will discuss later in Section 4.3 and and more profound implications discussed, for example, in [35,36].

I will simply quote the generalizations of these results to the non-Abelian case. The gauge transformation of a chiral superfield in the representation R of the gauge group is

$$\Phi \to e^{i\Lambda^a t^a}\Phi \qquad \Phi^\dagger \to \Phi^\dagger e^{-i\Lambda^{\dagger a} t^a} \ , \tag{110}$$

where Λ^a is a chiral superfield in the adjoint representation of G and t^a is is the representation of the generators of G in the representation R. The gauge transformation of the vector superfield is

$$e^{gV^a t^a} \to e^{i\Lambda^{\dagger a} t^a} e^{gV^a t^a} e^{-i\Lambda^a t^a} \tag{111}$$

Then the Lagrangian

$$\int d^2\theta d^2\overline{\theta} \, \Phi^\dagger e^{gV^a t^a} \Phi \tag{112}$$

is locally gauge-invariant. Carrying out the integrals in the gauge (103) reduces this Lagrangian to (84).

The form of the field strength superfield is rather more complicated than in the Abelian case,

$$W^a_\alpha t^a = -\frac{1}{8g}\overline{D}^2 e^{-gV^a t^a} (Dc)_\alpha e^{gV^a t^a} \tag{113}$$

In Wess-Zumino gauge, this formula does reduce to the non-Abelian version of (108),

$$W^a_\alpha(x,\theta) = \lambda^a_\alpha + \left[(i\sigma^{mn}F^a_{mn} + D^a)\theta\right]_\alpha + \theta^T c\theta \left[\mathcal{D}_m \lambda^{*a} i\overline{\sigma}^m c\right]_\alpha \ . \tag{114}$$

Then the Lagrangian

$$\int d^2\theta \, \mathrm{tr}[W^T c W] \tag{115}$$

reduces neatly to (82).

The most general renormalizable supersymmetric Lagrangian can be built out of these ingredients. We need to put together the Lagrangian (115), plus a term (112) for each matter chiral superfield, plus a superpotential Lagrangian to represent the scalar field potential energy. These formulae can be generalized to the case of a nonlinear sigma model on a Kähler manifold, with the gauge symmetry associated with an isometry of this target space. For the details, see [7].

2.6 R-Symmetry

The structure of the general superspace action for a renormalizable theory of scalar and fermion fields suggests that this theory has a natural continuous symmetry.

The superspace Lagrangian is

$$Ł = \int d^2\theta \, \mathrm{tr}[W^T c W] + \int d^4\theta \, \Phi^\dagger e^{gV\cdot t}\Phi + \int d^2\theta \, W(\Phi) + \int d^2\overline{\theta} \, (W(\Phi))^\dagger \ . \tag{116}$$

Consider first the case in which $W(\phi)$ contains only dimensionless parameters and is therefore a cubic polynomial in the scalar fields. Then Ł is invariant under the $U(1)$ symmetry

$$\Phi_k(x,\theta) \to e^{-i2\alpha/3}\Phi_k(x, e^{i\alpha}\theta) \ , \quad V^a(x,\theta,\overline{\theta}) \to V^a(x, e^{i\alpha}\theta, e^{-i\alpha}\overline{\theta}) \tag{117}$$

or, in components,

$$\phi_k \to e^{-i2\alpha/3}\phi_k \ , \quad \psi_k \to e^{i\alpha/3}\psi_k \ , \quad \lambda^a \to e^{-i\alpha}\lambda^a \ , \tag{118}$$

and the gauge fields are invariant. This transformation is called *R-symmetry*. Under R-symmetry, the charges of bosons and fermions differ by 1 unit, in such a way that that the gaugino and superpotential vertices have zero net charge.

Since all left-handed fermions have the same charge under (118), the R-symmetry will have an axial vector anomaly. It can be shown that the R-symmetry current (of dimension 3, spin 1) forms a supersymmetry multiplet together with the supersymmetry current (dimension $\frac{7}{2}$, spin $\frac{3}{2}$) and the energy-momentum tensor (dimension 4, spin 2) [37]. All three currents have perturbation-theory anomalies; the anomaly of the energy-momentum tensor is the trace anomaly, associated with the breaking of scalar invariance by coupling constant renormalization. The R-current anomaly is thus connected to the running of coupling constants and gives a useful formal approach to study this effect in supersymmetric models.

It is often possible to combine the transformation (117) with other apparent $U(1)$ symmetries of the theory to define a non-anomalous $U(1)$ R-symmetry. Under such a symmetry, we will have

$$\Phi_k(x,\theta) \to e^{-i\beta_k}\Phi_k(x, e^{i\alpha}\theta) \ , \quad \text{such that} \quad W(x,\theta) \to e^{2i\alpha}W(x, e^{i\alpha}\theta) \ . \tag{119}$$

Such symmetries also often arise in models in which the superpotential has dimensionful coefficients.

In models with extended, $N > 1$, supersymmetry, the R-symmetry group is also extended, to $SU(2)$ for $N = 2$ and to $SU(4)$ for $N = 4$ supersymmetry.

3 The Minimal Supersymmetric Standard Model

3.1 Particle Content of the Model

Now we have all of the ingredients to construct a supersymmetric generalization of the Standard Model. To begin, let us construct a version of the Standard Model with exact supersymmetry. To do this, we assign the vector fields in the Standard Model to vector supermultiplets and the matter fields of the Standard Model to chiral supermultiplets.

The vector supermultiplets correspond to the generators of $SU(3) \times SU(2) \times U(1)$. In these lectures, I will refer to the gauge bosons of these groups as A^a_m, W^a_m, and B_m, respectively. I will

represent the Weyl fermion partners of these fields as $\widetilde{g}^a$, $\widetilde{w}^a$, $\widetilde{b}$. I will call these fields the *gluino*, *wino*, and *bino*, or, collectively, *gauginos*. In the later parts of these lectures, I will drop the tildes over the gaugino fields when they are not needed for clarity.

I will assign the quarks and leptons to be fermions in chiral superfields. I will use the convention presented in Section 1.3 of considering left-handed Weyl fermions as the basic particles and right-handed Weyl fermions as their antiparticles. In the Standard Model, the left-handed fields in a fermion generation have the quantum numbers

$$L = \begin{pmatrix} \nu \\ e \end{pmatrix} \qquad \overline{e} \qquad Q = \begin{pmatrix} u \\ d \end{pmatrix} \qquad \overline{u} \qquad \overline{d} \tag{120}$$

The field $\overline{e}$ is the left-handed positron; the fields $\overline{u}$, $\overline{d}$ are the left-handed antiquarks. The right-handed Standard Model fermion fields are the conjugates of these fields. To make a generalization to supersymmetry, we will extend each of the fields in (120)—for each of the three generations—to a chiral supermultiplet. I will use the symbols

$$\widetilde{L} \qquad \widetilde{\overline{e}} \qquad \widetilde{Q} \qquad \widetilde{\overline{u}} \qquad \widetilde{\overline{d}} \tag{121}$$

to represent both the supermultiplets and the scalar fields in these multiplets. Again, I will drop the tilde if it is unambiguous that I am referring to the scalar partner rather than the fermion. The scalar particles in these supermultiplets are called *sleptons* and *squarks*, collectively, *sfermions*.

What about the Higgs field? The Higgs field of the Standard Model should be identified with a complex scalar component of a chiral supermultiplet. But it is ambiguous what the quantum numbers of this multiplet should be. In the Standard Model, the Higgs field is a color singlet with $I = \frac{1}{2}$, but we can take the hypercharge of this field to be either $Y = +\frac{1}{2}$ or $Y = -\frac{1}{2}$, depending on whether we take the positive hypercharge field or its conjugate to be primary. In a supersymmetric model, the choice matters. The superpotential is an analytic function of superfields, so it can only contain the field, not the conjugate. Then different Higgs couplings will be allowed depending on the choice that we make.

The correct solution to this problem is to include *both* possibilities, That is, we include a Higgs supermultiplet with $Y = +\frac{1}{2}$ and a second Higgs supermultiplet with $Y = -\frac{1}{2}$. I will call the scalar components of these multiplets H_u and H_d, respectively:

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} \qquad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} \tag{122}$$

I will refer to the Weyl fermion components with these quantum numbers as $\widetilde{h}_u$, $\widetilde{h}_d$. These fields or particles are called *Higgsinos*.

I will argue below that it is necessary to include both Higgs fields in order to obtain all of the needed couplings in the superpotential. However, there is another argument. The axial vector anomaly of one $U(1)$ and two $SU(2)$ currents (Fig. 5) must vanish to maintain the gauge invariance of the model. In the Standard Model, the anomaly cancels nontrivially between the quarks and the leptons. In the supersymmetric generalization of the Standard Model, each Higgsino makes a nonzero contribution to this anomaly. These contributions cancel if we include a pair of Higgsinos with opposite hypercharge.

3.2 Grand Unification

Before writing the Lagrangian in detail, I would like to point out that there is an interesting conclusion that follows from the quantum number assignments of the new particles that we have

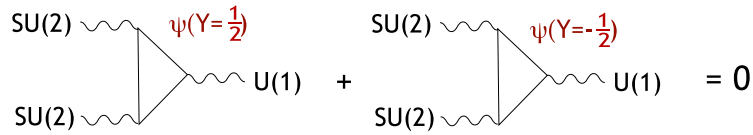

Figure 5: The anomaly cancellation that requires two doublets of Higgs fields in the MSSM.

introduced to make the Standard Model supersymmetric.

An attractive feature of the Standard Model is that the quarks and leptons of each generation fill out multiplets of the simple gauge group $SU(5)$. This suggests a very beautiful picture, called *grand unification*, in which $SU(5)$, or a group such as $SO(10)$ or E_6 for which this is a subgroup, is the fundamental gauge symmetry at very short distances. This unified symmetry will be spontaneously broken to the Standard Model gauge group $SU(3) \times SU(2) \times U(1)$.

For definiteness, I will examine the model in which the grand unified symmetry group is $SU(5)$. The generators of $SU(5)$ can be represented as 5×5 Hermitian matrices acting on the 5-dimensional vectors in the fundamental representation. To see how the Standard Model is embedded in $SU(5)$, it is convenient to write these matrices as blocks with 3 and 2 rows and columns. Then the Standard Model generators can be identified as

$$SU(3) \ : \ \begin{pmatrix} t^a & \\ & 0 \end{pmatrix} \ ; \quad SU(2) \ : \ \begin{pmatrix} 0 & \\ & \sigma^a/2 \end{pmatrix} \ ; \quad U(1) \ : \ \sqrt{\frac{3}{5}} \begin{pmatrix} -\frac{1}{3}\mathbf{1} & \\ & \frac{1}{2}\mathbf{1} \end{pmatrix} \ . \tag{123}$$

In these expressions, t^a is an $SU(3)$ generator, $\sigma^a/2$ is an $SU(2)$ generator, and all of these matrices are normalized to $\mathrm{tr}[T^A T^B] = \frac{1}{2}\delta^{AB}$. We should identify the last of these matrices with $\sqrt{3/5}\, Y$.

The symmetry-breaking can be caused by the vacuum expectation value of a Higgs field in the adjoint representation of $SU(5)$. The expectation value

$$\langle \Phi \rangle = V \cdot \begin{pmatrix} -\frac{1}{3}\mathbf{1} & \\ & \frac{1}{2}\mathbf{1} \end{pmatrix} \tag{124}$$

commutes with the generators in (123) and fails to commute with the off-diagonal generators. So this vacuum expectation value gives mass to the off-diagonal generators and breaks the gauge group to $SU(3) \times SU(2) \times U(1)$.

Matter fermions can be organized as left-handed Weyl fermions in the $SU(5)$ representations $\overline{5}$ and 10. The $\overline{5}$ is the conjugate of the fundamental representation of $SU(5)$; the 10 is the antisymmetric matrix with two 5 indices.

$$\overline{5} \ : \ \begin{pmatrix} \overline{d} \\ \overline{d} \\ \overline{d} \\ e \\ \nu \end{pmatrix}_L \ ; \qquad 10 \ : \ \begin{pmatrix} 0 & \overline{u} & \overline{u} & u & d \\ & 0 & \overline{u} & u & d \\ & & 0 & u & d \\ & & & 0 & \overline{e} \\ & & & & 0 \end{pmatrix}_L \tag{125}$$

It is straightforward to check that each entry listed has the quantum numbers assigned to that field in the Standard Model. To compute the hypercharges, we act on the $\overline{5}$ with (-1) times the hypercharge generator in (123), and we act on the 10 with the hypercharge generator on each index. This gives the standard results, for example, $Y = +\frac{1}{3}$ for the $\overline{d}$ and $Y = -\frac{1}{3} + \frac{1}{2} = \frac{1}{6}$ for u and d.

The $SU(5)$ covariant derivative is

$$\mathcal{D}_m = (\partial_m - ig_U A_m^A T^A) \ , \tag{126}$$

where g_U is the $SU(5)$ gauge coupling. There is only room for one value here. So this model predicts that the three Standard Model gauge couplings are related by

$$g_3 = g_2 = g_1 = g_U \ , \tag{127}$$

where

$$g_3 = g_s \qquad g_2 = g \qquad g_1 = \sqrt{\frac{5}{3}} g' \ . \tag{128}$$

Clearly, this prediction is not correct for the gauge couplings that we measure in particle physics.

However, there is a way to save this prediction. In quantum field theory, coupling constants are functions of length scale and change their values significantly from one scale to another by renormalization group evolution. It is possible that the values of g', g, and g_s that we measure could evolve at very short distances into values that obey (127).

I will now collect the formulae that we need to analyze this question. Let

$$\alpha_i = \frac{g_i^2}{4\pi} \tag{129}$$

for $i = 1, 2, 3$. The one-loop renormalization group equations for gauge couplings are

$$\frac{dg_i}{d\log Q} = -\frac{b_i}{(4\pi)^2} g_i^3 \qquad \text{or} \qquad \frac{d\alpha_i}{d\log Q} = -\frac{b_i}{(2\pi)} \alpha_i^2 \ . \tag{130}$$

For $U(1)$, the coefficient b_1 is

$$b_1 = -\frac{2}{3} \sum_f \frac{3}{5} Y_f^2 - \frac{1}{3} \sum_b \frac{3}{5} Y_b^2 \ , \tag{131}$$

where the two sums run over the multiplets of left-handed Weyl fermions and complex-valued bosons. The factors $\frac{3}{5}Y^2$ are the squares of the $U(1)$ charges defined by (123). For non-Abelian groups, the expressions for the b coefficients are

$$b = -\frac{11}{3} C_2(G) - \frac{2}{3} \sum_f C(r_f) - \frac{1}{3} \sum_b C(r_b) \ , \tag{132}$$

where $C_2(G)$ and $C(r)$ are the standard group theory coefficients. For $SU(N)$,

$$C_2(G) = C(G) = N \ , \quad C(N) = \frac{1}{2} \ . \tag{133}$$

The solution of the renormalization group equation (130) is

$$\alpha^{-1}(Q) = \alpha^{-1}(M) - \frac{b_i}{2\pi} \log \frac{Q}{M} \ . \tag{134}$$

Now consider the situation in which the three couplings g_i become equal at the mass scale M_U, the mass scale of $SU(5)$ symmetry breaking. Let α_U be the value of the α_i at this scale. Using (134),

we can then determine the Standard Model couplings at any lower mass scale. The three $\alpha_i(Q)$ are determined by two parameters. We can eliminate those parameters and obtain the relation

$$\alpha_3^{-1} = (1+B)\alpha_2^{-1} - B\alpha_1^{-1} \tag{135}$$

where

$$B = \frac{b_3 - b_2}{b_2 - b_1} \ . \tag{136}$$

The values of the α_i are known very accurately at $Q = m_Z$ [38]:

$$\alpha_3^{-1} = 8.50 \pm 0.14 \qquad \alpha_2^{-1} = 29.57 \pm 0.02 \qquad \alpha_1^{-1} = 59.00 \pm 0.02 \ . \tag{137}$$

Inserting these values into (135), we find

$$B = 0.716 \pm 0.005 \pm 0.03 \ . \tag{138}$$

In this formula, the first error is that propagated from the errors in (137) and the second is my estimate of the systematic error from neglecting the two-loop renormalization group coefficients and other higher-order corrections.

We can compare the value of B in (138) to the values of (136) from different models. The hypothesis that the three Standard Model couplings unify is acceptable only if the gauge theory that describes physics between m_Z and M_U gives a value of B consistent with (138). The minimal Standard Model fails this test. The values of the b_i are

$$\begin{aligned} b_3 &= 11 - \frac{4}{3}n_g \\ b_2 &= \frac{22}{3} - \frac{4}{3}n_g - \frac{1}{6}n_h \\ b_1 &= \qquad - \frac{4}{3}n_g - \frac{1}{10}n_h \end{aligned} \tag{139}$$

where n_g is the number of generations and n_h is the number of Higgs doublets. Notice that n_g cancels out of (136). This is to be expected. The Standard Model fermions form complete representations of $SU(5)$, and so their renormalization effects cannot lead to differences among the three couplings. For the minimal case $n_h = 1$ we find $B = 0.53$. To obtain a value consistent with (138), we need $n_h = 6$.

We can redo this calculation in the minimal supersymmetric version of the Standard Model. First of all, we should rewrite (132) for a supersymmetric model with one vector supermultiplet, containing a vector and a Weyl fermion in the adjoint representation, and a set of chiral supermultiplets indexed by k, each with a Weyl fermion and a complex boson. Then (132) becomes

$$\begin{aligned} b_i &= \frac{11}{3}C_2(G) - \frac{2}{3}C_2(G) - \left(\frac{2}{3} + \frac{1}{3}\right)\sum_k C(r_k) \\ &= 3C_2(G) - \sum_k C(r_k) \end{aligned} \tag{140}$$

The formula (131) undergoes a similar rearrangement. Inserting the values of the $C(r_k)$ for the fields of the Standard Model, we find

$$\begin{aligned} b_3 &= 9 - 2n_g \\ b_2 &= 6 - 2n_g - \frac{1}{2}n_h \\ b_1 &= \quad - 2n_g - \frac{3}{10}n_h \end{aligned} \tag{141}$$

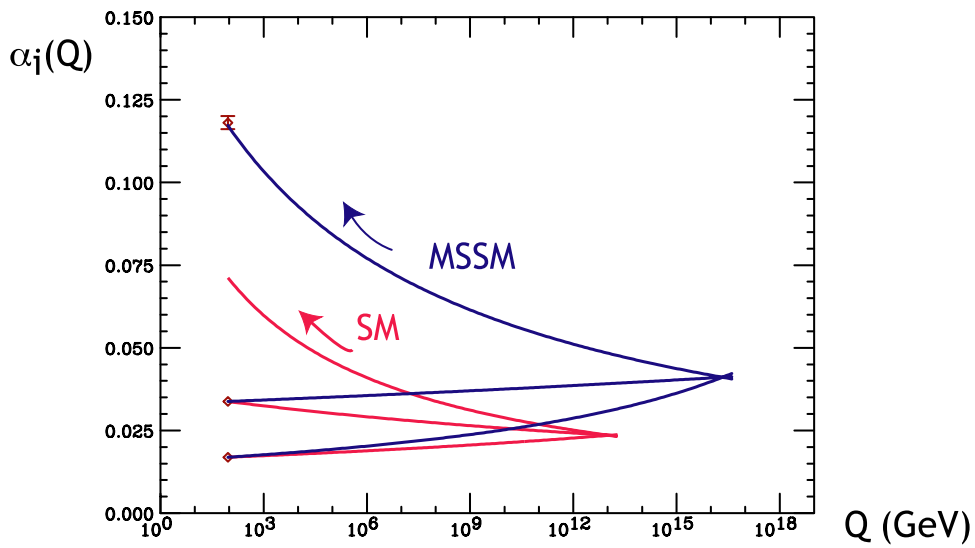

Figure 6: Prediction of the $SU(3)$ gauge coupling α_s from the electroweak coupling constants using grand unification, in the Standard Model and in the MSSM.

For the minimal Higgs content $n_h = 2$, this gives

$$B = \frac{5}{7} = 0.714 \tag{142}$$

in excellent agreement with (138).

In Fig. 6, I show the unification relation pictorially. The three data points on the the left of the figure represent the measured values of the three couplings (137). Starting from the values of α_1 and α_2, we can integrate (130) up to the scale at which these two couplings converge. Then we can integrate the equation for α_3 back down to $Q = m_Z$ and see whether the result agrees with the measured value. The lower set of curves presents the result for the Standard Model with $n_h = 1$. The upper set of curves shows the result for the supersymmetric extension of the Standard Model with $n_h = 2$. This choice gives excellent agreement with the measured value of α_s.

Actually, I slightly overstate the case for supersymmetry by ignoring two-loop terms in the renormalization group equations, and also by integrating these equations all the way down to m_Z even though, from searches at high-energy colliders, most of the squarks and gluinos must be heavier than 300 GeV. A more accurate prediction of $\alpha_s(m_Z)$ from the electroweak coupling constants gives a slightly higher value, 0.13 instead of 0.12. However, these corrections could easily be compensated by similar corrections to the upper limit of the integration, following the details of the particle spectrum at the grand unification scale. For a more detailed formal analysis of these corrections, see [39], and for a recent evaluation of their effects, see [40]. It remains a remarkable fact that the minimal supersymmetric extension of the Standard Model is approximately compatible with grand unification 'out of the box', with no need for further model-building.

3.3 Construction of the Lagrangian

Now I would like to write the full Lagrangian of the minimal supersymmetric extension of the Standard Model, which I will henceforth call the MSSM.

The kinetic terms and gauge couplings of the MSSM Lagrangian are completely determined by supersymmetry, the choice of the gauge group $SU(3) \times SU(2) \times U(1)$, and the choice of the quantum numbers of the matter fields. The Lagrangian is a sum of terms of the forms (82) and (84). Up to this point, the only parameters that need to be introduced are the gauge couplings g_1, g_2, and g_3.

Next, we need a superpotential W. The superpotential is the source of nonlinear fermion-scalar interactions, so we should include the appropriate terms to generate the Higgs Yukawa couplings needed to give mass to the quarks and leptons. The appropriate choice is

$$W_Y = y_d^{ij} \overline{d}^i H_{d\alpha} \epsilon_{\alpha\beta} Q_\beta^j + y_e^{ij} \overline{e}^i H_{d\alpha} \epsilon_{\alpha\beta} L_\beta^j - y_u^{ij} \overline{u}^i H_{u\alpha} \epsilon_{\alpha\beta} Q_\beta^j \ . \tag{143}$$

The notation for the quark and lepton multiplets is that in (120); the indices $i, j = 1, 2, 3$ run over the three generations. The indices $\alpha, \beta = 1, 2$ run over SU(2) isospin indices. Notice that the first two terms require a Higgs field H_d with $Y = -\frac{1}{2}$, while the third term requires a Higgs field H_u with $Y = \frac{1}{2}$. If we leave out one of the Higgs multiplets, some quarks or leptons will be left massless. This is the second argument that requires two Higgs fields in the MSSM.

I have written (143) including the most general mixing between left- and right-handed quarks and leptons of different generations. However, as in the minimal Standard Model, we can remove most of this flavor mixing by appropriate field redefinitions. The coupling constants y_d, y_e, y_u are general 3×3 complex-valued matrices. Any such matrix can be diagonalized using two unitary transformations. Thus, we can write

$$y_d = W_d Y_d V_d^\dagger \qquad y_e = W_e Y_e V_e^\dagger \qquad y_u = W_u Y_u V_u^\dagger \ , \tag{144}$$

with W_a and V_a 3×3 unitary matrices and Y_a real, positive, and diagonal. The unitary transformations cancel out of the kinetic energy terms and gauge couplings in the Lagrangian, except that the W boson coupling to quarks is transformed

$$g u^\dagger \overline{\sigma}^m d W_m^+ \to g u^\dagger \overline{\sigma}^m (V_u^\dagger V_d) d \, W_m^+ \ . \tag{145}$$

From this equation, we can identify $(V_d^\dagger V_u) = V_{CKM}$, the Cabibbo-Kobayashi-Maskawa weak interaction mixing matrix. The Lagrangian term (143) thus introduces the remaining parameters of the Standard Model, the 9 quark and lepton masses (ignoring neutrino masses) and the 4 CKM mixing angles. The field redefinition (144) can also induce or shift a QCD theta parameter, so the MSSM, like the Standard Model, has a strong CP problem that requires an axion or another model-building solution [41].

There are several other terms that can be added to W. One possible contribution is a pure Higgs term

$$W_\mu = -\mu H_{d\alpha} \epsilon_{\alpha\beta} H_{u\beta} \ . \tag{146}$$

The parameter μ has the dimensions of mass, and consequently this *mu term* provides a supersymmetric contribution to the masses of the Higgs bosons. Because this term is in the superpotential, it does not receive additive raditive corrections. Even in a theory that includes grand unification and energies scale of the order of 10^{16} GeV, we can set the parameter μ to be of order 100 GeV without finding this choice affected by large quantum corrections. We will see in Section 4.2 that the mu term is needed for phenomenological reasons. If $\mu = 0$, a Higgsino state will be massless and should have been detected already in experiments. It is odd that a theory whose fundamental mass scale

is the grand unification scale should require a parameter containing a weak interaction mass scale. I will present some models for the origin of this term in Section 3.5.

At this point, we have introduced two new parameters beyond those in the Standard Model. One is the value of μ. The other is the result of the fact that we have two Higgs doublets in the model. The ratio of the Higgs vacuum expectation values

$$\langle H_u \rangle \,/\, \langle H_d \rangle \equiv \tan\beta \tag{147}$$

will appear in many of the detailed predictions of the MSSM.

There are still more superpotential terms that are consistent with the Standard Model gauge symmetry and quantum numbers. These are

$$\begin{aligned} W_{\not R} = \eta_1 \epsilon_{ijk} \overline{u}_i \overline{d}_j \overline{d}_k + \eta_2 \overline{d} \epsilon_{\alpha\beta} L_\alpha Q_\beta \\ + \eta_3 \overline{e} \epsilon_{\alpha\beta} L_\alpha L_\beta + \eta_4 \epsilon_{\alpha\beta} L_\alpha H_{u\beta} \ . \end{aligned} \tag{148}$$

Here i, j, k are color indices, α, β are isospin indices, and arbitrary generation mixing is also possible. These terms violate baryon and lepton number through operators with dimensionless coefficients. In constructing supersymmetric models, it is necessary either to forbid these terms by imposing appropriate discrete symmetries or to arrange by hand that some of the dangerous couplings are extremely small [42].

If baryon number B and lepton number L are conserved in a supersymmetric model, this model respects a discrete symmetry called R-parity,

$$R = (-1)^{3B+L+2J} \ . \tag{149}$$

Here $(3B)$ is quark number and J is the spin of the particle. This quantity is constructed so that $R = +1$ on the particles of the Standard Model (including the Higgs bosons) and $R = -1$ on their supersymmetry partners. R acts differently on particles of different spin in the same supermultiplet, so R-parity is a discrete subgroup of a continuous R-symmetry.

In a model with grand unification, there will be baryon number and lepton number violation, and so B and L cannot be used as fundamental symmetries. However, we can easily forbid most of the superpotential terms (148) by introducing a discrete symmetry that distinguishes the field H_d from the lepton doublets L_i. A similar strategy can be used to forbid the first, 3-quark, term. With these additional discrete symmetries, the MSSM, including all other terms considered up to this point, will conserve R-parity.

3.4 The Lightest Supersymmetric Particle

If R-parity is conserved, the lightest supersymmetric particle will be absolutely stable. This conclusion has an important implication for the relation of supersymmetry to cosmology. If a supersymmetric particle is stable for a time longer than the age of the universe, and if this particle is electrically neutral, that particle is a good candidate for the cosmic dark matter. In Sections 6.3 and 6.4, I will discuss in some detail the properties of models in which the lightest Standard Model superpartner is the dark matter particle.

However, this is not the only possibility. Over times much longer than those of particle physics experiments—minutes, years, or billions of years—we need to consider the possibility that the lightest Standard Model superpartner will decay to a particle with only couplngs of gravitational strength. Complete supersymmetric models of Nature must include a superpartner of the graviton, a spin-$\frac{3}{2}$

particle called the *gravitino*. In a model with exact supersymmetry, the gravitino will be massless, but in a model with spontaneously broken supersymmetry, the gravitino acquires a mass through an analogue of the Higgs mechanism. If the supersymmetry breaking is induced by one dominant F-term, the value of this mass is [43]

$$m_{3/2} = \frac{8\pi}{3} \frac{\langle F \rangle}{m_{\rm Pl}} \ . \tag{150}$$

This expression is of the same order of magnitude as the expressions for Standard Model superpartner masses that I will give in Section 3.6. In string theory and other unified models, there may be additional Standard Model singlet fields with couplings of gravitation strength, called *moduli*, that might also be light enough that long-lived Standard Model superpartners could decay to them.

Supersymmetric models with R-parity conservation and dark matter, then, divide into two classes, according to the identity of the lightest supersymmetric particle—the LSP. On one hand, the LSP could be a Standard Model superpartner. Cosmology requires that this particle is neutral. Several candidates are available, including the fermionic partners of the photon, Z^0, and neutral Higgs bosons and the scalar partner of one of the neutrinos. In all cases, these particles will be weakly interacting; when they are produced at high-energy colliders, they should not make signals in a particle detector. On the other hand, the LSP could be the gravitino or another particle with only gravitational couplings. In that case, the lightest Standard Model superpartner could be a charged particle. Whether this particle is visible or neutral and weakly interacting, its decay should be included in the phenomenology of the model.

3.5 Models of Supersymmetry Breaking

There is still one important effect that is missing in our construction of the MSSM. The terms that we have written so far preserve exact supersymmetry. A fully supersymmetric model would contain a massless fermionic partner of the photon and a charged scalar particle with the mass of the electron. These particles manifestly do not exist. So if we wish to build a model of Nature with supersymmetry as a fundamental symmetry, we need to arrange that supersymmetry is spontaneously broken.

From the example of spontaneous symmetry breaking in the Standard Model, we would expect to do this by including in the MSSM a field whose vacuum expectation value leads to supersymmetry breaking. This is not as easy as it might seem. To explain why, I will first present some models of supersymmetry breaking.

The simplest model of supersymmetry breaking is the O'Raifeartaigh model [44], with three chiral supermultiplets ϕ_0, ϕ_1, ϕ_2 interacting through the superpotential

$$W = \lambda\phi_0 + m\phi_1\phi_2 + g\phi_0\phi_1^2 \ . \tag{151}$$

This superpotential implies the $F = 0$ conditions

$$\begin{aligned} 0 &= F_0^* = \lambda + g\phi_1^2 \\ 0 &= F_1^* = m\phi_2 + 2g\phi_0\phi_1 \\ 0 &= F_2^* = m\phi_1 \end{aligned} \tag{152}$$

The first and third equations contradict one another. It is impossible to satisfy both conditions, and so there is no supersymmetric vacuum state. This fulfils the condition for spontaneous supersymmetry breaking that I presented in Section 2.2.

This mechanism of supersymmetry breaking has an unwanted corollary. Because one combination of the scalar fields appears in two different constraints in (152), there must be an orthogonal combination that does not appear at all. This means that the F-term potential V_F has a surface of degenerate vacuum states. To see this explicitly, pick a particular vacuum solution

$$\phi_0 = \phi_1 = \phi_2 = 0 \ . \tag{153}$$

and expand the potential V_F about this point. There are 6 real-valued boson fields with masses

$$0 \ , \quad 0 \ , \quad m \ , \quad m \ , \quad \sqrt{m^2 - 2\lambda g} \ , \quad \sqrt{m^2 + 2\lambda g} \ . \tag{154}$$

These six fields do not pair into complex-valued fields; that is already an indication that supersymmetry is broken. The fermion mass term in (38) gives one Dirac fermion mass m and leaves one Weyl fermion massless. This massless fermion is the Goldstone particle associate with spontaneous supersymmetry breaking.

A property of these masses is that the sum rule for fermion and boson masses

$$\mathrm{str}[m^2] = \sum m_f^2 - \sum m_b^2 = 0 \tag{155}$$

remains valid even when supersymmetry is broken. This sum rule is the coefficient of the one-loop quadratic divergence in the vacuum energy. Since supersymmetry breaking does not affect the ultraviolet structure of the theory, this coefficient must cancel even if supersymmetry is spontaneously broken [45]. In fact, if Q is a conserved charge in the model, the sum rule is valid in each charge sector $Q = q$:

$$\mathrm{str}_q[m^2] = 0 \ . \tag{156}$$

In the O'Raifeartaigh model, supersymmetry is spontaneously broken by a nonzero expectation value of an F term. It is also possible to break supersymmetry with a nonzero expectation value of a D term. The D-term potential V_D typically has zeros. For example, in an $SU(3)$ supersymmetric Yang-Mills theory,

$$V_D = \frac{1}{2} \left(\sum_{3} \phi^\dagger t^a \phi - \sum_{\bar{3}} \overline{\phi} t^a \overline{\phi}^\dagger \right)^2 \tag{157}$$

and it is easy to find solutions in which the terms in parentheses sum to zero. However, it is not difficult to arrange a V_F such that the solutions of the $F = 0$ conditions do not coincide with the solutions of the $D = 0$ conditions. This leads to spontaneous symmetry breaking, again with the sum rule (156) valid at tree level.

Unfortunately, the sum rule (156) is a disaster for the prospect of finding a simple model of spontaneously broken supersymmetry that extends the Standard Model. For the charge sector of the d squarks, we would need all down-type squarks to have masses less than 5 GeV. For the charge sector of the charged leptons, we would need all sleptons to have masses less than 2 GeV.

3.6 Soft Supersymmetry Breaking

The solution to this problem is to construct models of spontaneously broken supersymmetry using a different strategy from the one that we use for electroweak symmetry breaking in the Standard Model. To break electroweak symmetry, we introduce a Higgs sector whose mass scale is the same as the scale of the fermion and gauge boson masses induced by the symmetry breaking. To break supersymmetry, however, we could introduce a new sector at a much higher mass scale, relying on a

weak coupling of the new sector to the Standard Model particles to communicate the supersymmetry breaking terms. In principle, a weak gauge interaction could supply this coupling. However, the default connection is through gravity. Gravity and supergravity couple to all fields. It can be shown that supersymmetry breaking anywhere in Nature is communicated to all other sectors through supergravity couplings.

We are thus led to the following picture, which produces a phenomenologically reasonable supersymmetric extension of the Standard Model: We extend the Standard Model fields to supersymmetry multiplets in the manner described in Section 3.1. We also introduce a *hidden sector* with no direct coupling to quark, leptons, and Standard Model gauge bosons. Supersymmetry is spontaneously broken in this hidden sector. A weak interaction coupling the two sectors then induces a supersymmetry-breaking effective interaction for the Standard Model particles and their superpartners. If Λ is the mass scale of the hidden sector, the supersymmetry breaking mass terms induced for the Standard Model sector are of the order of

$$m \sim \frac{\langle F \rangle}{M} \sim \frac{\Lambda^2}{M} \ ; \tag{158}$$

where M is the mass of the particle responsible for the weak connection between the two sectors. M is called the *messenger scale*. By default, the messenger is supergravity. Then $M = m_{\rm Pl}$ and $\Lambda \sim 10^{11}$ GeV. In this scenario, the superpartners acquire masses of the order of the parameter m in (158).

It remains true that the quarks, leptons, and gauge bosons cannot obtain mass until $SU(2) \times U(1)$ is broken. It is attractive to think that the symmetry-breaking terms that give mass to the superpartners cause $SU(2) \times U(1)$ to be spontaneously broken, at more or less the same scale. I will discuss a mechanism by which this can happen in Section 6.1. The weak interaction scale would then not be a fundamental scale in Nature, but rather one that arises dynamically from the hidden sector and its couplings.

The effective interaction that are generated by messenger exchange generally involve simple operators of low mass dimensions, to require the minimal number of powers of M in the denominator. These operators are *soft* perturbations of the theory, and so we say that the MSSM is completed by including *soft supersymmetry-breaking interactions*.

However, the supersymmetry-breaking terms induced in this model will not include all possible low-dimension operators. Since these interactions arise by coupling into a supersymmetry theory, they are formed by starting with a supersymmetric effective action and turning on F and D expectation values as spurions. Only a subset of the possible supersymmetry-breaking terms can be formed in this way [46]. By replacing a superfield Φ by $\theta^T c\theta \, \langle F \rangle$, we can convert

$$\begin{aligned} \int d^4\theta \, K(\Phi, \phi) &\to m^2 \phi^\dagger \phi \\ \int d^2\theta \, f(\Phi) W^T c W &\to m \lambda^T c \lambda \\ \int d^2 \, W(\Phi, \phi) &\to B\phi^2 + A\phi^3 \end{aligned} \tag{159}$$

However, as long as the ϕ theory is renormalizable, we cannot generate the terms

$$m\psi^T c\psi \ , \quad C\phi^* \phi^2 \ , \tag{160}$$

by turning on expectation values for F and D fields. Thus, we cannot generate supersymmetry-breaking interactions that are mass terms for the fermion field of a chiral multiplet or non-holomorphic cubic terms for the scalar fields.

There is another difficulty with terms of the form (160). In models with Standard Model singlet scalar fields, which typically occur in concrete models, these two interactions can generate new quadratic divergences when they appear in loop diagrams [46].

Here is the most general supersymmetry-breaking effective Lagrangian that can be constructed following the rule just given that is consistent with the gauge symmetries of the Standard Model:

$$\begin{aligned} Ł_{soft} = & -M_f^2|\widetilde{f}|^2 - \frac{1}{2}m_i\lambda_i^{Ta}c\lambda_i^a \\ & -(A_d y_d \widetilde{\overline{d}} H_{d\alpha}\epsilon_{\alpha\beta}\widetilde{Q}_\beta + A_e y_e \widetilde{\overline{e}} H_{d\alpha}\epsilon_{\alpha\beta}\widetilde{L}_\beta \\ & -A_u y_u \widetilde{\overline{u}} H_{u\alpha}\epsilon_{\alpha\beta}\widetilde{Q}_\beta - B\mu H_{d\alpha}\epsilon_{\alpha\beta}H_{u\beta}) - h.c. \end{aligned} \tag{161}$$

I have made the convention of scaling the A terms with the corresponding Yukawa couplings and scaling the B terms with μ. The parameters A and B then have the dimensions of mass and are expected to be of the order of m in (158).

For most of the rest of these lectures, I will represent the effects of the hidden sector and supersymmetry breaking simply by adding (161) to the supersymmetric Standard Model. I will then consider the MSSM to be defined by

$$Ł = Ł_F + Ł_D + Ł_W + Ł_{soft} \tag{162}$$

combining the pieces from (82), (84), (143), (146), and (161).

There are two problems with this story. The first is the μ term in the MSSM superpotential. This a supersymmetric term, and so μ can be arbitrarily large. To build a successful phenomenology of the MSSM, however, we need to have μ of the order of the weak scale. Ideally, μ should be parametrically equal to (158).

There are simple mechanisms that can solve this problem. A fundamental theory that leads to the renormalizable Standard Model at low energies can also contain higher-dimension operators suppressed by the high-energy mass scale. Associate this scale with the messenger scale. Then a supersymmetric higher-dimension operator in the superpotential

$$\int d^2\theta \, \frac{1}{M} S^2 H_d H_u \tag{163}$$

leads to a μ term if S acquires a vacuum expectation value. If S is a hidden sector field, we could find [47]

$$\mu = \frac{\langle S^2 \rangle}{M} \sim \frac{\Lambda^2}{M} \ , \tag{164}$$

A supersymmetric higher dimension contribution to the Kähler potential

$$\int d^4\theta \, \frac{1}{M} \Phi^\dagger H_d H_u \tag{165}$$

leads to a μ term if Φ acquires a vacuum expectation value in its F term. If Φ is a hidden sector field, we could find [48]

$$\mu = \frac{\langle F_\Phi \rangle}{M} \sim \frac{\Lambda^2}{M} \ , \tag{166}$$

In models with weak-coupling dynamics, higher-dimension operators are associated with the string or Planck scale; then, these mechanisms work most naturally if supergravity is the mediator. However, it is also possible to apply these strategies in models with strong-coupling dynamics in the hidden sector at an intermediate scale.

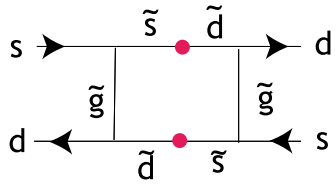

Figure 7: A dangerous contribution to K-$\overline{K}$ mixing involving gluino exchange and flavor mixing in the squark mass matrix.

Generating the μ term typically requires breaking all continuous R-symmetries of the model. This is unfortunate, because an R-symmetry might be helpful phenomenologically, for example, to keep gaugino masses small while allowing sfermion masses to become large, or because it might be difficult to break an R-symmetry using a particular explicit mechanism of supersymmetry breaking. In this case, it is necessary to add Standard Model singlet fields to the MSSM to allow all gaugino and Higgsino fields to acquire nonzero masses. Models of this type are presented in [49,50].

The second problem involves the flavor structure of the soft supersymmetry breaking terms. In writing (161), I did not write flavor indices. In principle, these terms could have flavor-mixing that is arbitrary in structure and different from that in (143). Then the flavor-mixing would not be transformed away when (143) is put into canonical form. However, flavor-mixing from the soft supersymmetry breaking terms is highly constrained by experiment. Contributions such as the one shown in Fig. 7 give contributions to K^0, D^0, and B^0 mixing, and to $\tau \to \mu\gamma$ and $\mu \to e\gamma$, that can be large compared to the measured values or limits. Theories of the origin of the soft terms in models of supersymmetry breaking should address this problem. For example, the models of *gauge-mediated* [52] and *anomaly-mediated* [53,54] supersymmetry breaking induce soft terms that depend only on the $SU(2) \times U(1)$ quantum number and are therefore automatically diagonal in flavor. A quite different solution, based on a extension of the MSSM with a continuous R-symmetry, is presented in [51].

If I assume that the soft supersymmetry-breaking Lagrangian is diagonal in flavor but is otherwise arbitrary, it introduces 22 new parameters. With arbitrary flavor and CP violation, it introduces over 100 new parameters. This seems a large amount of parameter freedom. I feel that it is not correct, though, to think of these as new fundamental parameters in physics. The soft Lagrangian is computed from the physics of the hidden sector, and so we might expect that these parameters are related to one another as a part of a theory of supersymmetry breaking. Indeed, the values of these parameters are the essential data from which we will infer the properties of the hidden sector and its new high energy interactions.

If supersymmetry is discovered at the weak interaction scale, it will be a key problem to measure the coefficients in the soft Lagrangian and to understand their pattern and implications. Most of my discussion in the next two sections will be devoted to the question of how the soft parameters can be determined from data at the LHC and ILC.

4 The Mass Spectrum of the MSSM

4.1 Sfermion Masses

Our first task in this program is to ask how the parameters of the MSSM Lagrangian are reflected in the mass spectrum of the superparticles. The relation between the MSSM parameters and the particle masses is surprisingly complicated, even at the tree level. For each particle, we will need to collect all of the pieces of the Lagrangian (162) that can contribute to the mass term. Some of these will be direct mass contributions; others will contain Higgs fields and contribute to the masses when these fields obtain their vacuum expectation values. In this discussion, and in the remainder of these lectures, I will ignore all flavor-mixing.

Begin with the squark and slepton masses. For light quarks and leptons, we can ignore the fermion masses and Higgs couplings. Even with this simplification, though, there are two sources for the scalar masses. One is the soft mass term

$$Ł_{soft} = -M_f^2 |\widetilde{f}|^2 \ . \tag{167}$$

The other comes from the D-term potential. The $SU(2)$ and $U(1)$ potentials contain the cross terms between the Higgs field and sfermion field contributions

$$\begin{aligned} V_D = & \frac{g^2}{2} \cdot 2 \cdot (H_d^\dagger \frac{\sigma^3}{2} H_d + H_u^\dagger \frac{\sigma^3}{2} H_u) \cdot (\widetilde{f}^* t^3 \widetilde{f}) \\ & + \frac{g'^2}{2} \cdot 2 \cdot (-\frac{1}{2} H_d^\dagger H_d + \frac{1}{2} H_u^\dagger H_u) \cdot (\widetilde{f}^* Y \widetilde{f}) \ . \end{aligned} \tag{168}$$

To evalute this expression, we must insert the vacuum expectation values of the two Higgs fields. In terms of the angle β defined in (147), these are

$$\langle H_u \rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} v \sin\beta \end{pmatrix} \qquad \langle H_d \rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} v \cos\beta \\ 0 \end{pmatrix} \ , \tag{169}$$

where $v = 246$ GeV so that $m_W = gv/2$.

Inserting the Higgs vevs into the potential (168), we find

$$\begin{aligned} V_D &= \widetilde{f}^* \big[\frac{v^2}{4} (\cos^2\beta - \sin^2\beta)(g^2 I^3 - g'^2 Y) \big] \widetilde{f} \\ &= \widetilde{f}^* \big[\frac{(g^2 + g'^2) v^2}{4} \ \cos 2\beta \, (I^3 - s_w^2 (I^3 + Y)) \big] \widetilde{f} \\ &= \widetilde{f}^* \big[m_Z^2 \cos 2\beta (I^3 - s_w^2 Q) \big] \widetilde{f} \ . \end{aligned} \tag{170}$$

Then, if we define

$$\Delta_f = (I^3 - s_w^2 Q) \cos 2\beta \, m_Z^2 \ , \tag{171}$$

the mass of a first- or second-generation sfermion takes the form

$$m_f^2 = M_f^2 + \Delta_f \tag{172}$$

when contributions proportional to fermion masses can be neglected. The D-term contribution can have interesting effects. For example, $SU(2)$ invariance of M_f^2 implies that

$$m^2(\widetilde{e}) - m^2(\widetilde{\nu}) = |\cos 2\beta| \ m_Z^2 > 0 \ . \tag{173}$$

For some choices of parameters, the measurement of this mass difference is a good way to determine $\tan\beta$ [55].

For third-generation fermions, the contributions to the mass term from Yukawa couplings and from A terms can be important. For the $\widetilde{b}$ and $\widetilde{\bar{b}}$, these contributions come from the terms in the effective Lagrangian

$$\begin{aligned} |F_b|^2 + |F_{\bar{b}}|^2 &= |y_b \langle H_d^0 \rangle \widetilde{b}|^2 + |y_b \widetilde{\bar{b}} \langle H_d^0 \rangle|^2 = m_b^2(|\widetilde{b}|^2 + |\widetilde{\bar{b}}|^2) \\ |F_{Hd}|^2 &= (-\mu \langle H_d^0 \rangle)^* (y_b \widetilde{\bar{b}} \widetilde{b}) + h.c. = -\mu m_b \tan\beta \, \widetilde{\bar{b}} \widetilde{b} + h.c. \\ -Ł_{soft} &= A_b y_b \langle H_d^0 \rangle \widetilde{\bar{b}} \widetilde{b} = A_b m_b \widetilde{\bar{b}} \widetilde{b} \ . \end{aligned} \tag{174}$$

In all, we find a mass matrix with mixing between the two scalar partners of the b quark,

$$\begin{pmatrix} \widetilde{b}^* & \widetilde{\bar{b}}^* \end{pmatrix} \mathcal{M}_b^2 \begin{pmatrix} \widetilde{b} \\ \widetilde{\bar{b}} \end{pmatrix} \ , \tag{175}$$

with

$$\mathcal{M}_b^2 = \begin{pmatrix} M_b^2 + \Delta_b + m_b^2 & m_b(A_b - \mu\tan\beta) \\ m_b(A_b - \mu\tan\beta) & \mathcal{M}_{\bar{b}}^2 + \Delta_{\bar{b}} + m_b^2 \end{pmatrix} \tag{176}$$

The mass matrix for $\widetilde{\tau}$, $\widetilde{\bar{\tau}}$ has the same structure. For $\widetilde{t}$, $\widetilde{\bar{t}}$, replace $\tan\beta$ by $\cot\beta$.

The mixing terms in the mass matrices of the third-generation sfermions often play an important role in the qualitative physics of the whole SUSY model. Because of the mixing, one sfermion eigenstate is pushed down in mass. This state is often the lightest squark or even the lightest superparticle in the theory.

4.2 Gaugino and Higgsino Masses

In a similar way, we can compute the mass terms for the gauginos and Higgsinos. Since the gauginos and Higgsino have the same quantum numbers after $SU(2)\times U(1)$ breaking, they will mix. We have seen in Section 2.4 that this mixing plays an essential role in the working of the Higgs mechanism in the limit where soft supersymmetry breaking terms are turned off.

The charged gauginos and Higgsinos receive mass from three sources. First, there is a soft SUSY breaking term

$$-Ł_{soft} = m_2 \widetilde{w}^{-T} c \widetilde{w}^+ \ . \tag{177}$$

The μ superpotential term contributes

$$-Ł_W = \mu \widetilde{h}_d^{-T} c \widetilde{h}_u^+ \ . \tag{178}$$

The gauge kinetic terms contribute

$$-Ł = \sqrt{2}\frac{g}{\sqrt{2}}\left(\langle H_d^0 \rangle \, \widetilde{w}^{+T} c \widetilde{h}_d^- + \langle H_u^0 \rangle \, \widetilde{w}^{-T} c \widetilde{h}_u^+\right) \tag{179}$$

Inserting the Higgs field vevs from (169), we find the mass term

$$\begin{pmatrix} \widetilde{w}^{-T} & \widetilde{h}_d^{-T} \end{pmatrix} c \, m_C \begin{pmatrix} \widetilde{w}^+ \\ \widetilde{h}_u^+ \end{pmatrix} \ , \tag{180}$$

with

$$m_C = \begin{pmatrix} m_2 & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & \mu \end{pmatrix} \ . \tag{181}$$

The mass matrix for neutral gauginos and Higgsinos also receives contributions from these three sources. In this case, all four of the states

$$(\widetilde{b}, \widetilde{w}^0, \widetilde{h}_d^0, \widetilde{h}_u^0) \tag{182}$$

have the same quantum numbers after $SU(2) \times U(1)$ breaking and can mix together. The mass matrix is

$$m_N = \begin{pmatrix} m_1 & 0 & -m_Z c_\beta s_w & m_Z s_\beta s_w \\ 0 & m_2 & m_Z c_\beta c_w & -m_Z s_\beta c_w \\ -m_Z c_\beta s_w & m_Z c_\beta c_w & 0 & -\mu \\ m_Z s_\beta s_w & -m_Z s_\beta c_w & -\mu & 0 \end{pmatrix} . \tag{183}$$

The mass eigenstates in these systems are referred to collectively as *charginos* and *neutralinos*. The matrix (183) is complex symmetric, so it can be diagonalized by a unitary matrix V_0,[‡]

$$m_N = V_0^* D_N V_0^\dagger . \tag{184}$$

I will denote the neutralinos as $\widetilde{N}_i^0$, $i = 1, \ldots, 4$, in order of mass with $\widetilde{N}_1^0$ the lightest. Elsewhere in the literature, you will see these states called $\widetilde{\chi}_i^0$ or $\widetilde{Z}_i^0$. The mass eigenstates are related to the weak eigenstates by the transformation

$$\begin{pmatrix} \widetilde{b}^0 \\ \widetilde{w}^0 \\ \widetilde{h}_d^0 \\ \widetilde{h}_u^0 \end{pmatrix} = V_0 \begin{pmatrix} \widetilde{N}_1 \\ \widetilde{N}_2 \\ \widetilde{N}_3 \\ \widetilde{N}_4 \end{pmatrix} . \tag{185}$$

Note that the diagonal matrix D_N in (184) may have negative or complex-valued elements. If that is true, the physical fermion masses of the $\widetilde{N}_i$ are the absolute values of the corresponding elements of D_N. The phases will appear in the three-point couplings of the $\widetilde{N}_i$ and can lead to observable interference effects. Complex phases in D_N would provide a new source of CP violation.

The chargino mass matrix (181) is not symmetric, so in general it is diagonalized by two unitary matrices

$$m_C = V_-^* D_C V_+^\dagger . \tag{186}$$

I will denote the charginos as $\widetilde{C}_i^\pm$, $i = 1, 2$, in order of mass with $\widetilde{C}_1^\pm$ the lighter. Elsewhere in the literature, you will see these states called $\widetilde{\chi}_i^\pm$ or $\widetilde{W}_i^\pm$. The mass eigenstates are related to the weak eigenstates by the transformation

$$\begin{pmatrix} \widetilde{w}^+ \\ \widetilde{h}_u^+ \end{pmatrix} = V_+ \begin{pmatrix} \widetilde{C}_1^+ \\ \widetilde{C}_2^+ \end{pmatrix} , \qquad \begin{pmatrix} \widetilde{w}^- \\ \widetilde{h}_u^- \end{pmatrix} = V_- \begin{pmatrix} \widetilde{C}_1^- \\ \widetilde{C}_2^- \end{pmatrix} . \tag{187}$$

It should be noted that μ are must be nonzero. If $\mu = 0$, the determinant of (183) vanishes and so the lightest neutralino must be massless. This neutralino will also have a large Higgsino content and thus an order-1 coupling to the Z^0. It is excluded by searches for an excess of invisible Z^0 decays and for $Z^0 \to \widetilde{N}_1 \widetilde{N}_2$. The condition $\mu = 0$ also implies that the lightest chargino has a mass below the current limit of about 100 GeV.

Often, one studies models for which m_1, m_2, and μ are all large compared to m_W and m_Z. The off-diagonal elements that mix the gaugino and Higgsino states are of the order of m_W and m_Z.

[‡]Note that this formula is different from that which diagonalizes a Hermitian matrix. A detailed discussion of the diagonalization of mass matrices appearing in SUSY can be found in the Appendix of [56].

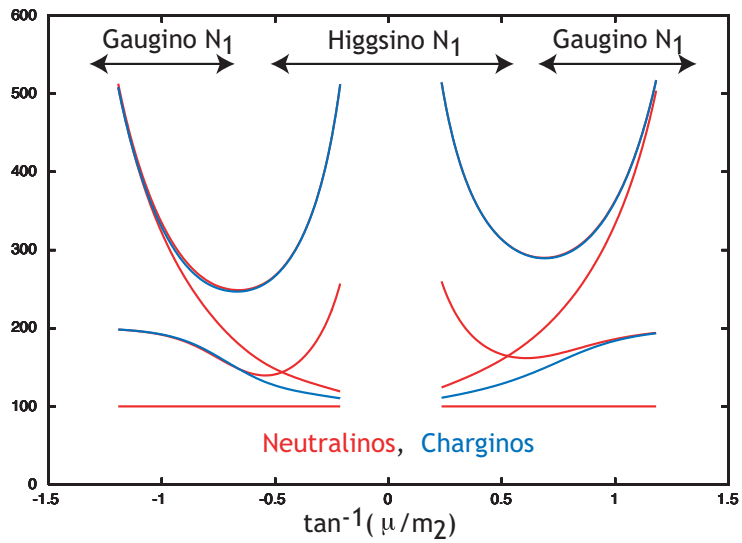

Figure 8: Masses of the four neutralinos and two charginos along a line in the SUSY parameter space on which $m(\widetilde{N}_1^0) = 100$ GeV while the parameter μ moves from large negative to large positive values. The parameter m_1 is set to $m_1 = 0.5m_2$. Note the approximate degeneracies in the extreme limits of the gaugino and Higgsino regions.

Thus, if the scale of masses generated by the SUSY breaking terms is large, the mixing is small and the individual eigenstates are mainly gaugino or mainly Higgsino. However, there are two distinct cases. The first is the *gaugino region*, where $m_1, m_2 < |\mu|$. In this region of parameter space, the lightest states $\widetilde{N}_1$, $\widetilde{C}_1$ are mainly gaugino, while the heavy neutralinos and charginos are mainly Higgsino. In the *Higgsino region*, $m_1, m_2 > |\mu|$, the situation is reversed and $\widetilde{N}_1$, $\widetilde{C}_1$ are mainly Higgsino. In this case, the two lightest neutralinos are almost degenerate. In Fig. 8, I show the mass eigenvalues as a function of the mass matrix parameters along a line in the parameter space on which the $\widetilde{N}_1$ has a fixed mass of 100 GeV. As we will see in Section 6.4, the exact makeup of the lightest neutralino as a mixture of gaugino and Higgsino components is important to the study of supersymmetric dark matter.

To summarize this discussion, I present in Fig. 9 the complete spectrum of new particles in the MSSM at a representative point in its parameter space. Notice that the third-generation sfermions are split off from the others in each group. Note also that the parameter point chosen is in the gaugino region. The lightest superparticle is the $\widetilde{N}_1$. I will discuss the spectrum of Higgs bosons in Section 6.2.

4.3 Renormalization Group Evolution of MSSM Parameters

The spectrum shown in Fig. 9 appears to have been generated by assigning random values to the soft SUSY breaking parameters. But, actually, I generated this spectrum by making very simple assumptions about the relationships of the soft parameters, at a high energy scale. Specifically, I assumed that the soft SUSY breaking gaugino masses and (separately) the sfermion masses were equal at the scale of grand unification. The structure that you see in the figure is generated by the renormalization group evolution of these parameters from the grand unification scale to the weak

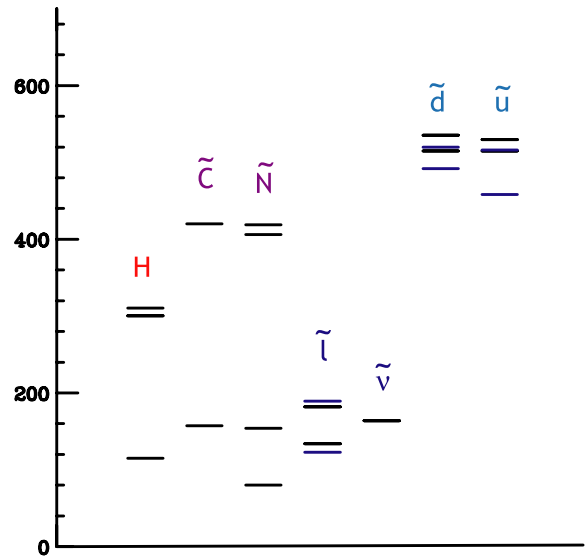

Figure 9: Illustrative spectrum of supersymmetric particles. The columns contain, from the left, the Higgs bosons, the four neutralinos, the two charginos, the charged sleptons, the sneutrinos, the down squarks, and the up squarks. The gluino, not shown, is at about 800 GeV.

scale.

The renomalization group (RG) evolution of soft parameters is likely to play a very important role in the interpretation of measurements of the SUSY particle masses. Essentially, after measuring these masses, it will be necessary to decode the results by running the effective mass parameters up to a higher energy at which their symmetries might become more apparent. The situation is very similar to that of the Standard Model coupling constants, where a renormalization group analysis told us that the apparently random values (137) for the coupling constants at the weak scale actually corresponds to a unification of couplings at a much higher scale.

In this section, I will write the most basic RG equations for the soft gaugino and sfermion masses. One further effect, which involves the Yukawa couplings and is important for the third generation, will be discussed later in Section 6.1.

The RG equation for the gaugino masses is especially simple. This is because both the gaugino masses and the gauge couplings arise from the superpotential term (115), with the supersymmetry breaking terms arising as shown in (159). As I have already noted, this F-term receives a radiative correction proportional to the β function as a consequence of the trace anomaly [34,36]. The corrections are the same for the gauge boson field strength and the gaugino mass. Thus, if gaugino masses and couplings are generated at the scale M, they have the relation after RG running to the scale Q:

$$\frac{m_i(Q)}{m_i(M)} = \frac{\alpha_i(Q)}{\alpha_i(M)} \ . \tag{188}$$

If the F term that generates the soft gaugino masses is an $SU(5)$ singlet, the soft gaugino masses will be grand-unified at M. Then, running down to the weak scale, they will have the relation

$$m_1 \ : \ m_2 \ : \ m_3 \ = \ \alpha_1 \ : \ \alpha_2 \ : \ \alpha_3 \ = \ 0.5 \ : \ 1 \ : \ 3.5 \ . \tag{189}$$

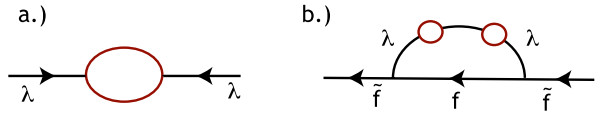

Figure 10: Diagrams that generate the soft mass parameters in gauge mediated supersymmetry breaking: (a.) gaugino masses; (b.) sfermion masses.

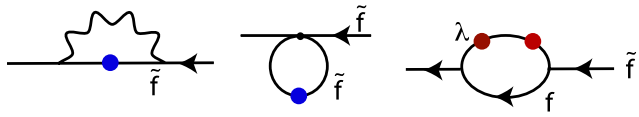

Figure 11: Diagrams that generate the renormalization group evolution of the soft sfermion mass parameters M_f^2.

This relation of soft gaugino masses is known as *gaugino unification*.

There are other models of the soft gaugino masses that also lead to gaugino unification. In *gauge-mediated SUSY breaking*, the dynamics responsible for SUSY breaking occurs at a scale much lower than the scale associated with mediation by supergravity. At this lower scale M_g (for example, 1000 TeV), some heavy particles with nontrivial $SU(3) \times SU(2) \times U(1)$ quantum numbers acquire masses from SUSY breaking. These fields then couple to gauginos and generate SUSY breaking masses for those particles through the diagram shown in Fig. 10(a). The heavy particles must fall into complete $SU(5)$ representations; otherwise, the coupling constant renormalization due to these particles between M_g and the grand unification scale would spoil the grand unification of the gauge couplings. Then the diagram in Fig. 10(a) generates soft gaugino masses proportional to $\alpha(M_g)$. Running these parameters down to the weak scale, we derive the relation (189) from this rather different mechanism.

Now let us turn to the RG running of soft scalar masses. In principle, there are two contributions, one from the RG rescaling of the soft mass term M_f^2 and one from RG evolution generating M_f^2 from the gaugino mass. The Feynman diagrams that contribute to the RG coefficients are shown in Fig. 11. The two one-loop diagrams proportional to M_f^2 cancel. The third diagram, involving the gaugino mass, gives the RG equation

$$\frac{dM_f^2}{d\log Q} = -\frac{2}{\pi}\sum_i \alpha_i(Q) C_2(r_i) m_i^2(Q) \ , \tag{190}$$

with $i = 1, 2, 3$ and $C_2(r_i)$ the squared charge in the fermion representation r_i under the gauge group i. This equation leads to a positive contribution to M_f^2 as one runs the RG evolution from the messenger scale down to the weak scale. The effect is largest for squarks, for which the SUSY breaking mass is induced from the gluino mass.

As an example of this mechanism of mass generation, assume gaugino unification and assume that $M_f^2 = 0$ for all sfermions at the grand unification scale. Then the weak scale sfermion masses will be in the ratio

$$\begin{aligned} M(\widetilde{e}) \ &: \ M(\widetilde{e}) \ : \ M(\widetilde{\overline{d}}) \ : \ M(\widetilde{\overline{u}}) \ : \ M(\widetilde{d}, \widetilde{u}) \ : \ m_2 \\ &= \ 0.5 \ : \ 0.9 \ : \ 3.09 \ : \ 3.10 \ : \ 3.24 \ : \ 1 \end{aligned} \tag{191}$$

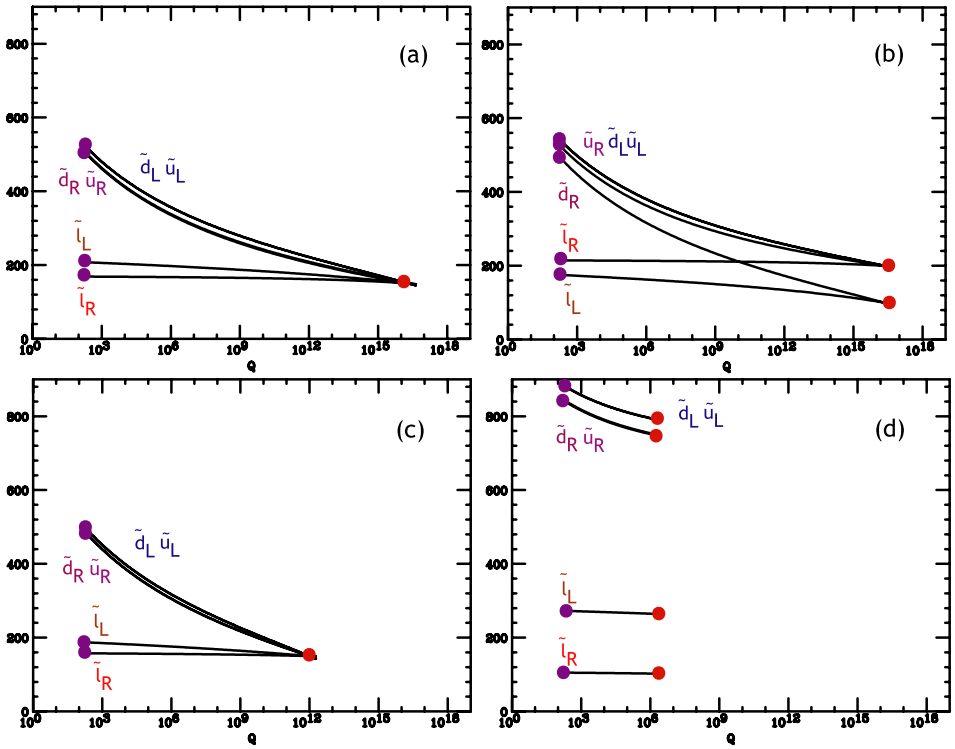

Figure 12: Evolution of squark and slepton masses from the messenger scale down to the weak scale, for four different models of supersymmetry breaking: (a.) universal sfermion masses at the grand unification scale M_U; (b.) sfermion masses at M_U that depend on the $SU(5)$ representation; (c.) universal sfermion masses at an intermediate scale; (d.) gauge mediation from a sector of mass about 1000 TeV.

This model of fermion mass generation is called *no-scale* SUSY breaking. It has the danger that the lightest stau mass eigenstate could be lighter than than the $\widetilde{N}_1$, leading to problems for dark matter. This problem can be avoided by RG running above the GUT scale [57]. Alternatively, it might actually be that the lightest Standard Model superpartner is a long-lived stau that eventually decays to a tau and a gravitino [58,59].

In gauge-mediated SUSY breaking, the diagram shown in Fig. 10(b) leads to the qualitatively similar but distinguishable formula

$$M_f^2 = 2\sum_i \alpha_i^3(M) C_2(r_i) \cdot \left(\frac{m_2}{\alpha_2}\right)^2 . \tag{192}$$

Each model of SUSY breaking leads to its own set of relations among the various soft SUSY breaking parameters. In general, the relations are predicted for the parameters defined at the messenger scale and must be evolved to the weak scale by RG running to be compared with experiment. Fig. 12 shows four different sets of high-scale boundary conditions for the RG evolution, and the corresponding evolution to the weak scale. If we can measure the weak-scale values, we could try to undo the evolution and recognize the pattern. This will be a very interesting study for the era in which superparticles are observed at high energy colliders.

There are some features common to these spectra that are general features of the RG evolution of soft parameters:

1. The pairs of sleptons $\widetilde{\widetilde{e}}$ and $\widetilde{e}$ can easily acquire a significant mass difference from RG evolution, and they might also have a different initial condition. It is important to measure the mass ratio $m(\widetilde{\widetilde{e}})/m(\widetilde{e})$ as a diagnostic of the scheme of SUSY breaking.

2. Gaugino unification is a quantitative prediction of certain schemes of SUSY breaking. It is important to find out whether this relation is correct or not for the real spectrum of superparticles in Nature.

3. When the RG effects on the squark masses dominate the values of M_f^2 from the initial condition, the various species of squark have almost the same mass and are much heavier than the sleptons. It is important to check whether most or all squarks appear at the same threshold.

5 The Measurement of Supersymmetry Parameters

5.1 Measurements of the SUSY Spectrum at the ILC

Now that we have discussed the physics that determines the form of the spectrum of superparticles, we turn to the question of how we would determine this spectrum experimentally. This is not as easy as it might seem. In this section, I will consider only models in which the dark matter particle is the $\widetilde{N}_1$, and all other SUSY particles decay to the $\widetilde{N}_1$. This neutral and weakly interacting particle would escape a collider detector unseen. Nevertheless, methods have been worked out not only to measure the masses of superparticles but also to determine mixing angles and other information needed to convert these masses to values of the underlying parameters of the MSSM Lagrangian.

Similar methods apply to other scenarios. For example, in models in which the neutralino decays to a particle with gravitational interactions, one would add that decay, if it is visible, to the analyses that I will present. It is possible in models of this type that the lightest Standard Model superpartner would be a charged slepton that is stable on the time scale of particle physics experiments. That scenario would produce very striking and characteristic events [58].

Most likely, this experimental study of the SUSY spectrum will begin in the next few years with the LHC experiments. However, at a hadron collider like the LHC, much of the kinematic information on superparticle production is missing and so special tricks are needed even to measure the spectrum. The study of supersymmetry should be much more straightforward at an e^+e^- collider such as the planned International Linear Collider (ILC). For this reason, I would like to begin my discussion of the experiments in this section by discussing SUSY spectrum measurements at e^+e^- colliders. More complete reviews of SUSY measurements at linear colliders can be found in [60,61].

I first discuss slepton pair production, beginning with the simplest process, $e^+e^- \to \widetilde{\mu}^+\widetilde{\mu}^-$ and considering successively the production of $\widetilde{\tau}$ and $\widetilde{e}$. Each step will bring in new complexities and will allow new measurements of the SUSY parameters.

The process $e^+e^- \to \widetilde{\mu}^+\widetilde{\mu}^-$, where $\widetilde{\mu}$ is the partner of either the left- or right-handed μ, can be analyzed with the simple formulae for scalar particle-antiparticle production. The cross section for pair production from polarized initial electrons and positrons to final-state scalars with definite $SU(2) \times U(1)$ quantum numbers is given by

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\beta^3 \sin^2\theta \, |f_{ab}|^2 \ , \tag{193}$$

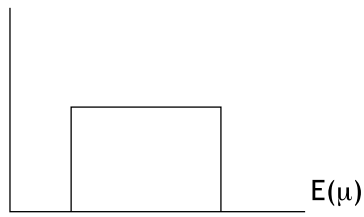

Figure 13: Schematic energy distribution of final-state muons in $e^+e^- \to \widetilde{\mu}^+\widetilde{\mu}^-$.

where

$$f_{ab} = 1 + \frac{(I_e^3 + s_w^2)(I_\mu^3 + s_w^2)}{c_w^2 s_w^2} \frac{s}{s - m_Z^2} \tag{194}$$

and, in this expression, $I^3 = -\frac{1}{2}, 0$ for $a, b = L, R$. For the initial state, $a = L$ denotes the state $e_L^- e_R^+$ and $a = R$ denotes $e_R^- e_L^+$. For the final state, $b = L$ denotes the $\widetilde{\mu}$, $b = R$ the $\widetilde{\overline{\mu}}$. Notice that this cross section depends strongly on the polarization states:

$$\begin{aligned} |f_{ab}|^2 = & \quad 1.69 \quad e_R^- e_L^+ \to \widetilde{\overline{\mu}}^+ \widetilde{\overline{\mu}}^- \\ = & \quad 0.42 \quad e_L^- e_R^+ \to \widetilde{\overline{\mu}}^+ \widetilde{\overline{\mu}}^- \\ = & \quad 0.42 \quad e_R^- e_L^+ \to \widetilde{\mu}^+ \widetilde{\mu}^- \\ = & \quad 1.98 \quad e_L^- e_R^+ \to \widetilde{\mu}^+ \widetilde{\mu}^- \end{aligned} \tag{195}$$

The angular distribution is characteristic of pair-production of a spin 0 particle; the normalization of the cross sections picks out the the correct set of $SU(2) \times U(1)$ quantum numbers.

If the smuon is light, its only kinematically allowed decay might be $\widetilde{\mu} \to \mu \widetilde{N}_1^0$. Even if the smuon is heavy, if the $\widetilde{N}_1$ is mainly gaugino, this decay should be important. As noted above, I am assuming that R-parity is conserved and that the $\widetilde{N}_1$ is the lightest particle in the superparticle spectrum. Then events with this decay on both sides will appear as

$$e^+e^- \to \mu^+\mu^- + (\text{missing } E \text{ and } p) \tag{196}$$

The spectrum of the observed muons is very simple. Since the $\widetilde{\mu}$ has spin 0, it decays isotropically in its own rest frame. In e^+e^- production at a definite center of mass energy, the $\widetilde{\mu}$ is produced at a definite energy, and thus with a definite boost, in the lab. The boost of an isotropic distribution is a flat distribution in energy. So, the muon energy distribution should be flat, between endpoints determined by kinematics, as shown in the idealized Fig. 13.

The endpoint positions are simple functions of the mass of the $\widetilde{\mu}$ and the mass of the $\widetilde{N}_1$,

$$E_\pm = \gamma(1 \pm \beta) \, \frac{m^2(\widetilde{\mu}) - m^2(\widetilde{N}_1)}{2m(\widetilde{\mu})} \ , \tag{197}$$

where $\gamma = E_{\rm CM}/2m(\widetilde{\mu})$, $\beta = (1 - 4m^2(\widetilde{\mu})/E_{\rm CM}^2)^{1/2}$. If we can identify both endpoint positions, we can solve for the two unknown masses. Figure 14 shows a simulation of the reconstructed smuon energy distribution from $\widetilde{\overline{\mu}}$ pair production at the ILC [63]. The high-energy edges of the distributions are rounded because of initial-state radiation in the e^+e^- collision. The experimenters expect to be able to measure this effect and correct for it. Then they should obtain values of the smuon mass to an accuracy of about one hundred MeV, or one part per mil.

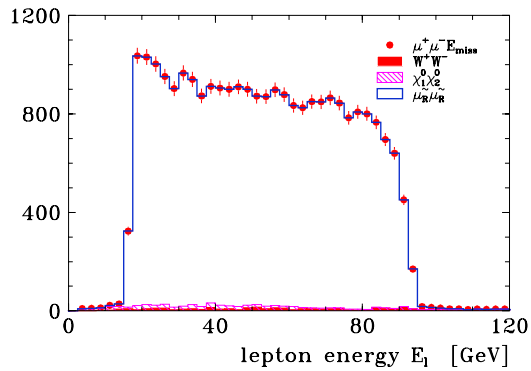

Figure 14: Energy distribution of muons from $e^+e^- \to \widetilde{\overline{\mu}}^-\widetilde{\overline{\mu}}^+$ at the ILC, in a simulation by Blair and Martyn that includes realistic momentum resolution and beam effects [63].

A similar analysis applies to $e^+e^- \to \widetilde{\tau}^+\widetilde{\tau}^-$, but there are several complications. First, for the τ system, mixing between the $\widetilde{\tau}$ and the $\widetilde{\overline{\tau}}$ might be important, especially if $\tan\beta$ is large. The production cross sections are affected directly by the mixing. For example, to compute the pair-production of the lighter $\widetilde{\tau}$ mass eigenstate from a polarized initial state, $e^-_R e^+_L \to \widetilde{\tau}^-_1\widetilde{\tau}^+_1$, we must generalize (193) to

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\beta^3\sin^2\theta\,|f_{R1}|^2 \ , \tag{198}$$

where

$$f_{R1} = f_{RR}\cos^2\theta_\tau + f_{RL}\sin^2\theta_\tau \tag{199}$$

and θ_τ is the mixing angle associated with the diagonalization of the $\widetilde{\tau}$ case of (176).

Second, while the $\widetilde{\overline{\tau}}^-$ can decay to $\tau^-_R\widetilde{b}$ through gauge couplings, this weak eigenstate can also decay to $\tau^-_L\widetilde{h}_d$ through terms proportional to the Yukawa coupling. Both decay amplitudes contribute to the observable decay $\widetilde{\tau}_1 \to \tau\widetilde{N}^0_1$. With the $\widetilde{\tau}$ mixing angle fixed from the measurement of the cross section, the τ polarization in $\widetilde{\tau}$ decays can be used to determine the mixing angles in the diagonalization of the neutralino mass matrix (183) [62].

In Fig. 15, I show the distribution of total visible energy in $\widetilde{\tau} \to 3\pi + \nu + \widetilde{N}^0_1$ at the ILC. Though there is no longer a sharp feature at the kinematic endpoint, it is still possible to accurately determine the $\widetilde{\tau}$ mass by fitting the shape of this distribution.

The physics of $e^+e^- \to \widetilde{e}^+\widetilde{e}^-$ brings in further new features. In this case, there is a new Feyman diagram, involving t-channel neutralino exchange. The two diagrams contributing to the cross section for this process are shown in Fig. 16. The t-channel diagram turns out to be the more important one, dominating the s-channel gauge boson exchange and generating a large forward peak in selectron production. The cross section for $e^-_R e^+_L \to \widetilde{\overline{e}}^-\widetilde{\overline{e}}^+$ is given by another generalization of (193),

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\beta^3\sin^2\theta\,|\mathcal{F}_{RR}|^2 \ , \tag{200}$$

where

$$\mathcal{F}_{RR} = f_{RR} - \sum_i \left|\frac{V_{01i}}{c_w}\right|^2 \frac{s}{m_i^2 - t} \ , \tag{201}$$

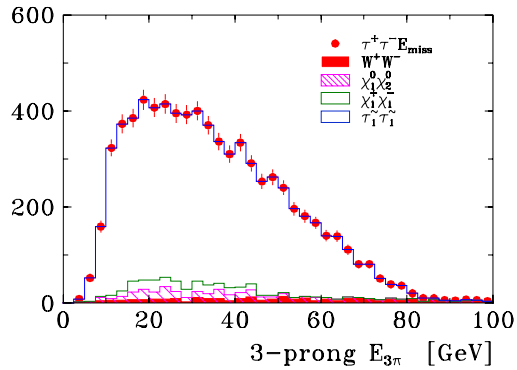

Figure 15: Energy distribution of the three-pion system from $e^+e^- \to \widetilde{\tau}_1^-\widetilde{\tau}_1^+$ at the ILC, with a τ decay to 3π, in a simulation by Blair and Martyn that includes realistic momentum resolution and beam effects. [63].

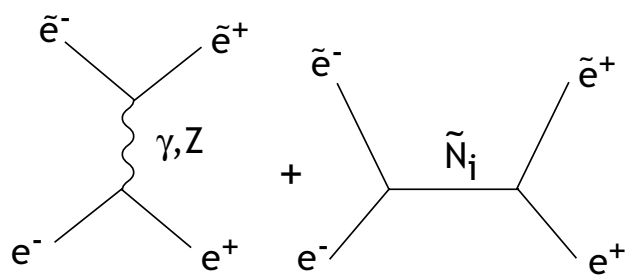

Figure 16: Feynman diagrams contributing to $e^+e^- \to \widetilde{e}^-\widetilde{e}^+$.

with the sum running over neutralino mass eigenstates. The factor V_{01i} is a matrix element of the unitary matrix introduced in (184).

The t-channel diagram also allows new processes such as $e_L^-e_L^+ \to \widetilde{e}^-\widetilde{e}^+$. Note the correlation of the initial-state electron and position spins with the identities of the final-state selectrons. A complete set of polarized cross sections for selectron pair production in e^+e^- and e^-e^- collisions can be found in [64].

The cross sections for chargino and neutralino pair production in e^+e^- collisions are somewhat more complicated, but still there are interesting things to say about these processes. Chargino pair production is given by the Feynman diagrams shown in Fig. 17. These diagrams are just the supersymmetric analogues of the diagrams for $e^+e^- \to W^+W^-$. As in that process, the most charcteristic final states are those with a hadronic decay on one side of the event and a leptonic decay on the other side, for example,

$$\widetilde{C}_1^+ \to \ell^+\nu\widetilde{N}_1^0 \ , \quad \widetilde{C}_1^- \to d\overline{u}\widetilde{N}_1^0 \ . \tag{202}$$

A typical event of this kind is shown in Fig. 18.

The chargino and neutralino production cross sections have a strong dependence on the mixing angles in (184) and (186) and offer a number of strategies for the determination of these mixing angles. Let me present one such strategy here. Consider the reaction from a polarized initial

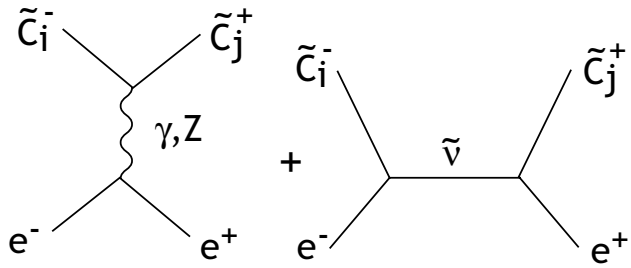

Figure 17: Feynman diagrams contributing to $e^+e^- \to \widetilde{C}_i^- \widetilde{C}_j^+$.

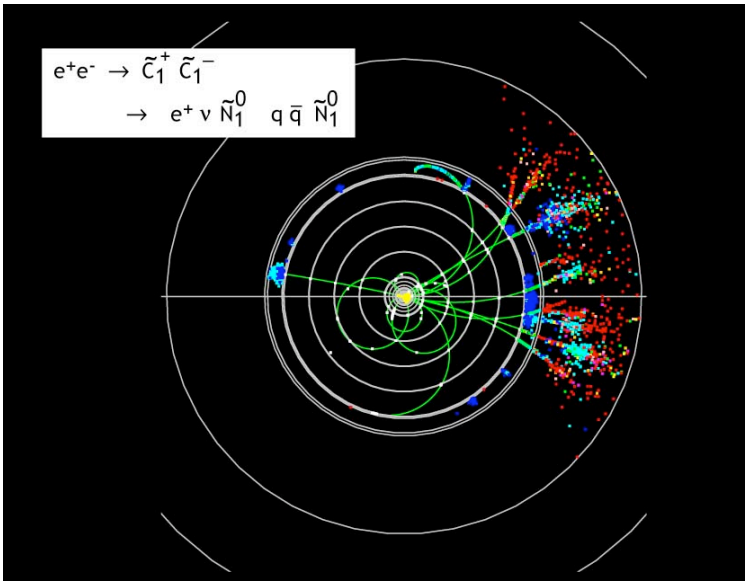

Figure 18: A simulated chargino pair production event at the ILC [65].

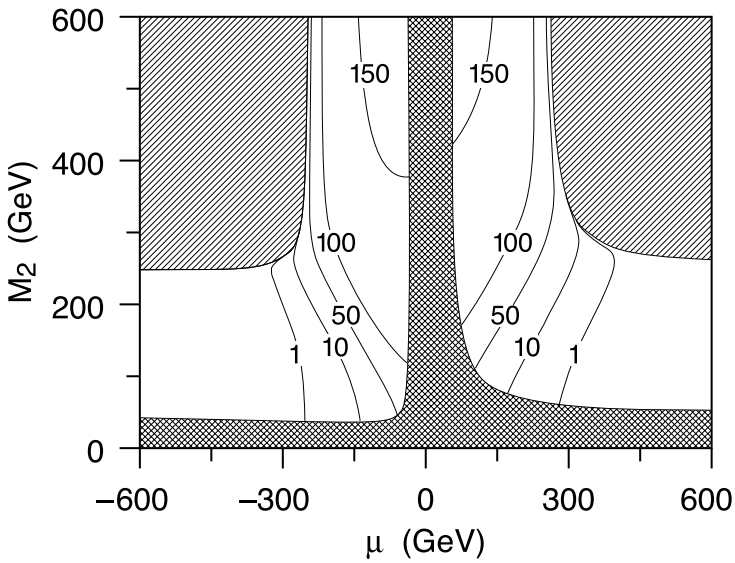

Figure 19: Contours of constant cross section for the process $e_R^- e_L^+ \to C_1^- C_1^+$ (in fb, for $E_{\rm CM} = 500$ GeV), as a function of the underlying SUSY parameters [66]. The region shown is that in which the lightest chargino mass varies from 50 to 200 GeV. For fixed $\widetilde{C}_1^+$ mass, the cross section increases from zero to about 150 fb as we move from the gaugino region into the Higgsino region.

state $e_R^- e_L^+ \to \widetilde{C}_1^- \widetilde{C}_1^+$. Since we have an initial e_R^-, the t-channel diagram vanishes because the right-handed electron does not couple to the neutrino. Now simplify the s-channel diagram by considering the limit of high energies, $s \gg m_Z^2$. In this limit, it is a good approximation to work with weak gauge eigenstates (B^0, W^0) rather than the mass eigenstates (γ, Z^0). The weak eigenstate basis gives a nice simplification. The initial e_R^- couples only to B^0. But $\widetilde{w}^\pm$ couple only to W^0, so at high energy the s-channel diagram gets contributions only from the Higgsino components of the $\widetilde{C}_1^-$ and $\widetilde{C}_1^+$ eigenstates. If we go to still higher energies, $s \gg m(\widetilde{C}_1)^2$, there is a further simplification. The cross section for $\widetilde{h}_R^- \widetilde{h}_L^+$ production is forward-peaked, and the cross section for $\widetilde{h}_L^- \widetilde{h}_R^+$ production is backward-peaked. Then, the cross section for $e_R^- e_L^+ \to \widetilde{C}_1^- \widetilde{C}_1^+$ takes the form

$$\frac{d\sigma}{d\cos\theta} \sim \frac{\pi\alpha^2}{8c_w^2 s}\left[|V_{+21}|^4(1+\cos\theta)^2 + |V_{-21}|^2(1-\cos\theta)^2\right] . \tag{203}$$

In this limit, it is clear that we can read off both of the mixing angles in (186) from the shape of this cross section.

The use of high-energy limits simplified this analysis, but the sentivity of this cross section to the chargino mixing angles is not limited to high energy. Even relatively close to threshold, the polarized cross sections for chargino production depend strongly on the chargino mixing angles and can be used to determine their values. In Fig. 19, I show contours of constant cross section for $e_R^- e_L^+ \to \widetilde{C}_1^- \widetilde{C}_1^+$ in the (m_2, μ) plane (for $\tan\beta = 4$ and assuming gaugino unification) [66]. The value of this cross section is always a good measure of whether the SUSY parameters in Nature put us in the gaugino or the Higgsino region of Fig. 8.

5.2 Observation of SUSY at the LHC

Now we turn to supersymmetry production processes at the LHC. This subject, though more difficult, has immediate importance, since the LHC experiments are just about to begin.

The reactions that produce superparticles are typically much more complicated at hadron colliders than at lepton colliders. This is true for several reasons. High energy collisions of hadrons are intrinsically more complicated because the final states include the fragments of the initial hadrons that do not participate in the hard reaction. More importantly, the dominant reactions at hadron colliders are those that involve strongly interacting superparticles. This means that the primary particles are typically the heavier ones in the spectrum, which then decay in several steps. In addition, large backgrounds from QCD obscure the signatures of supersymmetric particle production in many channels.

Because of these difficulties, there is some question whether SUSY particle production can be observed at the LHC. However, as I will explain, the signatures of supersymmetry are still expected to be striking and characteristic. It is not so clear, though, to what extent it is possible to measure the parameters of the SUSY Lagrangian, as I have described can be done from ILC experiments. This is an important study that still offers much room for new ideas.

The discovery of SUSY particles at the LHC and the measurement of SUSY parameters has been analyzed with simulations at a number of parameter points. Collections of interesting studies can be found in [63,67,68].

The dominant SUSY production processes at the LHC are

$$gg \to \widetilde{g}\widetilde{g} \ , \ \widetilde{q}\widetilde{q}^* \qquad gq \to \widetilde{g}\widetilde{q} \tag{204}$$

These cross sections are large—tens of pb in typical cases. The values of numerous SUSY production cross sections at the LHC are shown in Fig. 20 [70].

We have seen that the squarks and gluinos are typically the heaviest particles in the supersymmetry spectrum. The gluinos and squarks thus will decay to lighter superparticles. Some of these decays are simple, *e.g.*,

$$\widetilde{\overline{q}} \to \overline{q}\widetilde{N}_1^0 \ . \tag{205}$$

However, other decays can lead to complex decay chains such as

$$\widetilde{q} \to qN_2^0 \to q(\ell^+\ell^-)\widetilde{N}_1^0 \ , \qquad \widetilde{g} \to u\overline{d}C_1^+ \to u\overline{d}W^+\widetilde{N}_1^0 \ . \tag{206}$$

With the assumptions that R-parity is conserved and that the N_1^0 is the LSP, all SUSY decay chains must end with the N_1^0, which is stable and very weakly interacting. SUSY production processes at hadron colliders then have unbalanced visible momentum, accompanied by multiple jets and, possibility, isolated leptons or W and Z bosons. Momentum balance along the beam direction cannot be checked at hadron colliders, because fragments of the initial hadrons exit along the beam directions, but an imbalance of transverse momentum will be visible and can be a characteristic signature of new physics. SUSY events contain this signature and the general large activity characteristic of heavy particle production. A simulated event of this type is shown in Fig. 21.

Figure 22 shows a set of estimates given by Tovey and the ATLAS collaboration of the discovery potential for SUSY as a function of the LHC luminosity [71]. The most important backgrounds come from processes that are themselves relatively rare Standard Model reactions with heavy particle production,

$$pp \to (W, Z, t\overline{t}) \ + \ \text{jets} \ . \tag{207}$$

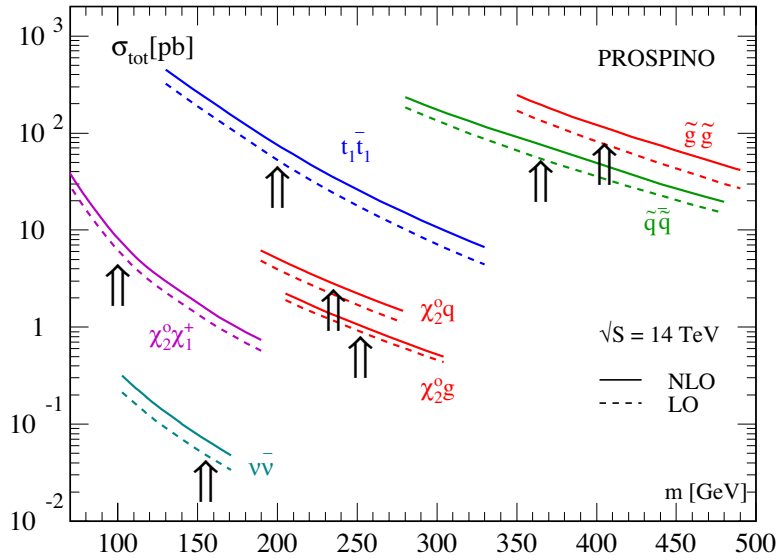

Figure 20: Cross sections for the pair-production of supersymmetric particles at the LHC, from [70].

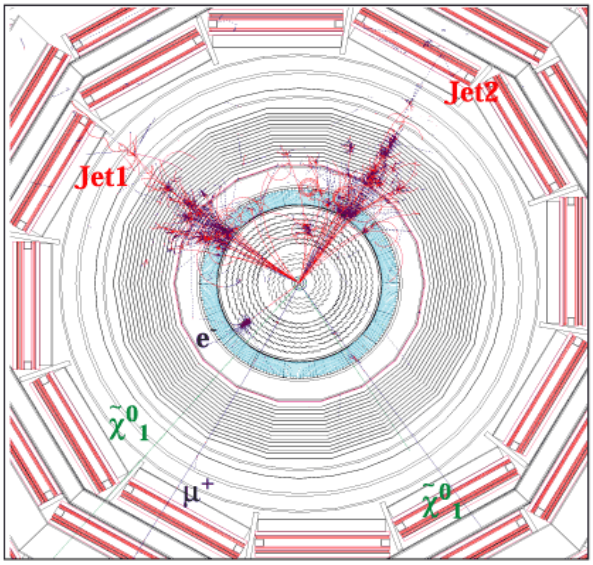

Figure 21: Simulated SUSY particle production event in the CMS detector at the LHC [69].

With some effort, we can experimentally normalize and control these backgrounds and reliably discovery SUSY production as a new physics process. In the figure, the contours for 5σ excesses of events above these backgrounds for various signatures of SUSY events are plotted as a function of the so-called 'mSUGRA' parameters. The SUSY models considered are defined as follows: Assume gaugino unification with a universal gaugino mass $m_{1/2}$ at the grand unification scale. Assume also that all scalar masses, including the Higgs boson mass parameters, are unified at the grand unification scale at the value m_0. Assume that the A parameter is universal at the grand unification scale; in the figures, the value $A = 0$ is used. Fix the value of $\tan\beta$ at the weak scale. Then it is possible to solve for μ and B, up to a sign, from the condition that electroweak symmetry is broken in such a way as to give the observed value of the Z^0 mass. (I will describe this calculation in Section 6.1.) This gives a 4-parameter subspace of the full 24-dimensional parameter space of the CP- and flavor-conserving MSSM, with the parameters

$$m_0 \ , \ m_{1/2} \ , \ A \ , \ \tan\beta \ , \ \mathrm{sign}(\mu) \ . \tag{208}$$

This subspace is often used to express the results of phenomenological analyses of supersymmetry. In interpreting such results, one should remember that this choice of parameters is used for simplicity rather than being motivated by physics.

The figure shows contours below which the various signatures of supersymmetry significantly modify the Standard Model expectations. For clarity, the contours of constant squark and gluino mass are also plotted. The left-hand plot shows Tovey's results for the missing transverse momentum plus multijets signature at various levels of LHC integrated luminosity. It is remarkable that, in the models in which the squark or gluino mass is below 1 TeV, SUSY should be discoverable with a data sample equivalent to a small fraction of a year of running. The right-hand plot shows the contours for the discovery of a variety of SUSY signals, with up to three leptons plus jets plus missing transverse momentum, with roughly one year of data at the initial design luminosity. The signals are, as I have described, relatively robust with repect to uncertainties in the Standard Model backgrounds. This makes it very likely that, if SUSY is really present in Nature as the explanation of electroweak symmetry breaking, we will discover it at the LHC.

The general characteristics of SUSY events also allow us to estimate the SUSY mass scale in a relatively straightforward way. In Fig. 23, I show a correlation pointed out by Hinchliffe and collaborators [72] between the lighter of the squark and gluino masses and the variable

$$M_{eff} = \not{E}_T + \sum_1^4 E_{Ti} \tag{209}$$

given by the sum of the transverse momenta of the four highest E_T jets together with the value of the missing transverse momentum. The correlation applies reasonably well to mSUGRA models. In other models with smaller mass gaps between the squarks and the lightest neutralino, this relation can break down, but M_{eff} still measures the mass difference between the squark or gluino and the $\widetilde{N}_1^0$ [73]. Some more sophisticated techniques for determining mass scales in SUSY models from global kinematic variables are described in [74].

5.3 Measurements of the SUSY Spectrum at the LHC

So far, I have only discussed the observation of the qualitative features of the SUSY model from global measures of the properties of events. Now I would like to give some examples of analyses in which specific details of the SUSY spectrum are measured with precision at the LHC. The examples that I will discuss involve the decay chain

$$\widetilde{q} \to q\widetilde{N}_2^0 \ , \ \widetilde{N}_2^0 \to \widetilde{N}_1^0 \ell^+\ell^- \ , \tag{210}$$

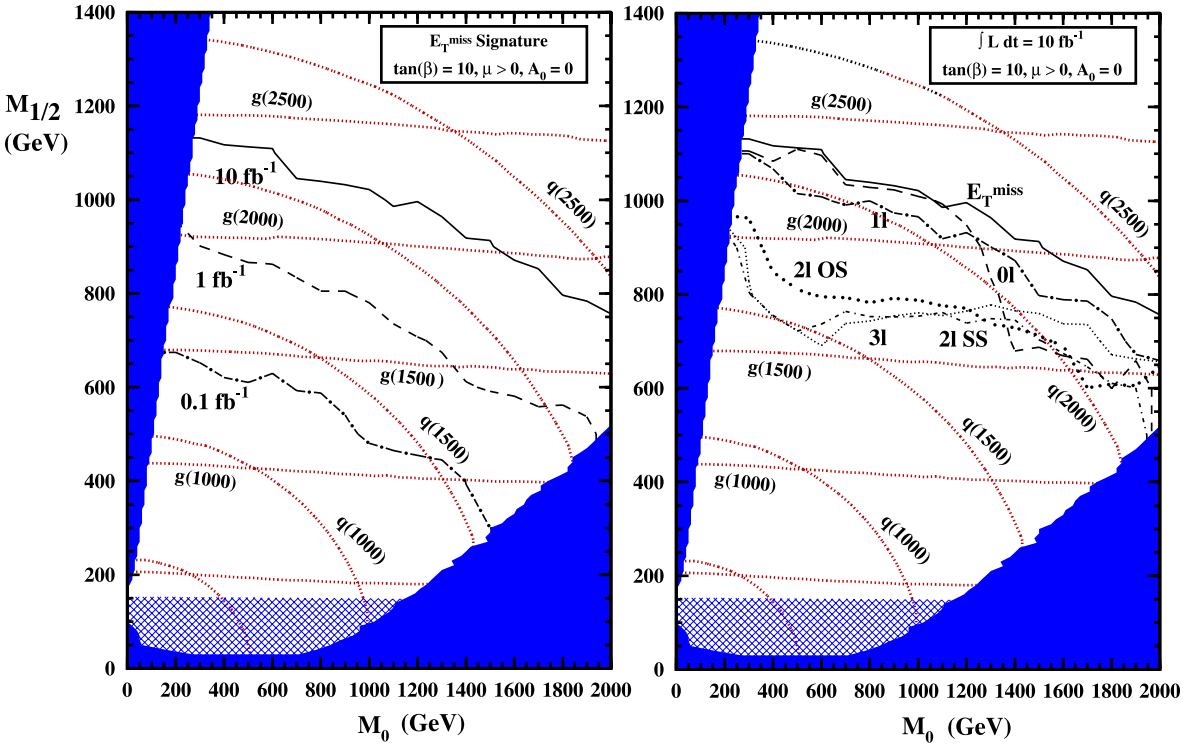

Figure 22: Estimates by the ATLAS collaboration of the observability of various signatures of SUSY at the LHC. The plots refer to models with grand unification and universal sfermion and gaugino masses M_0 and $M_{1/2}$. The left-hand plot shows the region of this parameter space in which it is possible to detect the signature of missing E_T plus multiple jets at various levels of integrated luminosity. The right-hand plot shows the region of this parameter space in which it is possible to detect an excess of events with one or more leptons in addition to jets and missing E_T [71].

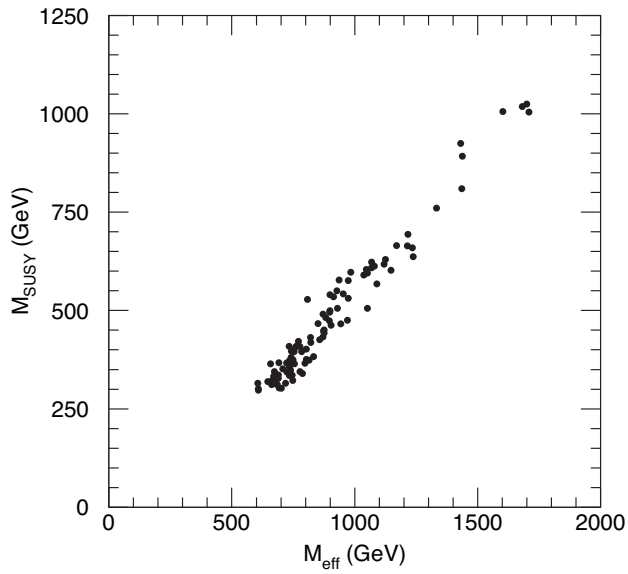

Figure 23: Correlation between the value of the observable (209) and the lighter of the squark and gluino masses, from [72].

which is typically seen in models in which the gluino is heavier than the squarks and the LSP is gaugino-like.

The decay of the N_2^0 can proceed by any of the mechanisms:

$$\begin{aligned} &\widetilde{N}_2^0 \to \ell^\pm + \widetilde{\ell}^\mp \ , \ \widetilde{\ell}^\mp \to \ell^\mp \widetilde{N}_1^0 \\ &\widetilde{N}_2^0 \to \widetilde{N}_1^0 Z^0 \ , \ Z^0 \to \ell^+\ell^- \\ &\widetilde{N}_2^0 \to \widetilde{N}_1^0 Z^{0*} \ , \ Z^{0*} \to \ell^+\ell^- \ . \end{aligned} \tag{211}$$

The last line indicates a virtual Z^0, decaying off-shell. In a model with gaugino unification and heavy Higgsinos, $\widetilde{N}_2$ is mainly $\widetilde{w}^0$ and $\widetilde{N}_1$ is mainly $\widetilde{b}^0$. Then these modes are preferred in the order listed as long as they are kinematically allowed. If the slepton decay is allowed, this is the dominant model. Otherwise, the decay to $\widetilde{N}_1 Z^0$ or other open two-body decays dominate. If no two-body decays are open, the $\widetilde{N}_2$ must decay through three-body processes such as the last line of (211).

The decay to an on-shell Z^0 is hard to work with [75], but the other two cases can be explored in depth. It is useful to begin with the *Dalitz plot* associated with the 3-body $(\widetilde{N}_1, \ell^+, \ell^-)$ system. Let

$$x_0 = \frac{2E(\widetilde{N}_1)}{m(\widetilde{N}_2)} \ , \quad x_+ = \frac{2E(\ell^+)}{m(\widetilde{N}_2)} \ , \quad x_- = \frac{2E(\ell^-)}{m(\widetilde{N}_2)} \ , \tag{212}$$

where the energies are measured in the rest frame of the N_2. The three variables are related by

$$x_0 + x_1 + x_2 = 2 \ . \tag{213}$$

The three-body decay phase space is given by

$$\int d\Pi_3 = \frac{m^2(\widetilde{N}_2)}{128\pi^3} \int dx_+ \, dx_- \ ; \tag{214}$$

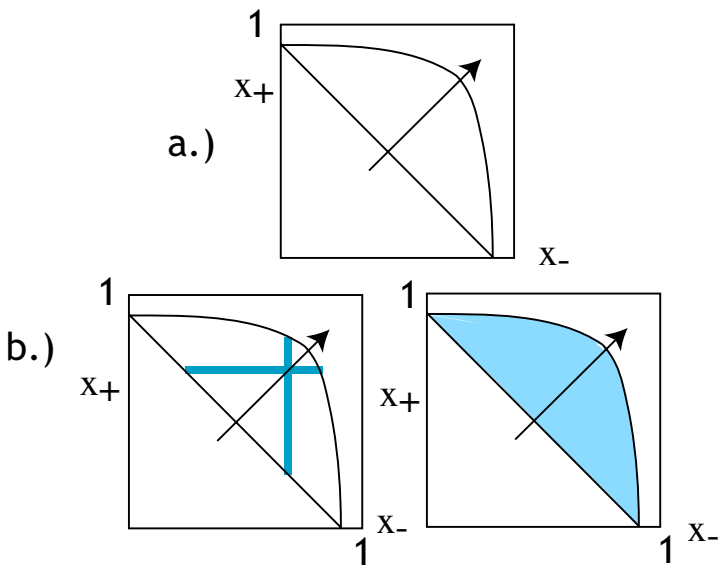

Figure 24: The Dalitz plot describing 3-body neutralino decays, $\widetilde{N}_2^0 \to \widetilde{N}_1^0 \ell^+ \ell^-$.

that is, phase space is flat in the variables (212). The basic kinematic identities involving the Dalitz plot variables are straightforward to work out, especially if we ignore the masses of the leptons. The kinematically allowed region is a wedge of the (x_+, x_-) plane bounded by the curves

$$\begin{aligned} x_+ + x_- &= 1 - (m(\widetilde{N}_1)/m(\widetilde{N}_2))^2 \\ (1-x_+)(1-x_-) &= (m(\widetilde{N}_1)/m(\widetilde{N}_2))^2 \ , \end{aligned} \tag{215}$$

as shown in Fig. 24(a). The invariant masses of two-body combinations are given in terms of the x_a by

$$\frac{m^2(\widetilde{N}_1 \ell^\pm)}{m^2(\widetilde{N}_2)} = (1 - x_\mp) \ , \quad \frac{m^2(\ell^+\ell^-)}{m^2(\widetilde{N}_2)} = (1 - \frac{m(\widetilde{N}_1)^2}{m(\widetilde{N}_2)^2}) \ . \tag{216}$$

I am assuming that the $\widetilde{N}_1$ is stable and weakly interacting. In this case, the $\widetilde{N}_1$ will not be observed in the LHC experiments, and also the frame of the $\widetilde{N}_2$ cannot be readily determined. The only property of this system that is straightforward to measure is the two-body invariant mass $m(\ell^+\ell^-)$. So it is interesting to note that the distribution of this quantity distinguishes the first and third cases in (211), in the manner shown in Fig. 24(b). In the case of a two-body decay to an intermediate slepton, the decays populate two lines on the Dalitz plot, leading to a sharp discontinuity at the kinematic endpoint. In the case of a three-body decay, the events fill the whole Dalitz plot, producing a distribution with a slope at the endpoint. With a good understanding of the detector resolution in the dilepton invariant mass, these cases can be distinguished experimentally.

In the three-body case, the endpoint of the dilepton mass distribution is exactly

$$m(\widetilde{N}_2) - m(\widetilde{N}_1) \ , \tag{217}$$

so the observable mass distribution gives a precise measurement of this SUSY mass difference. The shape of the spectrum has more information. For example, for heavy slepton masses, the shape

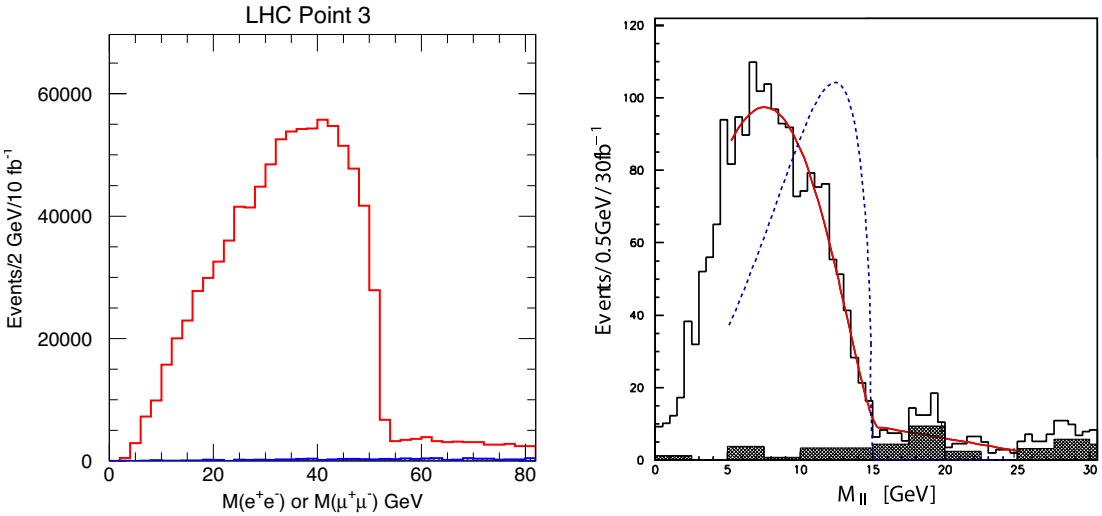

Figure 25: Distribution of the dilepton invariant mass in two supersymmetry models with 3-body neutralino decays: (a.) a model with gaugino-like neutralinos [72], (b.) a model with Higgsino-like neutralinos [73]. In the second figure, the dashed curve indicates the $m(\ell^+\ell^-)$ spectrum expected for gaugino-like neutralinos with the same mass splitting.

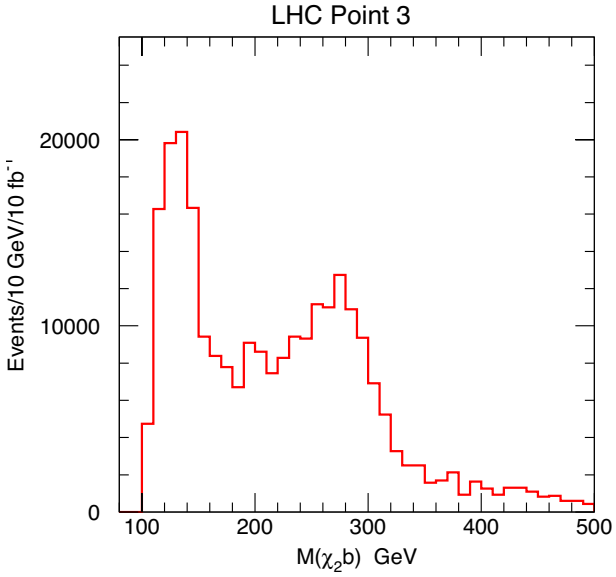

Figure 26: Reconstruction of a squark in the model of Fig. 25(a) by combining a dilepton pair at the endpoint of the $m(\ell^+\ell^-)$ distribution, the $\widetilde{N}_1^0$ in the same frame with mass determined from kinematics, and a b-tagged quark jet.

is distinctly different for gaugino-like or Higgsino-like neutralinos. Figure 25(a) shows the dilepton mass distribution for an mSUGRA parameter set for which the lightest two neutralinos are gaugino-like [72]. Figure 25(b) shows this distribution for a parameter set in which the two lightest neutralinos are Higgsino-like [73].

At the endpoint, the dilepton mass is maximal, and this requires that both the dilepton pair and the N_1 are at rest in the frame of the N_2. By measuring the four-vectors of the leptons, we would then know the N_1 and N_2 four-vectors, up to knowledge of the N_1 mass. It is possible to obtain this mass approximately from other measurements, for example, from the kinematics of $\widetilde{\widetilde{q}}$ decays directly to N_1. With this information, we could determine the N_2 four-vector. Now the problem of missing momentum is solved. By adding observed jets to the N_2 four-vector, it is possible to find squarks as resonances [72]. Figure 26 shows the result of such an analysis for the SUSY parameter set of Fig. 25. The peak just below 300 GeV is a reconstructed $\widetilde{b}$ squark.

The two-body case of $\widetilde{N}_2$ decay is even nicer. In this case, we can see from the right-hand figure in Fig. 24(b) that the endpoint of the dilepton mass distribution is not located at the mass difference (217) but instead at the smaller value

$$m(\ell^+\ell^-) = m(\widetilde{N}_2)\sqrt{1-\frac{m^2(\widetilde{\ell})}{m^2(\widetilde{N}_2)}}\sqrt{1-\frac{m^2(\widetilde{N}_1)}{m^2(\widetilde{\ell})}}\ . \tag{218}$$

Figure 27 shows an example of the dilepton spectrum from a SUSY parameter point in this region [63] The decay $\widetilde{q} \to qN_2$ is also a two-body decay, and there are similar kinematic relations for the upper and lower endpoints of the $(q\ell)$ and $(q\ell\ell)$ invariant mass distributions. These endpoints are likely to be visible in the collider data. Figure 28 shows two jet-lepton mass distributions from a similar analysis presented in [76]. In that analysis, it was possible to identify five well-measured kinematic endpoints, from which it was possible to solve (in an overdetermined way) for the four masses $m(N_1)$, $m(\widetilde{\ell})$, $m(N_2)$, $m(\widetilde{q})$.

There is one more case of an $\widetilde{N}_2 \to \widetilde{N}_1$ decay that should be mentioned. If two-body decays of $\widetilde{N}_2$ to sleptons are not kinematically allowed but the decay to $\widetilde{N}_1 h^0$ is permitted, this decay to a Higgs boson will be the dominant $\widetilde{N}_2$ decay. In this case, supersymmetry can provide a copious source of Higgs bosons. Figure 29 shows an analysis of a SUSY model in this parameter region [67]. Events with multijets and missing transverse energy are selected. In this sample, the mass distribution of two b-quark-tagged jets is shown. The signature of SUSY selects a sample of events in which the Higgs boson is visible in its dominant decay to $b\bar{b}$.

There is much more to say about the measurement of SUSY parameters at the LHC. Some more sophisticated sets of variables are introduced and applied in [76,77]. The question of measuring the spins of superparticles is discussed in [78,79,80,81]. And, we have not touched on alternative possibilities for the realization of SUSY, with R-parity violation or charged superparticles that are observed in the LHC experiments as stable particles. A broader overview of SUSY phenomenology at the LHC can be found in the references cited at the beginning of this section.

6 Electroweak Symmetry Breaking and Dark Matter in the MSSM

6.1 Electroweak Symmetry Breaking in the MSSM

In Section 1.2, I motivated the introduction of SUSY with the claim that SUSY could give an explanation of electroweak symmetry breaking, and for the presence of weakly interacting dark

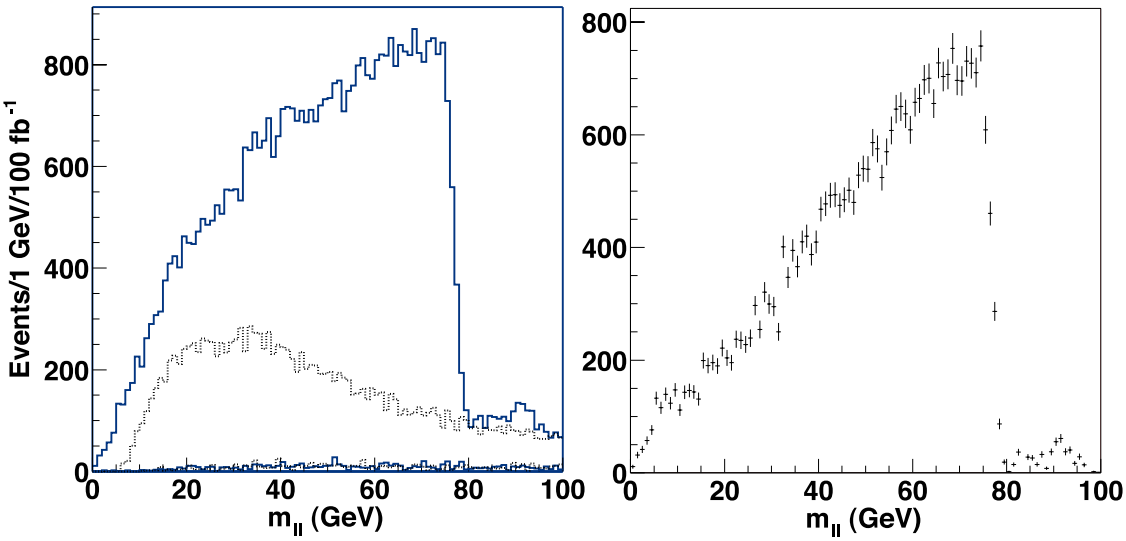

Figure 27: Dilepton mass distribution in a model with two-body $\widetilde{N}_2$ decays, from [63]. The left-hand plot shows the dilepton mass distributions for opposite-sign same-flavor dileptons (solid) and for opposite-sign opposite-flavor dileptons (dashed). The lower histograms give the estimates of the Standard Model background. The right-hand plot shows the difference of the two distributions.

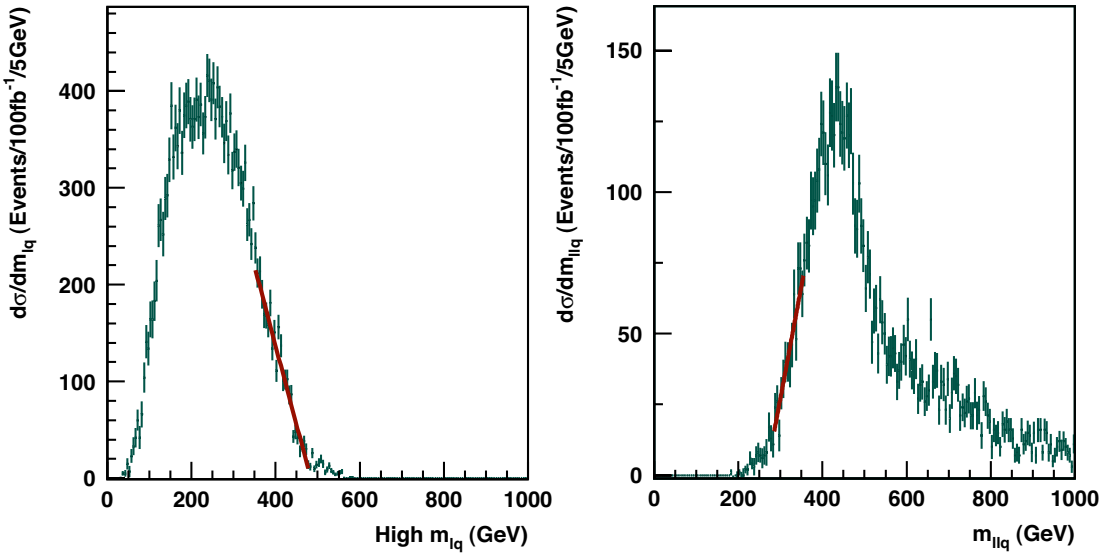

Figure 28: Distributions of mass combinations of leptons and high-p_T jets showing kinematic endpoints in the analysis of [76]: (a.) the higher $m(q\ell)$ combination; (b.) the $m(q\ell^+\ell^-)$ distribution.

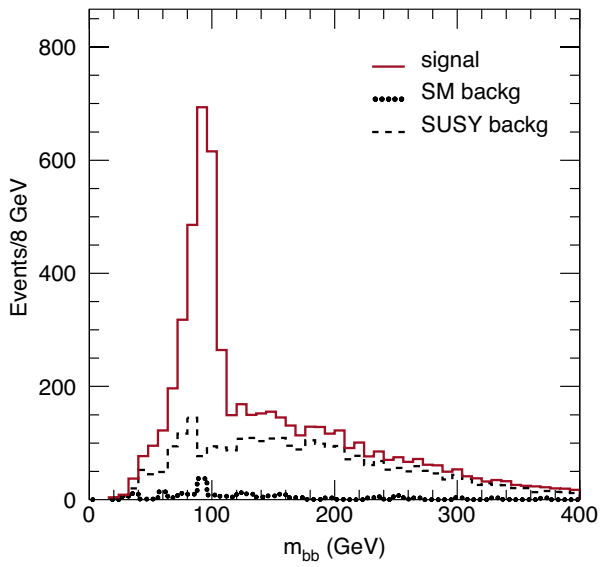

Figure 29: The dijet mass distribution for 2 b-tagged jets at a point in the SUSY parameter space where the decay $\widetilde{N}_2^0 \to h^0 \widetilde{N}_1^0$ is dominant, from [67].

matter in the universe. Now that we have a detailed understanding of the structure of the MSSM, it is time to come back and discuss these issues.

To present the mechanism of electroweak symmetry breaking in the MSSM, I need to add a term to one of the equations that I derived in Section 4.3. In (190), I presented the RG equation for the soft SUSY breaking scalar mass parameters, including renormalization effects from gauge interactions. I remarked that the contributions to this equation from Higgs Yukawa couplings are small for the scalars of the first and second generations. However, for the scalars of the third generation, these corrections can plan an important role.

The F-term interaction

$$Ł = -\left|y_t H_u \cdot \widetilde{t}\right|^2 \tag{219}$$

leads to a contribution to the RG equations for M_t, the mass parameter of $\widetilde{t}$, proportional to M_{Hu}^2, from the diagram shown in Fig. 30. The value of the diagram is

$$-\, i y_t^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2} (-i M_{Hu}^2) \frac{i}{k^2} = \frac{i}{(4\pi)^2} y_t^2 M_{Hu}^2 \log \Lambda^2 \ . \tag{220}$$

A scalar self-energy diagram is interpreted as $-i\delta m^2$, so this is a *negative* contribution to M_t^2. Each of the scalar fields $(H_u, \widetilde{t}, \widetilde{\bar{t}})$ gives a similar contribution that renomalizes the soft mass parameter of each of the others. For each correction, there is a counting factor from the number of color or $SU(2)$ degrees of freedom that run around the loop. There is also a correction to each of the scalar masses from the top quark A term. We must also remember that all of these terms add to the positive mass correction from the gaugino loops in Fig. 11, of which the gluino loop correction is the most important.

Taking all of these effects into account, we find for the RG equations of the soft mass parameters

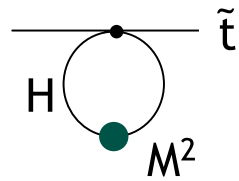

Figure 30: Diagram contributing a term to the renormalization group equation for the soft mass parameter of $\widetilde{t}$ proportional to the soft mass parameter for H_u.

of H_u, t, and $\bar{t}$

$$\begin{aligned}
\frac{dM_t^2}{d\log Q} &= \frac{2}{(4\pi)^2}\cdot 1\cdot y_t^2[M_t^2+M_{\widetilde{t}}^2+M_{Hu}^2+A_t^2]-\frac{8}{3\pi}\alpha_3 m_3^2+\cdots \\
\frac{dM_{\widetilde{t}}^2}{d\log Q} &= \frac{2}{(4\pi)^2}\cdot 2\cdot y_t^2[M_t^2+M_{\widetilde{t}}^2+M_{Hu}^2+A_t^2]-\frac{8}{3\pi}\alpha_3 m_3^2+\cdots \\
\frac{dM_{Hu}^2}{d\log Q} &= \frac{2}{(4\pi)^2}\cdot 3\cdot y_t^2[M_t^2+M_{\widetilde{t}}^2+M_{Hu}^2+A_t^2]m_3^2+\cdots
\end{aligned} \tag{221}$$

The structure is very interesting. The three scalar fields H_u, $\widetilde{t}$, and $\widetilde{\bar{t}}$ all receive negative corrections to their mass terms as these equations are integrated in the direction of decreasing $\log Q$. If any of these mass terms were to become negative, the corresponding field would have an instability to develop a vacuum expectation value, and the symmetry of the MSSM would be spontaneously broken. The symmetry-breaking we want is that associated with $\langle H_u\rangle \neq 0$. However, it seems equally possible that we could generate $\langle\widetilde{\bar{t}}\rangle \neq 0$, which would break color $SU(3)$, or $\langle\widetilde{t}\rangle \neq 0$, which would break both $SU(2)$ and $SU(3)$.

If the three mass parameters have similar values at a high mass scale, they race toward negative values according to (221). But H_u wins the race, and so the theory predicts the symmetry breaking pattern that is the one observed. In this way, the MSSM leads naturally to electroweak symmetry breaking and realizes the idea that electroweak symmetry breaking is connected to the large value of the top quark-Higgs coupling.

6.2 Higgs Boson Masses in the MSSM

Once we expect that $M_u^2 < 0$ at the weak scale, we can work out the details of the Higgs boson spectrum. First, we should write the potential for the Higgs fields H_u, H_d. As in the discussion of Sections 4.1 and 4.2, a number of terms need to be collected from the various pieces of the Lagrangian. The F terms contriubute

$$V_F = \mu^2(H_u^{0*}H_u^0 + H_d^{0*}H_d^0) \tag{222}$$

The D terms contribute

$$V_D = \frac{g^2+g'^2}{8}(H_u^{0*}H_u^0 - H_d^{0*}H_d^0)^2 \tag{223}$$

The soft SUSY breaking terms contribute

$$V_{soft} = M_{Hu}^2 H_u^{0*}H_u^0 + M_{Hd}^2 H_d^{0*}H_d^0 - (B\mu H_u^0 H_d^0 + h.c.) \tag{224}$$

The sum of these terms gives the complete tree-level Higgs potential. Differentiating this potential with respect to H_u^0 and H_d^0, we obtain the equations that determine the Higgs field vacuum expectation values. If we write these equations with the parametrization of the vacuum expectation values given in (169), we find

$$\begin{aligned} \mu^2 + M_{Hu}^2 &= B\mu \cot\beta + \frac{1}{2} m_Z^2 \cos 2\beta \\ \mu^2 + M_{Hd}^2 &= B\mu \tan\beta - \frac{1}{2} m_Z^2 \cos 2\beta \ , \end{aligned} \tag{225}$$

where $m_Z^2 = (g^2 + g'^2)v^2/4$. This system of equations can be solved for μ to give

$$\mu^2 = \frac{M_{Hd}^2 - \tan^2\beta M_{Hu}^2}{\tan^2\beta - 1} - \frac{1}{2} m_Z^2 \tag{226}$$

This is, for example, the way that we would determine μ in the mSUGRA parameter space described in Section 5.2.

It is interesting to turn this equation around and write it as an equation for m_Z in terms of the SUSY parameters,

$$m_Z^2 = 2\,\frac{M_{Hd}^2 - \tan^2\beta M_{Hu}^2}{\tan^2\beta - 1} - 2\mu^2 \ . \tag{227}$$

From this equation, a small value of m_Z would require a cancellation between the Higgs soft mass parameters and μ. The parameter μ sets the mass scale of the Higgsinos, and the Higgs soft mass parameters might be related to other masses of the SUSY scalar particles. Thus, if the masses of the charginos and neutralinos and, perhaps also, the sleptons are not close to m_Z, that disparity must be associated with an apparently unnatural cancellation between different SUSY parameters.

If we prohibit a delicate cancellation in (227), we put an upper bound on the SUSY partner masses. To avoid cancellations in more than two decimal places, μ must be less than 700 GeV. Similarly, we find bounds on the Higgs soft masses, and on the parameters that contribute to these masses through the RG equation. This consideration turns out to give a constraint on the gluino mass, $m_3 < 800$ GeV. Assuming gaugino universality, this becomes a condition $m_2 < 250$ GeV that restricts the chargino and neutralino masses. A variety of similar naturalness arguments that constrain the SUSY scale can be found in [82,83,84]. Though the logic is that of an estimate rather than a rigorous bound, this analysis strongly supports the idea that SUSY partners should be light enough to be discovered at the LHC and at the ILC.

Once we have the Higgs potential and the conditions for the Higgs vacuum expectation values, we can work out the masses of the Higgs bosons by expanding the potential around its minimum. A first step is to identify the combinations of Higgs fields that correspond to physical Higgs bosons. Look first at the charged Higgs bosons. There are two charged Higgs fields in the multiplets H_u, H_d. One linear combination of these fields is the Goldstone boson that is eaten by the W boson as it obtains mass through the Higgs mechanism. The orthogonal linear combination is a physical charged scalar field. If we decompose

$$\begin{aligned} H_u^+ &= \cos\beta H^+ + \sin\beta G^+ \\ H_d^- &= \sin\beta H^- + \sin\beta G^- \end{aligned} \tag{228}$$

where $H^- = (H^+)^*$, $G^- = (G^+)^*$, and β is precisely the mixing angle in (169), it can be seen that $G^\pm$ are the Goldstone bosons and $H^\pm$ are the physical scalar states.

A similar analysis applies to the neutral components of H_u^0 and H_d^0. These are complex-valued fields. It is appropriate to decomposed them as

$$H_u^0 = \frac{1}{\sqrt{2}}(v\sin\beta + \sin\alpha H^0 + \cos\alpha h^0 + i\cos\beta A^0 + i\sin\beta G^0)$$
$$H_d^0 = \frac{1}{\sqrt{2}}(v\cos\beta + \cos\alpha H^0 - \sin\alpha h^0 + i\sin\beta A^0 - i\cos\beta G^0) \tag{229}$$

The components H^0, h^0 are even under CP; the fields A^0, G^0 are odd under CP. The componet G^0 is the Goldstone boson eaten by the Z^0. The other three fields create physical scalar particles.

Having identified these fields, we can compute their masses. The formulae for the Higgs masses take an especially simple form when they are expressed in terms of the mass of the A^0. For the charged Higgs boson

$$m_{H+}^2 = m_A^2 + m_W^2 \ . \tag{230}$$

For the CP-even scalars, one finds a mass matrix

$$\begin{pmatrix} m_A^2\sin^2\beta + m_Z^2\cos^2\beta & -(m_A^2+m_Z^2)\sin\beta\cos\beta \\ -(m_A^2+m_Z^2)\sin\beta\cos\beta & m_A^2\cos^2\beta + m_Z^2\sin^2\beta \end{pmatrix} \tag{231}$$

The physical scalar masses m_h^2 and m_H^2 are the eigenvalues of this matrix, defined in such a way that $m_h^2 < m_H^2$. The angle α in (229) is the mixing angle that defines these eigenstates.

Taking the trace of (231), we find the relation

$$m_h^2 + m_H^2 = m_A^2 + m_Z^2 \ . \tag{232}$$

We can also obtain an upper bound on the lighter Higgs mass m_h^2 by taking the matrix element of (231) in the state $(\cos\beta, \sin\beta)$. The bound is a very strong one:

$$m_h^2 \le m_Z^2\cos^2\beta \ < m_Z^2 \ . \tag{233}$$

This seems inconsistent with lower bounds on the Higgs boson mass from LEP 2, which exclude $m_h < 114$ GeV for the Standard Model Higgs and for most scenarios of SUSY Higgs bosons [85].[§] However, the one-loop corrections to the tree-level result (231) give a significant positive correction

$$\delta m_h^2 = \frac{3}{\pi}\frac{m_t^4}{m_W^2}\sin^4\beta\,\log\frac{m_{\tilde{t}}m_{\tilde{\bar{t}}}}{m_t^2} \ . \tag{234}$$

This correction can move the mass of the h^0 up to about 130 GeV. The detailed summary of the radiative corrections to the h^0 mass in the MSSM is presented in [88]. A very clear and useful accounting of the major corrections can be found in [89].

It is possible to raise the mass of the h^0 by going outside the MSSM and adding additional $SU(2)$ singlet superfields to the model. However, this strategy is limited by a general constraint coming from grand unification. The requirement that the Higgs couplings do not become strong up to the grand unification scale limit the mass of the Higgs to about 200 GeV [90]. It is possible to raise the mass of the Higgs further only by enlarging the Standard Model gauge group or adding new thresholds that affect unification [91,92].

In the MSSM, we can easily have the situation in which $m_A \gg m_h$. In this limit, the couplings of the h^0 are very close to those of the Standard Model Higgs boson, and the H^0, A^0, and $H^\pm$ are almost degenerate. If $\tan\beta \gg 1$, the heavy neutral Higgs bosons decay dominantly to $b\bar{b}$ and $\tau^+\tau^-$.

Much more about the phenomenology of Higgs bosons in supersymmetry can be found in [93,94].

[§]Some exceptional Higgs decay schemes that escape these bounds are considered in [86,87].

6.3 WIMP Model of Dark Matter

Now we turn to the second problem highlighted in the Introduction, the problem of dark matter in the universe. It has been known from many astrophysical measurements that the universe contains enormous amounts of invisible, weakly interacting matter. For an excellent review of the classic astrophysical evidence for this dark matter, see [95].

In the past few years, measurements of the cosmic microwave background have given a new source of evidence for dark matter. Since this data comes from an era in the early universe before the formation of any structure, it argues strongly that the invisible matter is not made of rocks or brown dwarfs but is actually a new, very weakly interacting form of matter. These measurements also determine quite accurately the overall amount of conventional and dark matter in the universe. Let ρ_b, ρ_N, and ρ_Λ be the large-scale energy densities of the universe from baryons, dark matter, and the energy of the vacuum. The data from the microwave background tells us that $\rho_b + \rho_N + \rho_\Lambda = \rho_c$, the 'closure density' corresponding in general relativity to a flat universe, to about 1% accuracy. If $\Omega_i = \rho_i/\rho_c$, the most recent data from the WMAP experiment and other sources gives [96,97]

$$\Omega_b = 0.042 \pm 0.003 \quad \Omega_N = 0.20 \pm 0.02 \quad \Omega_\Lambda = 0.74 \pm 0.02 \ . \tag{235}$$

These results present a double mystery. We do not know what particle the dark matter is made of, and we do not have any theory that explains the observed magnitude of the vacuum energy or 'dark energy'.

I believe that supersymmetry will eventually play an essential role in solving the problem of dark energy. In ordinary quantum field theory, the value of the vacuum energy is quartically divergent, so the problem of computing the vacuum energy is not even well-posed. In supersymmetry, there is at least a well-defined zero of the energy associated with exact supersymmetry, which implies $\langle 0| H |0\rangle = 0$. Unfortunately, in most of today's models of supersymmetry, the vacuum energy is set by the SUSY breaking scale. This gives $\Lambda \sim (10^{11}\ \text{GeV})^4$, about 80 orders of magnitude larger than the observed value of the vacuum energy. From this starting point, Λ must be fine-tuned to the scale of eV^4. This is an important problem that needs new insights which, however, I will not provide here.

On the other hand, supersymmetry offers a very definite solution to the problem of the origin of dark matter. We have already noted in Section 3.4 that it is straightforward to arrange that the lightest supersymmetric particle can be absolutely stable. If this particle were produced in the early universe, some density of this type of matter should still be present. In most, but not all, regions of parameter space, the lightest supersymmetric particle is neutral. Candidates include the lightest neutralino, the lightest sneutrino, and the gravitino. In the remainder of these lectures, I will concentrate on the case in which the lightest neutralino is the dark matter particle. For a discussion of the other candidates, see [98].

To begin our discussion, I would like to estimate the cosmic density of dark matter in a more general context. Let me make the following minimal assumptions about the nature of dark matter, that the dark matter particle is stable, neutral, and weakly interacting. To these properties, I would like to add one more, that dark matter particles can be created in pairs at sufficiently high temperature, and that, at some time in the early universe, dark matter particles were in thermal equilibrium. I will refer to a particle satisfying these assumptions as a 'weakly interacting massive particle' or WIMP. The assumption of thermal equilibrium is a strong one that is not satisfied even in many models of supersymmetric dark matter. For some exceptions, see [99,100]. However, let us see what implications follow from these assumptions.

The assumption that WIMPs were once in thermal equilibrium provides a definite initial condition

from which to compute the current density of dark matter. In thermal equilibrium at temperture T, we have for the number density of dark matter particles

$$n_{eq} = \frac{g}{(2\pi)^{3/2}}(mT)^{3/2}e^{-m/T} \ . \tag{236}$$

where g is the number of spin degrees of freedom of the massive particle. As the universe expands, the temperature of the universe deccreases and the rate of WIMP pair production becomes very small. But the rate of dark matter pair annihilation also becomes small as the WIMPs separate from one another.

The expansion of the universe is governed by the Hubble constant $H = \dot{a}/a$,where a is the scale factor. Einstein's equations imply that

$$H^2 = \frac{8\pi}{3}\frac{\rho}{m_{\rm Pl}^2} \ . \tag{237}$$

In a radiation-dominated universe where g_* is the number of relativistic degrees of freedom, $\rho = \pi^2 g_* T^4/30$. Then H is proportional to T^2. In a radiation-dominated universe, the temperature red-shifts as the universe expands, so that $T \sim a^{-1}$. Combining this relation with the equation $H = \dot{a}/a \sim T^2$, we find $t \sim T^{-2} \sim a^2$, that is, $a \sim t^{1/2}$ or $\dot{a}/a = 1/2t$. Setting this expression equal to the explict form of H in (237), we find a detailed formula for the time since the start of the radiation-dominated era for cooling to a temperature T,

$$t = \left(\frac{16\pi^3 g_*}{45}\right)^{-1/2}\frac{m_{\rm Pl}}{T^2} \ . \tag{238}$$

The evolution of the WIMP density is described by the Boltzmann equation

$$\frac{dn}{dt} = -3Hn - \langle\sigma v\rangle\left(n^2 - n_{eq}^2\right) \ , \tag{239}$$

where H is the Hubble constant, σ is the $\widetilde{N}\widetilde{N}$ annihilation cross section—which appears thermally averaged with the relative velocity of colliding WIMPs—and n_{eq} is the equilibrium WIMP density (236). Assume, just for the sake of argument, that the temperature T is of the order of 100 GeV. At this temperature, the Hubble constant has the magnitude $H \sim 10^{-17}T$, so the expansion of the universe is very slow on the scale of typical elementary particle reactions. However, when T becomes less than the WIMP mass m, the WIMP density is exponentially suppressed and so the collision term in the Boltzmann equation is also very small. These two terms are of the same size at the *freezeout* temperature T_F satisfying

$$e^{-m/T_F} \sim \frac{1}{m_{\rm Pl} m\langle\sigma v\rangle} \ . \tag{240}$$

At temperatures below T_F, we may neglect the production of WIMPs in particle collisions. The WIMP density is then determined by the expansion of the universe and the residual rate of WIMP pair annihilation. Maybe it is more appropriate to think of T_F as the temperature at which a WIMP density is frozen *in*. To determine the freezeout temperature, we take the logarithm of the right-hand side of (240). The result depends only on the order of magnitude of the annihilation cross section. For any interaction of electroweak strength,

$$\xi_F = T_F/m \sim 1/25 \ . \tag{241}$$

This physical picture suggests a way to estimate the cosmic density of WIMP dark matter. We can take as our initial condition the thermal density of dark matter at freezeout. We then integrate

the Boltzmann equation, ignoring the term proportional to n_{eq}^2 associated with the production of WIMP pairs [102].

In analyzing the Boltzmann equation, it is useful normalize the particle density n of dark matter to the density of entropy s. Since the universe expands very slowly, this expansion is very close to adiabatic. Then entropy is conserved,

$$\frac{ds}{dt} = -3Hs \ . \tag{242}$$

In a radiation-dominated universe, $s = 2\pi^2 g_* T^3/45$. Now define

$$Y = \frac{n}{s} \ , \qquad \xi = \frac{T}{m} \ , \tag{243}$$

the latter as in (241). Using the expression (238), we can convert the evolution in time to an evolution in temperature or in ξ. Applying these changes of variables and dropping the n_{eq}^2 term, the Boltmann equation (239) rearranges to the form

$$\frac{dY}{d\xi} = C \langle \sigma v \rangle Y^2 \ , \tag{244}$$

where

$$C = \left(\frac{\pi g_*}{45}\right)^{1/2} m m_{\rm Pl} \ . \tag{245}$$

Let Y_F be the value of Y at $\xi = \xi_F$. If we assume that $\langle \sigma v \rangle$ is approximately constant, since we are at temperatures close to threshold, it is straightforward to integrate this equation to $\xi = 0$, corresponding to late times.

$$Y^{-1} = Y_F^{-1} + C\xi_F \langle \sigma v \rangle \ . \tag{246}$$

The second term typically dominates the first. Then we can put back the value of C in (245) and write the final answer in terms of the ratio of the mass density of dark matter to the closure density $\Omega_N = n m_N/\rho_c$. In this way, we find

$$\Omega_N = \frac{s_0}{\rho_c}\left(\frac{45}{\pi g_*}\right)^{1/2} \frac{1}{\xi_F m_{\rm Pl}} \frac{1}{\langle \sigma v \rangle} \ , \tag{247}$$

where s_0 is the current entropy density of the universe. Turner and Scherrer observed that this formula gives a value of Ω_N that is usually within 10% of the result from exact integration of the Boltzmann equation [102]. If $\langle \ sigmav \rangle$ has a significant dependence on temperature, the derivation is still correct with the replacement

$$\xi \langle \sigma v \rangle \to \int_0^{\xi_f} d\xi \, \langle \sigma v \rangle (\xi) \tag{248}$$

in the denominator of the last term in (247).

This is a remarkable relation. Almost every factor in this relation is known from astrophysical measurements. The left-hand side is given by (235). On the right-hand side, the entropy density of the universe is dominated by the entropy of the microwave background photons and can be computed from the microwave background temperature. The closure density is known from the measurement of the Hubble constant and the observation that the universe is flat. The parameters g_* and ξ_F are relatively insensitive to the strength of the annihilation cross section, with values $g_* \sim 100$, $\xi_F \sim 1/25$. The mass of the WIMP does not appear explicitly in (247). We can then solve for $\langle \sigma v \rangle$. The result is

$$\langle \sigma v \rangle = 1 \text{ pb} \ . \tag{249}$$

This is the value of a typical electroweak cross section at energies of a few hundred GeV. If we convert this value to a mass M of an exchanged particle using the formula

$$\langle \sigma v \rangle = \frac{\pi \alpha^2}{8M^2} \ , \tag{250}$$

the value (249) corresponds to $M = 100$ GeV.

I consider this a truly remarkable result. From a purely astrophysical argument, relying on quite weak and general assumptions, we arrive at the conclusion that there must be new particles at the hundred GeV energy scale. It is probably not a concidence that this argument leads us back to the mass scale of electroweak symmetry breaking.

In our study of supersymmetry, we have found an argument from the physics of electroweak symmetry breaking that predicts the existence of dark matter. As I discussed at the beginning of these lectures, models that explain electroweak symmetry breaking are complex. They typically involve many new particles. It is easily arranged that the lightest of the new particles is neutral. In supersymmetry, there is a reason why the new particles are likely to carry a conserved quantum number (149). Other models of electroweak symmetry breaking, such as the extra dimensional and little Higgs models discussed in Section 1.2, have their own reasons to have a complex particle spectrum and discrete symmetries. Then these models lead in their own ways to WIMPs at the hundred GeV mass scale.

A slight extension of this argument adds more interest. In supersymmetry, the sector of new particles includes particles with QCD color. Since the top quark probably plays an essential role in the mechanism of electroweak symmetry breaking, it is very likely that, in any model, some of the new particles will carry color. If these particles have masses below 1 TeV, they have large (10 pb) pair-production cross sections at the LHC. These particles will then decay to the dark matter particle, producting complex events with several hard jets and missing transverse momentum. These mild assumptions thus lead to the conclusion, from any model that follows this general line of argument, that *we should expect exotic events with multiple jets and missing transverse momentum to appear with pb cross sections at the LHC.*

6.4 Dark Matter Annihilation in the MSSM

This argument of the previous section gives a very optimistic conclusion for the discovery of new physics at the LHC. However, we have already discussed that the first observation of supersymmetry or another model of new physics will only be the first step in a lengthy experimental program. Once we know that superparticles or other new particles exist, we will need to study them in detail to learn their detailed interactions and, eventually, to work out the underlying Lagrangian that governs their behavior. As we have already discussed in Section 3.5 and 4.3, this Lagrangian can give us a clue to the nature of the ultimate theory at very short distances.

The study of dark matter intersects this program in an interesting way. In principle, once we have discovered supersymmetric particles, we can try to measure their properties and see if these coincide with the properties required from astrophysical detections of dark matter. As we have seen in Section 5.3, the LHC experiments expect to measure the mass of the LSP to about 10% accuracy. These measurements can hopefully be compared to mass measurements at the 20% level that can be expected from astrophysical dark matter detection experiments [103,104]. We would also wish to find out whether the annihilation cross section $\langle \sigma v \rangle$ that is predicted from the supersymmetry parameters measured at colliders agrees with the value (249) required to predict the observed WIMP relic density. This comparison turns out to depend in a complex way on the parameters of the underlying supersymmetry theory.

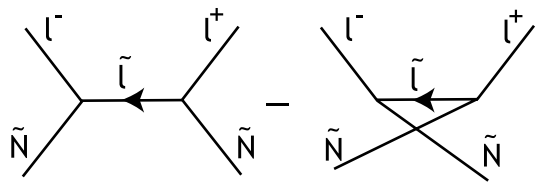

Figure 31: Diagrams giving the simplest scheme of neutralino pair annihilation, leading to the annihilation cross section (251).

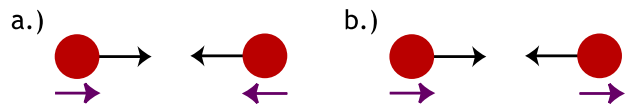

Figure 32: Two possible spin configurations for neutralino annihilation: (a.) spin 0; (b.) spin 1. Because of Fermi statistics, the latter state does not exist in the S-wave.

To begin our discussion of the annihilation cross section, we can make a simple model of neutralino annihilation and see how well it works. We have seen in Section 4.3 that the right-handed sleptons are often the lightest charged particles in the supersymmetry spectrum. Consider, then, an idealized parameter set in which the neutralino is a pure bino and pair annihilation is dominated by the slepton exchange diagrams shown in Fig. 31. (Away from the pure bino case, there are also s-channel diagrams with Z^0, h^0, H^0, A^0.) In this special limit, the annihilation cross section is given by

$$v\frac{d\sigma}{d\cos\theta} = \pi\alpha^2 m_N^2 \left|\frac{1}{c_w}\right|^2 \left|\frac{1}{m_{\widetilde{\ell}}^2 - t} - \frac{1}{m_{\widetilde{\ell}}^2 - u}\right|^2 , \tag{251}$$

where m_N is the $\widetilde{N}_1$ mass. The relative velocity v appears due to the flux factor in the cross section; this factor cancels in σv. I have ignored the lepton masses. This expression is of the order of (250) with $M \sim m_N$, except for one unfortunate feature: At threshold, $t = u$ and the cross section vanishes. This leads to a severe suppression, by a factor of

$$v^2 \cdot \left|\frac{m_N^2}{m_{\widetilde{\ell}}^2 + m_N^2}\right|^4 , \tag{252}$$

which is at least of order $\xi_f/16$. So the relic density estimated in this simple way is too large by about a factor of 10.

There is an interesting physics explanation for the vanishing of this cross section at threshold [105]. Neutralinos are spin-$\frac{1}{2}$ fermions, and we might guess from this that, near threshold, they would annihilate in the S-wave either in a spin 0 or in a spin 1 state. The two spin configurations are shown in Fig. 32. However, because the neutralino is a Majorana fermion and therefore its own antiparticle, an S-wave state of two neutralinos must be antisymmetric in spin. Hence, the spin 1 S-wave state does not exist However, as we know from pion decay, a spin 0 state can convert to a pair of light leptons only with a helicity flip. Thus, there is an annihilation cross section from the spin 0 S-wave only when lepton masses are included, and even then with the suppression factor m_ℓ^2/m_N^2, which is 10^{-4} even for $\tau^+\tau^-$ final states.

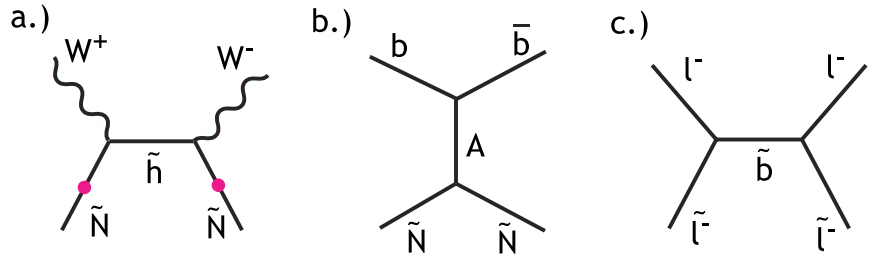

Figure 33: Three mechanisms for obtaining a sufficiently large annihilation cross section to give the observed density of neutralino dark matter: (a.) gaugino-Higgsino mixing, opening the annihilation channels to W^+W^- and Z^0Z^0, (b.) resonance annihilation through the Higgs boson A^0, (c.) co-annihilation with another supersymmetric particle, here taken to be a $\widetilde{\ell}$.

To obtain a realistic value for the neutralino relic density, we have to bring in more complicated mechanisms of neutralino annihilation. These mechanisms are not difficult to find in various regions of the large supersymmetry paramet er space [106,107,108]. We need to look for annihilation processes that can proceed in the S-wave with full strength. Three possible mechanisms are shown in Fig. 33.

Pairs of neutralinos can annihilate in the S-wave into vector bosons. The bino does not couple to W or Z pairs, but if the lightest neutralino has Higgsino or wino content, this reaction can be important. For charginos of mass about 200 GeV, this annihilation cross section can be 50 pb for a pure wino or Higgsino, so only a modest content of these states is needed to give a cross section of 1 pb.

The s-channel exchange of a Higgs boson can provide a mechanism for neutralino annihilation in the spin 0 S-wave. Because this state is CP-odd, it is the boson A^0 that is relevant here. If m_A is close to the neutralino threshold $2m_N$, the cross section has a resonant enhancement. Note that the $\widetilde{N}_1$ annihilation vertex to A arises as a Higgs-Higgsino-gaugino Yukawa term, so this vertex is nonzero only if $\widetilde{N}_1$ has both gaugino and Higgsino content. If $m_A = 2m_N$, the resonance enhancement is at full strength and the cross section can be as large as 50 pb. Thus, it is A boson masses about 20 GeV above or below the threshold that give the desired cross section (249).

The final mechanism shown in the figure is *coannihilation*. As we have discussed, the freezeout of the $\widetilde{N}_1$ occurs at a temperature given by $T/m_N \sim 1/25$. So if there is another particle in the supersymmetry spectrum that is within 4% of the $\widetilde{N}_1$ mass, this state will have a number density that remains in equilibrium with the number density of the $\widetilde{N}_1$. If this particle has S-wave annihilation reactions, those reactions can be the dominant mechanisms for the annihilation of supersymmetric particles. For a light slepton, the reactions

$$\widetilde{\ell}^- + \widetilde{N}_1^0 \to \ell^- + \gamma \ , \qquad \widetilde{\ell}^- + \widetilde{\ell}^- \to \ell^- + \ell^- \tag{253}$$

can give significant S-wave annihilation. In [106,109], the lighter stau is invoked as the coannihilating particle. In [110], the lighter top squark is invoked as the coannihilating state. If the lightest neutralinos and charginos are Higgsino-like, chargino coannihilation can also be important.

It is, then, a complex matter to predict the neutralino relic density from microscopic physics. We will first need to learn what particles in the supersymmetry spectrum play the dominant role as particle exchanged in annihilation reactions or as coannihilating species. We will then need to

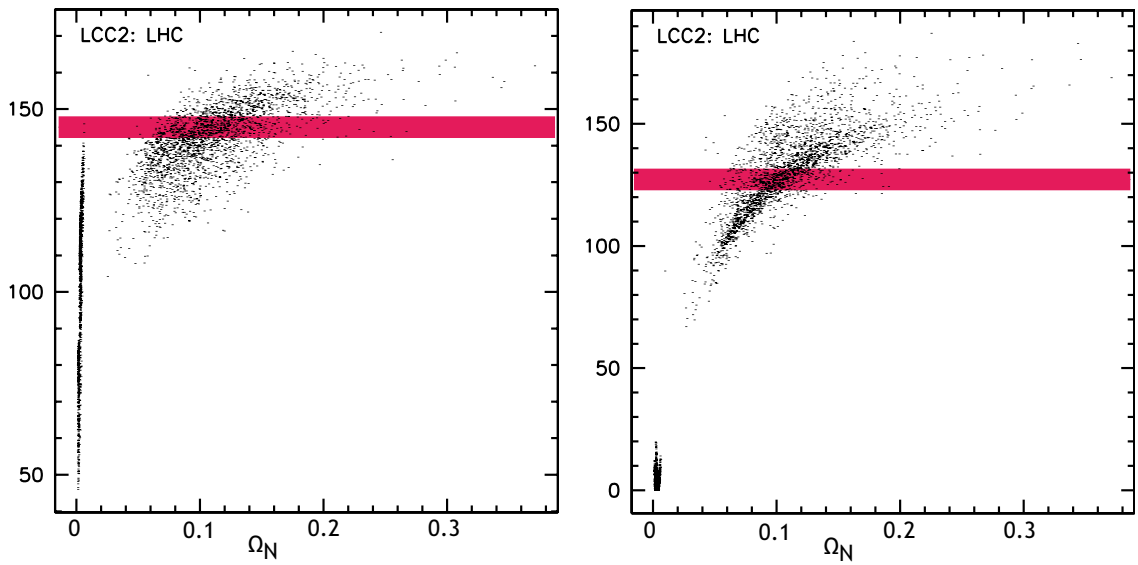

Figure 34: Scatter plot of SUSY parameter points consistent with data from the LHC in the analysis of the parameter set LCC2 from [111]. The horizontal axis show the value of Ω_N at each parameter point. The vertical axes show polarized-beam cross sections measurable at the ILC, in fb: (a.) $\sigma(e^-_R e^+_L \to \widetilde{C}^+_1 \widetilde{C}^-_1)$, (b.) $\sigma(e^-_R e^+_L \to \widetilde{N}^0_2 \widetilde{N}^0_3)$. The colored bands show the $\pm 1\sigma$ region allowed after the ILC cross section measurements.

measure the couplings and mixing angles of the important particles, since the dominant annihilation diagrams depend sensitively on these.

Some examples of how measurements at the LHC and ILC can accumulate the relevant information are described in [111]. Figure 34 shows a part of the analysis of this paper for a particular SUSY model in which the dominant annihilation reactions are $\widetilde{N}_1 \widetilde{N}_1 \to W^+W^-, Z^0Z^0$. As a first step, the authors constructed numerous supersymmetry parameter sets that were consistent with the mass spectrum of this model as it would be measured at the LHC. These parameter sets included a variety of models in which the LSP was dominantly bino and wino. The figure shows scatter plots of the predictions of these models with ILC cross sections for neutralino and chargino pair production on the vertical axis and Ω_N on the horizontal axis. The two cross sections clearly separate the bino- and wino-like solutions. The second of these cross sections is the polarized reaction of chargino pair production for which the cross section is displayed in Fig. 19. The horizontal lines represent the accuracy of the measurements of these cross sections expected at the ILC. These measurements select the bino solution and also play an important role in fixing the bino-Higgsino mixing angle which is a crucial input to the annihilation cross sections. In Fig. 35, I show the distribution of predictions for Ω_N expected for this model, in the analysis of [111], from the data on SUSY particles that would be obtained from the LHC, from the ILC at a center-of-mass energy of 500 GeV, and from the ILC at a center-of-mass energy of 1000 GeV.

The similar summary plot for another of the models considered in [111] is shown in Fig. 36. The model considered in this analysis is one in which the neutralino relic density is set by stau coannihilation. In this model, the stau would be discovered at the LHC, and the stau-neutralino mass difference would be measured to about 10% accuracy at the 500 GeV ILC. However, the annihilation reactions also depend on mixing angles and on the value of $\tan\beta$. In this scenario, these

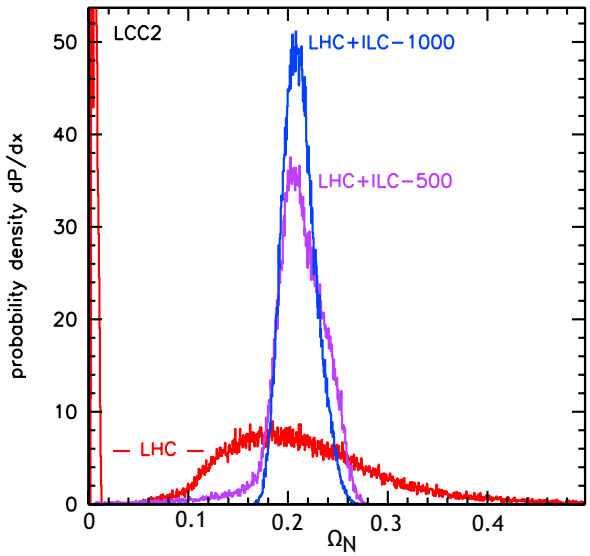

Figure 35: Summary plot for the prediction of Ω_N from collider data for the SUSY parameter set LCC2 considered in [111]. The three curves show the likelihood distributions for the prediction of Ω_N using data from the LHC, the ILC at 500 GeV, and the ILC at 1000 GeV.

are determined only by ILC measurements of some of the heavier states of the SUSY spectrum.

Collider measurements of the SUSY spectrum can also be used to constrain cross sections of the WIMP that are important for experiments that seek to detect dark matter, for example, the neutralino-proton cross section and the cross section for neutralino pair annihilation to gamma rays. If we can accurately predict these cross sections from collider data, the information about the SUSY spectrum that we learn from colliders will feed back into the astrophysics of dark matter. Some numerical examples that illustrate this are presented in [111].

7 Conclusions

In these lectures, I have given an overview of supersymmetry and its application to elementary particle physics. In the early sections of this review, I presented the formalism of SUSY and explained the rules for constructing supersymmetric Lagrangians. Our discussion then became more concrete, focusing on the mass spectrum of the MSSM and the properties of the particle states of the MSSM spectrum. This led us to a discussion of the experimental probes of this spectrum and the possibility of measurement of the parameters of the supersymmetric Lagrangian.

This possibility is now coming very near. As I have discussed in the last sections of this review, supersymmetry gives concrete answers to the major questions about elementary particle physics that we expect to be addressed at the hundred GeV scale—the questions of the origin of electroweak symmetry breaking and the identity of cosmic dark matter. In the next year, the LHC will begin to explore the physics of this mass scale. Supersymmetry is one candidate for what will be found. I hope that, after studying these lectures, you will agree that the picture provided by supersymmetry is highly plausible and even compelling.

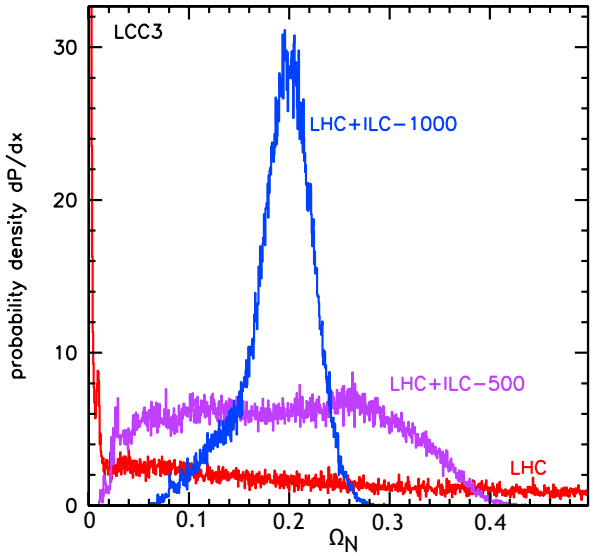

Figure 36: Summary plot for the prediction of Ω_N from collider data for the SUSY parameter set LCC3 of [111]. The notation is as in Fig. 35.

Whatever explanations we will learn from the LHC data, our investigation of it will follow the general paradigm that I have described here. In successive stages, we will use data from the LHC and the ILC to learn the mass spectrum of new particles that are revealed at the LHC, to determine their quantum numbers and couplings, and to reconstruct their underlying Lagrangian. On the basis of the detailed studies of this program that have been carried out for the MSSM, we have the expectation that we will be able to learn the underlying theory of the new particles and to test the specific explanations that this theory gives for the mysteries of the fundamental interactions.

Is supersymmetry just an attractive theory, or is it a part of the true description of elementary particles? We are about to find out.

ACKNOWLEDGEMENTS

I am grateful to Sally Dawson, Rabi Mohapatra and, especially, to K. T. Mahanthappa for organizing the 2006 TASI Summer School at which these lectures were presented. I thank Howard Haber and Thomas Dumitrescu for instructive comments on the manuscript. This work was supported by the US Department of Energy under contract DE–AC02–76SF00515.

References

[1] A. D. Linde, JETP Lett. **19**, 183 (1974) [Pisma Zh. Eksp. Teor. Fiz. **19**, 320 (1974)].

[2] L. Susskind, Phys. Rev. D **20**, 2619 (1979).

[3] S. Weinberg, Phys. Rev. D **19**, 1277 (1979).

[4] P. W. Higgs, Phys. Rev. Lett. **13**, 508 (1964).

[5] K. D. Lane, arXiv:hep-ph/9401324.

[6] S. P. Martin, arXiv:hep-ph/9709356.

[7] J. Wess and J. Bagger, *Supersymmetry and supergravity*. (Princeton U. Press, 1992).

[8] J. A. Bagger, arXiv:hep-ph/9604232.

[9] J. D. Lykken, arXiv:hep-th/9612114.

[10] G. L. Kane, arXiv:hep-ph/0202185.

[11] M. Drees, R. M. Godbole, and P. Roy, *Theory and Phenomenology of Sparticles*. (World Scientific, 2004).

[12] P. Binetruy, *Supersymmetry: Theory, Experiment, and Cosmology*. (Oxford U. Press, 2004).

[13] H. Baer and X. Tata, *Weak Scale Supersymmetry*. (Cambridge U. Press, 2006).

[14] H. Dreiner, H. E. Haber, and S. P. Martin, to appear.

[15] N. Arkani-Hamed, A. G. Cohen, E. Katz and A. E. Nelson, JHEP **0207**, 034 (2002) [arXiv:hep-ph/0206021].

[16] M. Schmaltz and D. Tucker-Smith, Ann. Rev. Nucl. Part. Sci. **55**, 229 (2005) [arXiv:hep-ph/0502182].

[17] For a pedagogical derivation of origin of the negative μ^2 in little Higgs models, see M. Perelstein, M. E. Peskin and A. Pierce, Phys. Rev. D **69**, 075002 (2004) [arXiv:hep-ph/0310039].

[18] H. C. Cheng, B. A. Dobrescu and C. T. Hill, Nucl. Phys. B **573**, 597 (2000) [arXiv:hep-ph/9906327]; N. Arkani-Hamed, H. C. Cheng, B. A. Dobrescu and L. J. Hall, Phys. Rev. D **62**, 096006 (2000) [arXiv:hep-ph/0006238];

[19] C. Macesanu, Int. J. Mod. Phys. A **21**, 2259 (2006) [arXiv:hep-ph/0510418].

[20] S. R. Coleman and J. Mandula, Phys. Rev. **159** (1967) 1251.

[21] R. Haag, J. T. Lopuszanski and M. Sohnius, Nucl. Phys. B **88**, 257 (1975).

[22] Yu. A. Golfand and E. P. Likhtman, JETP Lett. **13** (1971) 323 [Pisma Zh. Eksp. Teor. Fiz. **13** (1971) 452].

[23] D. V. Volkov and V. P. Akulov, JETP Lett. **16** (1972) 438 [Pisma Zh. Eksp. Teor. Fiz. **16** (1972) 621].

[24] J. Wess and B. Zumino, Nucl. Phys. B **70** (1974) 39.

[25] For a taste, see N. Beisert, Comptes Rendus Physique **5**, 1039 (2004) [arXiv:hep-th/0409147].

[26] K. Intriligator, N. Seiberg and D. Shih, JHEP **0604**, 021 (2006) [arXiv:hep-th/0602239].

[27] In a common alternative notation, ϕ is written $-A^*$. Then this equation becomes $F = mA$, the *Newton-Witten equation*. See [28].

[28] V. Gates, *et al.*, in *Proceedings of the Workshop on Unified String Theories*, M. Green and D. Gross, eds. (World Scientific, 1986).

[29] B. Zumino, Nucl. Phys. B **89**, 535 (1975).

[30] M. T. Grisaru, W. Siegel and M. Rocek, Nucl. Phys. B **159**, 429 (1979).

[31] I. Affleck, M. Dine and N. Seiberg, Phys. Rev. Lett. **52**, 1677 (1984).

[32] B. Zumino, Phys. Lett. B **87**, 203 (1979).

[33] J. Wess and B. Zumino, Nucl. Phys. B **78**, 1 (1974).

[34] M. T. Grisaru, B. Milewski and D. Zanon, Nucl. Phys. B **266**, 589 (1986).

[35] V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **229**, 381 (1983).

[36] N. Arkani-Hamed and H. Murayama, JHEP **0006**, 030 (2000) [arXiv:hep-th/9707133].

[37] S. Ferrara and B. Zumino, Nucl. Phys. B **87**, 207 (1975).

[38] I. Hinchliffe and J. Erler and P. Langacker, in W. M. Yao *et al.* [Particle Data Group], J. Phys. G **33**, 1 (2006).

[39] Y. Yamada, Z. Phys. C **60**, 83 (1993).

[40] M. L. Alciati, F. Feruglio, Y. Lin and A. Varagnolo, JHEP **0503**, 054 (2005) [arXiv:hep-ph/0501086].

[41] M. Dine, arXiv:hep-ph/0011376.

[42] S. Dimopoulos and L. J. Hall, Phys. Lett. B **207**, 210 (1988).

[43] S. Deser and B. Zumino, Phys. Rev. Lett. **38**, 1433 (1977).

[44] L. O'Raifeartaigh, Nucl. Phys. B **96**, 331 (1975).

[45] M. A. Luty, arXiv:hep-th/0509029.

[46] L. Girardello and M. T. Grisaru, Nucl. Phys. B **194**, 65 (1982).

[47] J. E. Kim and H. P. Nilles, Phys. Lett. B **138**, 150 (1984).

[48] G. F. Giudice and A. Masiero, Phys. Lett. B **206**, 480 (1988).

[49] L. J. Hall and L. Randall, Nucl. Phys. B **352**, 289 (1991).

[50] P. J. Fox, A. E. Nelson and N. Weiner, JHEP **0208**, 035 (2002) [arXiv:hep-ph/0206096]; A. E. Nelson, N. Rius, V. Sanz and M. Unsal, JHEP **0208**, 039 (2002) [arXiv:hep-ph/0206102].

[51] G. D. Kribs, E. Poppitz and N. Weiner, arXiv:0712.2039 [hep-ph].

[52] M. Dine, A. E. Nelson, Y. Nir and Y. Shirman, Phys. Rev. D **53**, 2658 (1996) [arXiv:hep-ph/9507378].

[53] G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, JHEP **9812**, 027 (1998) [arXiv:hep-ph/9810442].

[54] L. Randall and R. Sundrum, Nucl. Phys. B **557**, 79 (1999) [arXiv:hep-th/9810155].

[55] J. L. Feng and T. Moroi, Phys. Rev. D **56**, 5962 (1997) [arXiv:hep-ph/9612333].

[56] S. Y. Choi, H. E. Haber, J. Kalinowski and P. M. Zerwas, Nucl. Phys. B **778**, 85 (2007) [arXiv:hep-ph/0612218].

[57] M. Schmaltz and W. Skiba, Phys. Rev. D **62**, 095004 (2000) [arXiv:hep-ph/0004210].

[58] J. L. Feng and T. Moroi, Phys. Rev. D **58**, 035001 (1998) [arXiv:hep-ph/9712499].

[59] J. L. Feng, S. F. Su and F. Takayama, Phys. Rev. D **70**, 063514 (2004) [arXiv:hep-ph/0404198].

[60] T. Abe *et al.* [American Linear Collider Working Group], in *Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001)* ed. N. Graf [arXiv:hep-ex/0106056]

[61] J. L. Feng and M. M. Nojiri, arXiv:hep-ph/0210390.

[62] M. M. Nojiri, Phys. Rev. D **51**, 6281 (1995) [arXiv:hep-ph/9412374].

[63] G. Weiglein *et al.* [LHC/LC Study Group], Phys. Rept. **426**, 47 (2006) [arXiv:hep-ph/0410364].

[64] M. E. Peskin, Int. J. Mod. Phys. A **13**, 2299 (1998) [arXiv:hep-ph/9803279].

[65] I thank Norman Graf for providing this figure.

[66] J. L. Feng, M. E. Peskin, H. Murayama and X. R. Tata, Phys. Rev. D **52**, 1418 (1995) [arXiv:hep-ph/9502260].

[67] ATLAS Collaboration, *Detector and Physics Performance Technical Design Report*, vol.II. CERN/LHCC/99-14 (1999).

[68] A. Ball, M. Della Negra, A. Petrilli and L. Foa [CMS Collaboration], J. Phys. G **34**, 995 (2007).

[69] http://cmsinfo.cern.ch/outreach/CMSdetectorInfo/NewPhysics/

[70] W. Beenakker, R. Hopker, M. Spira and P. M. Zerwas, Nucl. Phys. B **492**, 51 (1997) [arXiv:hep-ph/9610490]; `http://www.ph.ed.ac.uk/~tplehn/prospino/`

[71] D. R. Tovey, Eur. Phys. J. direct C **4** (2002) N4.

[72] I. Hinchliffe, F. E. Paige, M. D. Shapiro, J. Soderqvist and W. Yao, Phys. Rev. D **55**, 5520 (1997) [arXiv:hep-ph/9610544].

[73] R. Kitano and Y. Nomura, Phys. Rev. D **73**, 095004 (2006) [arXiv:hep-ph/0602096].

[74] N. Arkani-Hamed, P. Schuster, N. Toro, J. Thaler, L. T. Wang, B. Knuteson and S. Mrenna, arXiv:hep-ph/0703088.

[75] Some models with on-shell gauge bosons in the final state of squark decays are analyzed in J. M. Butterworth, J. R. Ellis and A. R. Raklev, JHEP **0705**, 033 (2007) [arXiv:hep-ph/0702150].

[76] B. C. Allanach, C. G. Lester, M. A. Parker and B. R. Webber, JHEP **0009**, 004 (2000) [arXiv:hep-ph/0007009].

[77] C. G. Lester and D. J. Summers, Phys. Lett. B **463**, 99 (1999) [arXiv:hep-ph/9906349].

[78] A. J. Barr, Phys. Lett. B **596**, 205 (2004) [arXiv:hep-ph/0405052].

[79] T. Goto, K. Kawagoe and M. M. Nojiri, Phys. Rev. D **70**, 075016 (2004) [Erratum-ibid. D **71**, 059902 (2005)] [arXiv:hep-ph/0406317].

[80] J. M. Smillie and B. R. Webber, JHEP **0510**, 069 (2005); [arXiv:hep-ph/0507170]. C. Athanasiou, C. G. Lester, J. M. Smillie and B. R. Webber, JHEP **0608**, 055 (2006) [arXiv:hep-ph/0605286], arXiv:hep-ph/0606212.

[81] A. Alves, O. Eboli and T. Plehn, Phys. Rev. D **74**, 095010 (2006) [arXiv:hep-ph/0605067].

[82] J. R. Ellis, K. Enqvist, D. V. Nanopoulos and F. Zwirner, Mod. Phys. Lett. A **1**, 57 (1986).

[83] R. Barbieri and G. F. Giudice, Nucl. Phys. B **306**, 63 (1988).

[84] J. L. Feng, K. T. Matchev and T. Moroi, Phys. Rev. Lett. **84**, 2322 (2000) [arXiv:hep-ph/9908309]; Phys. Rev. D **61**, 075005 (2000) [arXiv:hep-ph/9909334].

[85] M. M. Kado and C. G. Tully, Ann. Rev. Nucl. Part. Sci. **52**, 65 (2002).

[86] R. Dermisek, J. F. Gunion and B. McElrath, Phys. Rev. D **76**, 051105 (2007) [arXiv:hep-ph/0612031].

[87] S. Chang and N. Weiner, arXiv:0710.4591 [hep-ph].

[88] G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich and G. Weiglein, Eur. Phys. J. C **28**, 133 (2003) [arXiv:hep-ph/0212020].

[89] H. E. Haber, R. Hempfling and A. H. Hoang, Z. Phys. C **75**, 539 (1997) [arXiv:hep-ph/9609331].

[90] N. Cabibbo, L. Maiani, G. Parisi and R. Petronzio, Nucl. Phys. B **158**, 295 (1979).

[91] P. Batra, A. Delgado, D. E. Kaplan and T. M. P. Tait, JHEP **0402**, 043 (2004) [arXiv:hep-ph/0309149].

[92] R. Harnik, G. D. Kribs, D. T. Larson and H. Murayama, Phys. Rev. D **70**, 015002 (2004) [arXiv:hep-ph/0311349].

[93] J. F. Gunion, H. E. Haber, G. Kane, and S. Dawson, *The Higgs Hunter's Guide*. (Addison-Wesley, 1990).

[94] M. S. Carena and H. E. Haber, Prog. Part. Nucl. Phys. **50**, 63 (2003) [arXiv:hep-ph/0208209].

[95] V. Trimble, Ann. Rev. Astron. Astrophys. **25**, 425 (1987).

[96] D. N. Spergel *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **170**, 377 (2007) [arXiv:astro-ph/0603449].

[97] O.Lahav and A. R. Liddle, in W. M. Yao *et al.* [Particle Data Group], J. Phys. G **33**, 1 (2006).

[98] G. Bertone, D. Hooper and J. Silk, Phys. Rept. **405**, 279 (2005) [arXiv:hep-ph/0404175].

[99] T. Moroi and L. Randall, Nucl. Phys. B **570**, 455 (2000) [arXiv:hep-ph/9906527].

[100] R. Kitano and Y. Nomura, Phys. Lett. B **632**, 162 (2006) [arXiv:hep-ph/0509221].

[101] M. Ibe and R. Kitano, JHEP **0708**, 016 (2007) [arXiv:0705.3686 [hep-ph]].

[102] R. J. Scherrer and M. S. Turner, Phys. Rev. D **33**, 1585 (1986) [Erratum-ibid. D **34**, 3263 (1986)].

[103] E. A. Baltz, J. E. Taylor and L. L. Wai, arXiv:astro-ph/0610731.

[104] A. M. Green, JCAP **0708**, 022 (2007) [arXiv:hep-ph/0703217].

[105] H. Goldberg, Phys. Rev. Lett. **50**, 1419 (1983).

[106] J. R. Ellis, K. A. Olive, Y. Santoso and V. C. Spanos, Phys. Lett. B **565**, 176 (2003) [arXiv:hep-ph/0303043].

[107] J. Edsjo, M. Schelke, P. Ullio and P. Gondolo, JCAP **0304**, 001 (2003) [arXiv:hep-ph/0301106].

[108] H. Baer, A. Belyaev, T. Krupovnickas and X. Tata, JHEP **0402**, 007 (2004) [arXiv:hep-ph/0311351].

[109] R. Arnowitt, B. Dutta and Y. Santoso, Nucl. Phys. B **606**, 59 (2001) [arXiv:hep-ph/0102181].

[110] C. Balazs, M. S. Carena and C. E. M. Wagner, Phys. Rev. D **70**, 015007 (2004) [arXiv:hep-ph/0403224].

[111] E. A. Baltz, M. Battaglia, M. E. Peskin and T. Wizansky, Phys. Rev. D **74**, 103521 (2006) [arXiv:hep-ph/0602187].