Cosmological Solutions with Torsion in a Model of de Sitter Gauge Theory of Gravity

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The torsion is shown to be vitally important in the explanation of the evolution of the universe in a large class of gravitational theories containing quadratic terms of curvature and torsion. The cosmological solutions with homogeneous and isotropic torsion in a model of de Sitter gauge theory of gravity are presented, which may explain the observation data for SN Ia when parameters are suitably chosen and supply a natural transit from decelerating expansion to accelerating expansion without the help of the introduction of other strange fields in the theory.

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I. INTRODUCTION

The observations on SN Ia [1] show that our universe is in a stage of accelerating expansion. In order to explain the accelerating expansion of the universe, many dark energy models are constructed. In many fashioned models, some strange fields are introduced to effectively describe the behaviors of the dark energy. There are alternative approaches to describe the dark energy. For example, the gravitational Lagrangian is supposed to be $f(R)$ and so on. However, such kind of the gravitational Lagrangian is also a phenomenological one. There is no fundamental principle to determine the function f.

On the other hand, since 1970s, some efforts to construct a new gravitational theory — the gravity with local de Sitter (dS) symmetry — have been made [2, 3, 10][1]. A model of dS gauge theory of gravity can be formulated inspired by the dS invariant special relativity [11–13] and the principle of localization [10]. This is just like the Poincaré gauge theory of gravity (see, e.g.[14]), which can be inspired by the Einstein special relativity and the localization of Poincaré symmetry. The principle of localization requires that the full symmetry — $ISO(1,3)$ for Einstein special relativity, $SO(1,4)$ for dS invariant special relativity and $SO(2,3)$ for anti-dS (AdS) invariant special relativity — as well as the laws of dynamics are both localized.

It has been shown that the model of dS gauge theory of gravity can be realized on a kind of totally umbilic submanifolds and under the dS-Lorentz gauge, the dS connection becomes the gravitational gauge potential of the model, which combines the Lorentz tetrad and connection together as an $\mathfrak{so}(1,4)$ valued connection. The gravitational action takes the form of Yang-Mills gauge theory. After variation of the action with respect to the Lorentz tetrad and connection, one may obtain the Einstein-like equations and Yang-like equations, respectively. The Yang-like equations are the generalization of the gravitational field equations in the theory of gravity proposed by Yang [15] and others [16]. They consist of a set of highly non-linear equations. To solve the equations is, in general, a very difficult task. Fortunately, it can be shown that all vacuum solutions of Einstein field equations with a cosmological constant are the vacuum solutions of the set of equations without torsion [10, 17]. In particular, Schwarzschild-dS and Kerr-dS metrics are two solutions of the model of dS gauge theory of gravity. Therefore, the model of dS gauge theory of gravity can pass all solar-system-scale observations and experimental tests for general relativity. In addition, there exist torsion-free and spin current-free, Big Bang solutions in the model of dS gauge theory of gravity [18]. Unfortunately, the equation of state (EoS) for the perfect fluid in the solutions has to take, up to a constant, the form of radiation, namely, $p = \rho/3 + c$. Obviously, such kind of cosmological solutions cannot explain the evolution of the universe. The stringent requirement comes from the nontrivial Yang equation in addition to Einstein-like equations.

One of the purposes of the present paper is to show that when an isotropic and homogeneous torsion is taken into account, an arbitrary form of EoS for matter may be supplemented to the Einstein-like equations and Yang-like equations and thus the diverse cosmological so-

[1] The same connection with different gravitational dynamics has also been studied (See, e.g. [4–9])

lutions may be constructed, which may be used to explain the evolution of the universe. For example, we may find a series of 'dust-dominated' cosmological solutions. In the cosmological solutions the contributions of the higher order of curvature term and torsion term automatically serve as the dark energy and dark radiation. Hence, it is not necessary to introduce strange fields to describe the dark energy. Another purpose of the paper is to investigate what kind of ingredients of gravitational dark energy in the model may fit the SN Ia observation data.

The paper is arranged as follows. We first review the model of dS gauge theory of gravity in the next section. In the third section, we show that the torsion-free and spin current-free solutions in a large class of gravitational theories containing quadratic terms of curvature and torsion cannot explain the evolution of the universe and that the torsion is necessary for the purpose, at least, in the model of dS gauge theory of gravity. In Sec. IV, we present some numerical solutions for the homogeneous and isotropic universe with torsion. Also in this section, we try to fix the ingredients of gravitational dark energy by comparing with the SN Ia observation data. We shall give some concluding remarks in the final section.

II. A REVIEW OF THE MODEL OF DS GAUGE THEORY OF GRAVITY

Like the Poincaré gauge theory of gravity [14, 19] in which the full Poincaré symmetry of a Minkowski spacetime is localized, the model of dS gauge theory of gravity can be stimulated by that gravity should be based on the idea of the localization of the full dS-symmetry of a dS spacetime and its dynamics is supposed to be governed by a gauge-like one with a dimensionless coupling constant g. In the following we shall review how to construct a model of dS gauge theory of gravity on a kind of totally umbilic submanifolds of local dS-invariance and in a special gauge called the dS-Lorentz gauge the dS connection becomes the one proposed earlier in [2, 3, 10].

A. Totally umbilic submanifold and dS connection

The model of dS gauge theory of gravity [2, 10] supposes that the spacetimes with gravity of local dS-invariance is described as a kind of $(1+3)$-dimensional totally umbilic submanifolds $\mathcal{H}^{1,4}$ as sub-manifolds of $(1+4)$-dimensional Riemann-Cartan manifolds $\mathcal{M}^{1,4}$. In surface theory [20], a surface is totally umbilic if the normal curvatures at each point are constants. A sphere in 3-dimensional Euclid space is an example of a totally umbilic surface. A totally umbilic surface has the property that its first fundamental form is proportional to its second fundamental form. This property serves as the definition of a nondegenerate totally umblic submanifold in a higher dimensional Riemann-Cartan space with an indefinite metric [21] we are interested in.

A totally umbilic submanifold with local dS symmetry is a $(1+3)$-dimensional Riemann-Cartan manifold with signature -2 . At each point p of the totally umbilic submanifold, there exists a local tangent dS space with an invariant dS radius R, embedded in the tangent space $T_p^{1,4} = T_p^{1,3} \times \mathbb{R}_p$ of $(1+4)$-dimensional Riemann-Cartan manifold $\mathcal{M}^{1,4}$ at p. Here, $T_p^{1,3}$

the tangent space of $(1+3)$-dimensional spacetime at p, while $\mathbb{R}_p$ is the tangent space at p orthorgonal to $T_p^{1,3}$ in $\mathcal{M}^{1,4}$. On the co-tangent space $T_p^{1,3*}$ at the point $p \in \mathcal{H}^{1,3}$ there is a Lorentz frame 1-form such that

$$\mathbf{b}^b = e^b_\mu dx^\mu, \quad \mathbf{b}^b(\partial_\mu) = e^b_\mu, \quad e^a_\mu e^\mu_b = \delta^a_b, \quad e^a_\mu e^\nu_a = \delta^\nu_\mu, \tag{1}$$

where ∂_μ and dx^μ are the coordinate bases of $T_p^{1,3}$ and $T_p^{1,3*}$. Their inner products define the metrics:

$$< \partial_\mu, \partial_\nu >= g_{\mu\nu}, \quad < e_a, e_b >= \eta_{ab} = \mathrm{diag}(1, -1, -1, -1). \tag{2}$$

Let the unit vector in $\mathbb{R}_p$ and unit 1-form in $\mathbb{R}_p^*$ be n and ν, respectively. Then, both $\{e_a, n; \mathbf{b}^b, \nu\}$ and $\{\partial_\mu, n; dx^\lambda, \nu\}$ span $T_p^{1,4} = T_p^{1,3} \times \mathbb{R}_p$ and $T_p^{1,4*} = T_p^{1,3*} \times {\mathbb{R}_p}^*$. They satisfy the following conditions in addition to (2)

$$n(\nu) = 1, \quad \mathbf{b}^b(n) = dx^\lambda(n) = 0, \quad \nu(e_a) = \nu(\partial_\mu) = 0; \tag{3}$$

$$< n, n >= -1, \quad < e_a, n > =< \partial_\mu, n > = 0. \tag{4}$$

The dS-Lorentz base $\{\hat{E}_A\}$ and their dual $\{\hat{\Theta}^B\}$ $(A, B = 0, \cdots, 4)$ can be defined as:

$$\{\hat{E}_A\} = \{e_a, n\}, \quad \{\hat{\Theta}^B\} = \{\mathbf{b}^b, \nu\}. \tag{5}$$

Eqs. (1)—(4) can be expressed as

$$\hat{\Theta}^B(\hat{E}_A) = \delta^B_A, \quad < \hat{E}_A, \hat{E}_B >= \eta_{AB} = \mathrm{diag}(1, -1, -1, -1, -1). \tag{6}$$

The transformations, which map $M_p^{1,4}$ to itself and preserve the inner products, are

$$\hat{E}_A \to E_A = (S^t)_A^{\ B} \hat{E}_B, \quad \hat{\Theta}^A \to \Theta = ((S)^{-1})^A_{\ B} \hat{\Theta}^B, \quad (S^t)_A^{\ C} \eta_{CD} S^D_{\ \ B} = \eta_{AB}, \tag{7}$$

where $S = (S^A_{\ B}) \in SO(1,4)$, the superscript t denotes the transpose.

In the tangent bundle $T\mathcal{H}^{1,3}$, there is a Lorentz covariant derivative a la Cartan:

$$\nabla_{e_a} e_b = \theta^c_{\ b}(e_a) e_c; \quad \theta^a_{\ b} = B^a_{\ b\mu} dx^\mu, \quad \theta^a_{\ b}(\partial_\mu) = B^a_{\ b\mu}. \tag{8}$$

$B^a_{\ c\mu} \in \mathfrak{so}(1,3)$ are connection coefficients of the Lorentz connection 1-form $\theta^a_{\ c}$. The torsion and curvature can be defined as

$$\begin{aligned} \Omega^a &= d\mathbf{b}^a + \theta^a_{\ b} \wedge \mathbf{b}^b = \frac{1}{2} T^a_{\ \mu\nu} dx^\mu \wedge dx^\nu \\ & \quad T^a_{\ \mu\nu} = \partial_\mu e^a_\nu - \partial_\nu e^a_\mu + B^a_{\ c\mu} e^c_\nu - B^a_{\ c\nu} e^c_\mu, \end{aligned} \tag{9}$$

$$\begin{aligned} \Omega^a_{\ b} &= d\theta^a_{\ b} + \theta^a_{\ c} \wedge \theta^c_{\ b} = \frac{1}{2} F^a_{\ b\mu\nu} dx^\mu \wedge dx^\nu; \\ & \quad F^a_{\ b\mu\nu} = \partial_\mu B^a_{\ b\nu} - \partial_\nu B^a_{\ b\mu} + B^a_{\ c\mu} B^c_{\ b\nu} - B^a_{\ c\nu} B^c_{\ b\mu}. \end{aligned} \tag{10}$$

They satisfy the corresponding Bianchi identities. Similarly, the dS-covariant derivative can be introduced

$$\hat{\nabla}_{E_A} E_B = \Theta^C_{\ B}(E_A) E_C. \tag{11}$$

$\Theta^A_{\ C} \in \mathfrak{so}(1,4)$ is the dS-connection 1-form. In the local coordinate chart $\{x^\mu\}$ on $\mathcal{H}^{1,3}$,

$$\hat{\nabla}_{\partial_\mu} E_B = \Theta^C_{\ B}(\partial_\mu) E_C = B^C_{\ B\mu} E_C, \quad \hat{\nabla}_n E_B = \Theta^C_{\ B}(n) E_C = B^C_{\ B4} E_C, \tag{12}$$

$B^A_{\ C\mu}$ and $B^A_{\ C4}$ denote the dS-connection coefficients on $\mathcal{H}^{1,3}$.

In the dS-Lorentz frame the dS-connection reads [2–10, 22, 23]

$$(\check{B}^{AB}{}_{\mu}) = \begin{pmatrix} B^{ab}{}_{\mu} & R^{-1}e^a_\mu \\ -R^{-1}e^b_\mu & 0 \end{pmatrix} \in \mathfrak{so}(1,4), \tag{13}$$

where $B^{AB}{}_{\mu} = \eta^{BC} B^A_{\ C\mu}$ and $B^{AB}{}_{4} = \eta^{BC} B^A_{\ C4}$. Its curvature reads

$$\check{\mathcal{F}}_{\mu\nu} = (\check{\mathcal{F}}^{AB}{}_{\mu\nu}) = \begin{pmatrix} F^{ab}{}_{\mu\nu} + R^{-2}e^{ab}{}_{\mu\nu} & R^{-1}T^a{}_{\mu\nu} \\ -R^{-1}T^b{}_{\mu\nu} & 0 \end{pmatrix} \in so(1,4), \tag{14}$$

where $e^a{}_{b\mu\nu} = e^a_\mu e_{b\nu} - e^a_\nu e_{b\mu}$, $e_{a\mu} = \eta_{ab}e^b_\mu$, $F^{ab}{}_{\mu\nu}$ and $T^a{}_{\mu\nu}$ are curvature (10) and torsion (9) of the Lorentz-connection.

B. A simple model of dS gauge theory of gravity

Now we consider a simple model of dS gauge theory of gravity with the dS connection[2]. The total action of the model with source is taken as

$$S_{\rm T} = S_{\rm GYM} + S_{\mathbf{M}}, \tag{15}$$

where $S_{\rm M}$ is the action of the source with minimum coupling to the gravitational fields, and $S_{\rm GYM}$ the gauge-like action in Lorentz gauge of the model as follows [2, 3, 10]:

$$\begin{aligned} S_{\rm GYM} &= \frac{1}{4g^2}\int_{\mathcal{M}} d^4x e \mathbf{Tr}_{dS}(\check{\mathcal{F}}_{\mu\nu}\check{\mathcal{F}}^{\mu\nu}) \\ &= -\int_{\mathcal{M}} d^4x e \left[\frac{\hbar}{4g^2}F^{ab}{}_{\mu\nu}F_{ab}{}^{\mu\nu} - \chi(F - 2\Lambda) - \frac{\chi}{2}T^a{}_{\mu\nu}T_a{}^{\mu\nu}\right]. \end{aligned} \tag{16}$$

Here, $e = \det(e^a_\mu)$, a dimensionless constant g should be introduced as usual in the gauge theory to describe the self-interaction of the gauge field, χ a dimensional coupling constant related to g and R, and $F = \frac{1}{2}F^{ab}{}_{\mu\nu}e_{ab}{}^{\mu\nu}$ the scalar curvature of the Cartan connection, the same as the action in the Einstein-Cartan theory. In order to make sense in comparing with the Einstein-Cartan theory, we should take $R = (3/\Lambda)^{1/2}$, $\chi = 1/(16\pi G)$ and $g^{-2} = 3\chi\Lambda^{-1}$.

The field equations can be given via the variational principle with respect to $e^a{}_\mu$, $B^{ab}{}_\mu$,

$$T_a{}^{\mu\nu}{}_{||\nu} - F^\mu_a + \frac{1}{2}Fe^\mu_a - \Lambda e^\mu_a = 8\pi G(T_{{\rm M}a}{}^\mu + T_{{\rm G}a}{}^\mu), \tag{17}$$

$$F_{ab}{}^{\mu\nu}{}_{||\nu} = R^{-2}(16\pi G S_{{\rm M}ab}{}^\mu + S_{{\rm G}ab}{}^\mu). \tag{18}$$

[2] The same dS-connection with similar or different dynamics has also been explored in Ref. [4]–[23]

In Eqs.(17) and (18), $||$ represents the covariant derivative compatible with Christoffel symbol $\{^{\mu}_{\nu\kappa}\}$ and spin connection $B^{a}{}_{b\mu}$. Besides, $F_{a}{}^{\mu} = -F_{ab}{}^{\mu\nu} e^{b}_{\nu}$, $F = F_{a}{}^{\mu} e^{a}_{\mu}$,

$$\begin{aligned} T_{\mathrm{M}a}{}^{\mu} &:= -\frac{1}{e}\frac{\delta S_{\mathrm{M}}}{\delta e^{a}_{\mu}}, && (19)\\ T_{\mathrm{G}a}{}^{\mu} &:= g^{-2} T_{\mathrm{F}a}{}^{\mu} + 2\chi T_{\mathrm{T}a}{}^{\mu}, && (20) \end{aligned}$$

are the tetrad form of the stress-energy tensors for matter and gravity, respectively, where

$$T_{\mathrm{F}a}{}^{\mu} := -\frac{1}{4e}\frac{\delta}{\delta e^{a}_{\mu}} \int d^4x e \mathrm{Tr}(F_{\nu\kappa} F^{\nu\kappa}) = e^{\kappa}_{a} \mathrm{Tr}(F^{\mu\lambda} F_{\kappa\lambda}) - \frac{1}{4} e^{\mu}_{a} \mathrm{Tr}(F^{\lambda\sigma} F_{\lambda\sigma}) \tag{21}$$

is the tetrad form of the stress-energy tensor for curvature and

$$T_{\mathrm{T}a}{}^{\mu} := -\frac{1}{4e}\frac{\delta}{\delta e^{a}_{\mu}} \int d^4x e T^{b}{}_{\nu\kappa} T_{b}{}^{\nu\kappa} = e^{\kappa}_{a} T_{b}{}^{\mu\lambda} T^{b}{}_{\kappa\lambda} - \frac{1}{4} e^{\mu}_{a} T_{b}{}^{\lambda\sigma} T^{b}{}_{\lambda\sigma} \tag{22}$$

is the tetrad form of the stress-energy tensor for torsion.

$$S_{\mathrm{M}ab}{}^{\mu} = \frac{1}{2\sqrt{-g}} \frac{\delta S_{\mathrm{M}}}{\delta B^{ab}{}_{\mu}} \tag{23}$$

and $S_{\mathrm{G}ab}{}^{\mu}$ are spin currents for matter and gravity, respectively. Especially, the spin current for gravity can be divided into two parts,

$$S_{\mathrm{G}ab}{}^{\mu} = S_{\mathrm{F}ab}{}^{\mu} + 2 S_{\mathrm{T}ab}{}^{\mu}, \tag{24}$$

where

$$\begin{aligned} S_{\mathrm{F}ab}{}^{\mu} &:= \frac{1}{2\sqrt{-g}} \frac{\delta}{\delta B^{ab}{}_{\mu}} \int d^4x \sqrt{-g} F = -e_{ab}{}^{\mu\nu}{}_{||\nu} = Y^{\mu}{}_{\lambda\nu} e_{ab}{}^{\lambda\nu} + Y^{\nu}{}_{\lambda\nu} e_{ab}{}^{\mu\lambda} && (25)\\ S_{\mathrm{T}ab}{}^{\mu} &:= \frac{1}{2\sqrt{-g}} \frac{\delta}{\delta B^{ab}{}_{\mu}} \frac{1}{4} \int d^4x \sqrt{-g} T^{c}{}_{\nu\lambda} T_{c}{}^{\nu\lambda} = T_{[a}{}^{\mu\lambda} e_{b]\lambda} && (26) \end{aligned}$$

are the spin current for curvature F and torsion T, respectively. Here,

$$Y^{\lambda}{}_{\mu\nu} = \frac{1}{2}(T^{\lambda}{}_{\nu\mu} + T_{\mu}{}^{\lambda}{}_{\nu} + T_{\nu}{}^{\lambda}{}_{\mu}). \tag{27}$$

is the contortion.

It has been shown that all solutions, including the dS, Schwarzschild-dS, Kerr-dS spacetimes, of vacuum Einstein equation with a non-zero cosmological constant also solve Eqs.(17) and (18) for the vacuum and torsion-free case [10, 17]. So, this simple model may pass the observation tests on solar system scale.

III. COSMOLOGICAL SOLUTIONS IN THE MODEL OF DS GAUGE THEORY OF GRAVITY

To deal with the cosmological solutions, we suppose, as usual, that the universe is homogeneous and isotropic and thus is described by the Friedmann-Robertson-Walker (FRW) metirc

$$ds^2 = dt^2 - a^2(t)[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)]. \tag{28}$$

The matter in the universe takes the perfect fluid,

$$T^{\mu\nu} = (\rho + p)U^\mu U^\nu - pg^{\mu\nu}, \tag{29}$$

where U^μ is 4-velocity of comoving observer, ρ energy density and p pressure.

A. Torsion-free cosmological model

When the spacetime is torsion-free and there is no spin current in it, Eqs.(17) and (18) reduce to [18]

$$\mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R}g_{\mu\nu} + \Lambda g_{\mu\nu} = -8\pi G(T_{\mu\nu} + T^{\rm R}_{\mu\nu}) \tag{30}$$

$$\mathcal{R}_{\mu\nu}{}^{\lambda\sigma}{}_{;\sigma} = 0 \tag{31}$$

where

$$T^{\rm R}_{\mu\nu} = \mathcal{R}^{\kappa\rho}{}_{\mu}{}^{\lambda}\mathcal{R}_{\kappa\rho\nu\lambda} - \frac{1}{4}g_{\mu\nu}\mathcal{R}^{\kappa\rho\lambda\sigma}\mathcal{R}_{\kappa\rho\lambda\sigma} = \frac{\mathcal{R}}{3}(\mathcal{R}_{\mu\nu} - \frac{1}{4}\mathcal{R}g_{\mu\nu}). \tag{32}$$

The first equation is the Einstein-like equation and the second one is the Yang equation [15]. The validity of the second equality in Eq.(32) is because FRW metric is conformally flat. Obviously, the trace of the Einstein-like equation is the same as that of Einstein equation.

$$\mathcal{R} = 8\pi GT + 4\Lambda. \tag{33}$$

From the Bianchi identity, the Yang equation becomes

$$\mathcal{R}^{\lambda}_{\mu;\nu} = \mathcal{R}^{\lambda}_{\nu;\mu}. \tag{34}$$

In particular,

$$\mathcal{R}^{\nu}_{\mu;\nu} = \mathcal{R}_{;\mu}. \tag{35}$$

On the other hand, the divergence-free of the Einstein tensor requires that

$$\mathcal{R}^{\nu}_{\mu;\nu} = \frac{1}{2}\mathcal{R}_{;\mu}. \tag{36}$$

Therefore,

$$\mathcal{R}^{\nu}_{\mu;\nu} = \mathcal{R}_{;\mu} = 0, \tag{37}$$

which results in

$$T_{;\mu} = 0, \qquad i.e. \qquad (\rho - 3p)^{\cdot} = 0, \tag{38}$$

where a dot represents the derivative with respect to cosmic time t. Thus, the EoS for the perfect fluid in this model has to be

$$p = \frac{1}{3}\rho + c. \tag{39}$$

The same result has been obtained by directly solving the equations in [18]. It is obvious that such kind of cosmological model cannot explain the evolution of the universe.

In fact, the conclusion can be generalized to the theory that the gravitational Lagrangian contains more terms such as,

$$e^{\lambda}_{a} e^{\sigma}_{b} T^{a}{}_{\mu\lambda} T^{b\mu}{}_{\sigma},\ e^{\sigma}_{a} e^{\mu}_{b} T^{a}{}_{\mu\lambda} T^{b\lambda}{}_{\sigma},\ e^{ab}{}_{\lambda\sigma} e^{cd}{}_{\mu\nu} F_{ab}{}^{\mu\nu} F_{cd}{}^{\lambda\sigma},\ e^{b}_{\sigma} e^{c}_{\mu} F_{ab}{}^{\mu}{}_{\nu} F_{ac}{}^{\nu\sigma},\ F_{a}{}^{\mu} F^{a}{}_{\mu},\ e^{a}_{\nu} e^{b}_{\mu} F_{a}{}^{\mu} F_{b}{}^{\nu}.$$

Obviously, the addition of the first two terms will not change the conclusion because they have no contribution to the torsion-free and spin-current-free field equations. In the above derivation, only the field equations (33) and (34) are used. Eq.(33) will not change because the trace of the stress-energy tensors for these gravitational terms all vanish. The middle two terms have the same contribution to (31) thus to (34) as the term $F_{ab}{}^{\mu\nu} F^{ab}{}_{\mu\nu}$ does. Therefore, they only alter the unimportant coefficients. The last two terms will contribute $(R^{\mu}_{[\lambda}\delta^{\nu}_{\sigma]})_{;\nu}$ terms to (31), thus the form of (34) will still remain. Hence, the torsion-free, spin-current-free, homogeneous, isotropic cosmological solutions with perfect fluids in a general $F - 2\Lambda + F^2$ theory including torsion-squared term cannot explain the evolution of our universe except the case that the coupling constants of these terms are suitably arranged so that Eq.(31) vanishes.

B. Field equations for the universe with isotropic and homogenous torsion

The reason that up to a constant, the EoS has to take almost radiation form is that the 24 components of torsion are set zero in the above model while the component equations obtained from the variation with respect to connection (or equivalently to contortion) do not disappear simultaneously. Thus, the constraint on the EoS appears. To remove the constraint, we should study the cosmological solutions in the model of dS gauge theory of gravity with isotropic and homogenous torsion. For simplicity, we still assume the spin currents are zero in the universe.

It can be shown that the isotropic and homogenous torsion takes the form of

$$\begin{cases} \mathbf{T}^0 = 0 \\ \mathbf{T}^1 = T_+\, \mathbf{b}^0 \wedge \mathbf{b}^1 + T_-\, \mathbf{b}^2 \wedge \mathbf{b}^3 \\ \mathbf{T}^2 = T_+\, \mathbf{b}^0 \wedge \mathbf{b}^2 - T_-\, \mathbf{b}^1 \wedge \mathbf{b}^3 \\ \mathbf{T}^3 = T_+\, \mathbf{b}^0 \wedge \mathbf{b}^3 + T_-\, \mathbf{b}^1 \wedge \mathbf{b}^2, \end{cases} \tag{40}$$

where $T_\pm$ is a function of time t and '+' and '-' represent the even and odd parity, respectively. Also, T_+ is the trace part of the torsion, $\frac{1}{3}T^a{}_{0a}$, while T_- is the traceless part of the torsion. Again, the matter in the universe is assumed to take the perfect fluid form Eq.(29).

The reduced Einstein-like equations are:

$$
\begin{aligned}
&-\frac{\ddot{a}^2}{a^2}-(\dot{T}_+ +2\frac{\dot{a}}{a}T_+ -2\frac{\ddot{a}}{a})\dot{T}_+ +\frac{1}{4}(\dot{T}_- +2\frac{\dot{a}}{a}T_-)\dot{T}_- +T_+^4-\frac{3}{2}T_+^2T_-^2+\frac{1}{16}T_-^4\\
&+(5\frac{\dot{a}^2}{a^2}+2\frac{k}{a^2}-\frac{3}{R^2})T_+^2-\frac{1}{2}(\frac{5}{2}\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}-\frac{3}{R^2})T_-^2+2\frac{\dot{a}}{a}(\frac{\ddot{a}}{a}-2\frac{\dot{a}^2}{a^2}-2\frac{k}{a^2}+\frac{3}{R^2})T_+\\
&-\frac{\dot{a}}{a}(4T_+^2-3T_-^2)T_+ +\frac{\dot{a}^2}{a^2}(\frac{\dot{a}^2}{a^2}+2\frac{k}{a^2}-\frac{2}{R^2})+\frac{k^2}{a^4}-\frac{2}{R^2}\frac{k}{a^2}+\frac{2}{R^4}=-\frac{2}{3}R^{-2}(8\pi GT_\mu^\nu e_0^\mu b_\nu^0),
\end{aligned}
\tag{41}
$$

$$
\begin{aligned}
&\frac{\ddot{a}^2}{a^2}+(\dot{T}_+ +2\frac{\dot{a}}{a}T_+ -2\frac{\ddot{a}}{a}+\frac{6}{R^2})\dot{T}_+ -\frac{1}{4}(\dot{T}_- +2\frac{\dot{a}}{a}T_-)\dot{T}_- -T_+^4+\frac{3}{2}T_+^2T_-^2-\frac{1}{16}T_-^4\\
&+\frac{\dot{a}}{a}(4T_+^2-3T_-^2)T_+ -(5\frac{\dot{a}^2}{a^2}+2\frac{k}{a^2}+\frac{3}{R^2})T_+^2+\frac{1}{2}(\frac{5}{2}\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}+\frac{3}{R^2})T_-^2\\
&-2\frac{\dot{a}}{a}(\frac{\ddot{a}}{a}-2\frac{\dot{a}^2}{a^2}-2\frac{k}{a^2}-\frac{6}{R^2})T_+ -\frac{4}{R^2}\frac{\ddot{a}}{a}-\frac{\dot{a}^2}{a^2}(\frac{\dot{a}^2}{a^2}+2\frac{k}{a^2}+\frac{2}{R^2})\\
&-\frac{k^2}{a^4}-\frac{2}{R^2}\frac{k}{a^2}+\frac{6}{R^4}-2\frac{k}{a^2}+\frac{3}{R^2})T_+ =-2R^{-2}(8\pi GT_\mu^\nu e_1^\mu b_\nu^1).
\end{aligned}
$$

The reduced Yang-like equations are

$$
\ddot{T}_- +3\frac{\dot{a}}{a}\dot{T}_- +(\frac{1}{2}T_-^2-6T_+^2+12\frac{\dot{a}}{a}T_+ +\frac{\ddot{a}}{a}-5\frac{\dot{a}^2}{a^2}-2\frac{k}{a^2}+\frac{6}{R^2})T_- =0,
\tag{42}
$$

$$
\begin{aligned}
&\ddot{T}_+ +3\frac{\dot{a}}{a}\dot{T}_+ -(2T_+^2-\frac{3}{2}T_-^2-6\frac{\dot{a}}{a}T_+ -\frac{\ddot{a}}{a}+5\frac{\dot{a}^2}{a^2}+2\frac{k}{a^2}-\frac{3}{R^2})T_+ -\frac{3}{2}\frac{\dot{a}}{a}T_-^2\\
&-\frac{\dddot{a}}{a}-\frac{\dot{a}\ddot{a}}{a^2}+2\frac{\dot{a}^3}{a^3}+2\frac{\dot{a}}{a}\frac{k}{a^2}=0.
\end{aligned}
\tag{43}
$$

Now, we have 4 independent gravitational equations for 5 independent variables: scale factor a, torsion components T_+ and T_-, energy density $\rho=T_\mu^\nu e_0^\mu b_\nu^0$, and pressure $p=T_\mu^\nu e_1^\mu b_\nu^1$. They, with the EoS of fluid, constitute the complete system of equations for the 5 variables. Namely, the constraint on EoS has been relieved and it is possible that the cosmological solutions with homogeneous and isotropic torsion may explain the evolution of the universe.

For the even parity of torsion, namely $T_-=0$, the independent component equations of Einstein-like and Yang-like equations further reduce to

$$
\begin{aligned}
&-\frac{\ddot{a}^2}{a^2}-(\dot{T}_+ +2\frac{\dot{a}}{a}T_+ -2\frac{\ddot{a}}{a})\dot{T}_+ +T_+^4-4\frac{\dot{a}}{a}T_+^3+(5\frac{\dot{a}^2}{a^2}+2\frac{k}{a^2}-\frac{3}{R^2})T_+^2\\
&+2\frac{\dot{a}}{a}(\frac{\ddot{a}}{a}-2\frac{\dot{a}^2}{a^2}-2\frac{k}{a^2}+\frac{3}{R^2})T_+ +\frac{\dot{a}^2}{a^2}(\frac{\dot{a}^2}{a^2}+2\frac{k}{a^2}-\frac{2}{R^2})+\frac{k^2}{a^4}-\frac{2}{R^2}\frac{k}{a^2}+\frac{2}{R^4}\\
&=-\frac{2}{3}R^{-2}(8\pi GT_\mu^\nu e_0^\mu b_\nu^0),
\end{aligned}
\tag{44}
$$

$$\frac{\ddot{a}^2}{a^2}+(\dot{T}_++2\frac{\dot{a}}{a}T_+-2\frac{\ddot{a}}{a}+\frac{6}{R^2})\dot{T}_+-T_+^4+4\frac{\dot{a}}{a}T_+^3-(5\frac{\dot{a}^2}{a^2}+2\frac{k}{a^2}+\frac{3}{R^2})T_+^2$$
$$-2\frac{\dot{a}}{a}(\frac{\ddot{a}}{a}-2\frac{\dot{a}^2}{a^2}-2\frac{k}{a^2}-\frac{6}{R^2})T_+-\frac{4}{R^2}\frac{\ddot{a}}{a}-\frac{\dot{a}^2}{a^2}(\frac{\dot{a}^2}{a^2}+2\frac{k}{a^2}+\frac{2}{R^2})-\frac{k^2}{a^4}-\frac{2}{R^2}\frac{k}{a^2}+\frac{6}{R^4}$$
$$=-2R^{-2}(8\pi GT_\mu^\nu e_1^\mu b_\nu^1), \tag{45}$$

$$\ddot{T}_++3\frac{\dot{a}}{a}\dot{T}_+-(2T_+^2-6\frac{\dot{a}}{a}T_+-\frac{\ddot{a}}{a}+5\frac{\dot{a}^2}{a^2}+2\frac{k}{a^2}-\frac{3}{R^2})T_+-\frac{\dddot{a}}{a}-\frac{\dot{a}\ddot{a}}{a^2}+2\frac{\dot{a}^3}{a^3}+2\frac{\dot{a}}{a}\frac{k}{a^2}=0. \tag{46}$$

The three equations with the EoS of fluid can determine the 4 variables $T_+,\ a,\ \rho$ and p. On the other hand, for the odd parity of torsion, namely $T_+=0$, the number of Einstein-like equations and Yang-like equations still remains 4. The 4 equations with EoS of fluid are the over-determined set of equations for the variables $T_-,\ a,\ \rho$ and p. Therefore, the cosmological model with odd parity of torsion in the model of dS gauge theory of gravity cannot explain the evolution of the universe either.

For simplicity, we will focus on even parity, in which case Eqs. (44) and (45) give rise to

$$\frac{\ddot{a}}{a}=-H^2-\frac{k}{a^2}+\frac{4}{3}\pi G(\rho-3p)+\frac{2}{R^2}+\frac{3}{2}(\dot{T}_++3HT_+-{T_+}^2), \tag{47}$$

where $H=\dot{a}/a$ is the Hubble parameter. With the help of Eq.(47), Eqs. (46) and (44) can be rewritten as

$$\begin{aligned}\ddot{T}_+ &= -3(H+\frac{3}{2}T_+)\dot{T}_++[\frac{13}{2}(T_+-3H)T_++6H^2-\frac{8}{R^2}+\frac{3k}{a^2}-\frac{28}{3}\pi G(\rho-3p)]T_+\\ &\quad -\frac{8}{3}\pi G(\rho-3p)^{\cdot},\end{aligned} \tag{48}$$

$$\begin{aligned}&\left[\frac{4}{3}\pi G(\rho-3p)\right]^2+\frac{4}{3}\pi G(\rho-3p)\left[\dot{T}_++7HT_+-3T_+^2-2\left(H^2+\frac{k}{a^2}-\frac{2}{R^2}\right)\right]-\frac{16}{3R^2}\pi G\rho\\ &+\left(\frac{2}{R^2}-H^2-\frac{k}{a^2}\right)\dot{T}_++\left(\frac{8}{R^2}-3H^2-\frac{3k}{a^2}\right)HT_++\left(\frac{37}{4}H^2+\frac{k}{a^2}-\frac{3}{R^2}\right)T_+^2+\frac{7}{2}HT_+\dot{T}_+\\ &-\frac{3}{2}T_+^2\dot{T}_+-\frac{13}{2}HT_+^3+\frac{1}{4}\dot{T}_+^2+\frac{5}{4}T_+^4-\frac{2}{R^2}\left(H^2+\frac{k}{a^2}-\frac{1}{R^2}\right)=0.\end{aligned} \tag{49}$$

In principle, one can solve Eqs.(47)—(49) and EoS to obtain the cosmological solution for $a(t)$, $\rho(t)$, $p(t)$ and T_+. In the following, we shall find some numerical solutions for the some reasonable initial parameters.

IV. THE EVOLUTION OF THE UNIVERSE

In the present stage of the universe the dominated matter has the EoS: $p=0$. For the convenience to compare our model with the ΛCDM model in GR, we rewrite Eq.(44) for $p=0$ as

$$1=\Omega_m+\Omega_\Lambda+\Omega_k+\Omega_{D_r}+\Omega_{D_1}, \tag{50}$$

where

$$\Omega_m = \frac{8\pi G\rho}{3H^2}, \qquad \Omega_\Lambda = \frac{\Lambda}{3H^2}, \qquad \Omega_k = \frac{-k}{a^2H^2}, \qquad \Omega_{D_r} = \frac{D_r}{H^2}, \qquad \Omega_{D_1} = \frac{D_1}{H^2}, \tag{51}$$

with

$$D_r := \frac{1}{2}R^2 T_{\mathrm{F}t}{}^t + T_{\mathrm{T}t}{}^t = \frac{3}{2}[R^2(B^2 - A^2) + T_+{}^2], \tag{52}$$

$$D_1 := -\frac{1}{3}T_t{}^{t\nu}{}_{||\nu} + 2HT_+ - T_+{}^2 = 3HT_+ - 2T_+{}^2, \tag{53}$$

where

$$A = \dot{T}_+ + HT_+ - \frac{\ddot{a}}{a}, \qquad B = 2HT_+ - T_+{}^2 - H^2 - \frac{k}{a^2}. \tag{54}$$

$D_r/(8\pi G)$ is the energy density of dark radiation contributed from both curvature and torsion and $D_1/(8\pi G)$ is the dark energy of the first part from the torsion. They are new contributions in comparison with the ΛCDM model in GR, in which $1 = \Omega_m + \Omega_\Lambda + \Omega_k$, and play the role of the dynamical dark energy.

By virtue of Eq.(50), Eq.(47) for $p = 0$ can be rewritten as

$$q = \frac{1}{2}\Omega_m - \Omega_\Lambda + \Omega_{D_r} + \frac{1}{2}(\Omega_{D_1} - \Omega_{D_2}), \tag{55}$$

where $\Omega_{D_2} = D_2/H^2$ and

$$D_2 = -T_r{}^{r\nu}{}_{||\nu} + 4HT_+ + 2\dot{T}_+ - T_+{}^2 = 3\dot{T}_+ + 6HT_+ - T_+{}^2. \tag{56}$$

$D_2/(8\pi G)$ is the dark energy of the second part from the torsion.

The dark radiation Ω_{D_r} and dark energy Ω_{D_1} and Ω_{D_2} as well as Ω_Λ are unobservable directly at present time. However, the present values of $\Omega_{D_1} + \Omega_{D_r}$ and $\Omega_{D_2} - \Omega_{D_r}$ can be determined from Eqs. (50) and (55) if the present values of q, Ω_m, Ω_Λ and Ω_k are known. This gives a chance to estimate 'the density of torsion' in the universe.

The behavior of the scale factor can be obtained by numerically integrating the above equations backward from today. The initial conditions for numerical calculation may be chosen based on the following facts. The kinematical analysis on the data of the SN Ia observations shows that the present deceleration parameter should be about $q_0 \approx -0.7$ — -0.81 [24–26]. (A subscript 0 denotes the present value as usual.) The density of matter (including baryonic and dark matter) on the scale of galaxy clusters is estimated between $\Omega_{m0} \approx 0.2$ — 0.3 [27], which is consistent with the cosmological estimate from the observation data of WMAP [28], SDSS [29], etc. in the framework of general relativity. The space of the universe is very flat, so we may suppose that $|\Omega_{k0}| \leq 0.02$.

Figure 1 shows the evolution of scale factor for $q_0 = -0.81$ as argued in [26] and $\Omega_{m0} = 0.24$. When q_0 and Ω_{m0} are fixed, there are still two degrees of freedom among Ω_{k0}, $\Omega_{\Lambda0}$, $\Omega_{D_r0} + \Omega_{D_10}$ and $\Omega_{D_20} - \Omega_{D_r0}$. We choose Ω_{k0} and $\Omega_{\Lambda0}$ as independent ones and fix Ω_{k0} first and then plot curves for different values of $\Omega_{\Lambda0}$. In the figure, the horizontal axis is time in the unit of H_0^{-1} and the vertical axis is a/a_0. From the figure, we can find

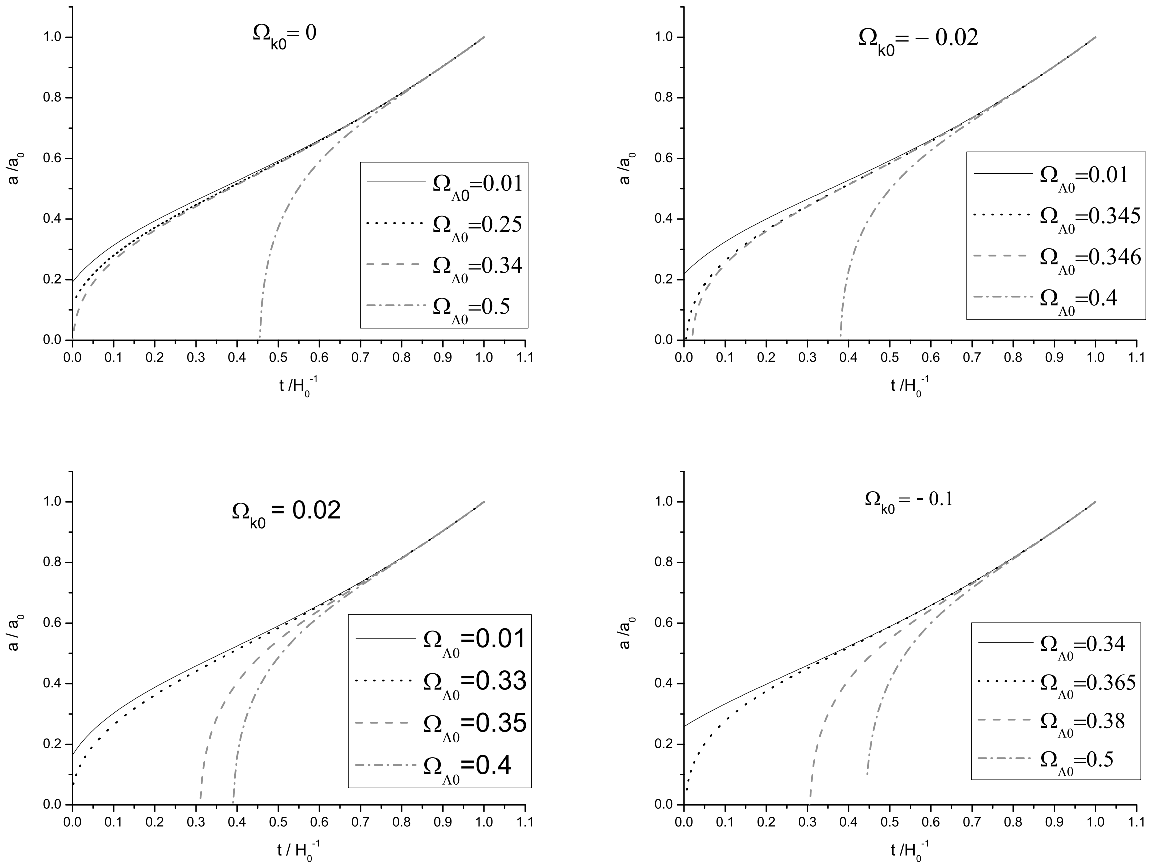

FIG. 1: Plots of the evolution of scale factor subject to different parameters. The horizontal axis is time in the unit of H_0^{-1}, while the vertical axis is the ratio of the scale factor to its present value. The upper left plot is for the flat universe. The lower left plot is for the slightly curved, open universe. The upper right plot is for the slightly curved, closed universe. The lower right plot is for the more curved, closed universe to show the role of the Ω_{k0}.

that in all cases considered, the larger the cosmological constant, the younger the universe is. Obviously, some models have been ruled out because they cannot explain the ages of the oldest globular clusters [30], which are between 10 and 13 Gyr. But, there are still wide parameter ranges (roughly speaking, $\Omega_{\Lambda 0} < 0.35$) for the models which might be used to explain the evolution of the universe. It is remarkable that the models supply a natural transit from the decelerating expansion to accelerating expansion without help of the strange fields such as quintessence, K-essence, phantom, quintom, etc. For example, when the space of the universe is a little bit curved so that $\Omega_{k0} = -0.02$, which has been indicated from the analysis of WMAP [28] and SDSS [29], the model with $\Omega_{\Lambda 0} = 0.345$ behaves as $a \to 0$ as $t \to 0$. In this case, $\Omega_{D_1 0} + \Omega_{D_r 0} = 0.435$, $\Omega_{D_2 0} - \Omega_{D_r 0} = 1.605$, $\frac{1}{2}(\Omega_{D_1 0} - \Omega_{D_2 0}) + \Omega_{D_r 0} = -0.585$ due to Eqs. (50) and (55). It means that on the large scale, the effect of torsion cannot be ignored. The ratio of the energy density of torsion to the critical energy density is even greater than those for cosmological constant and matter.

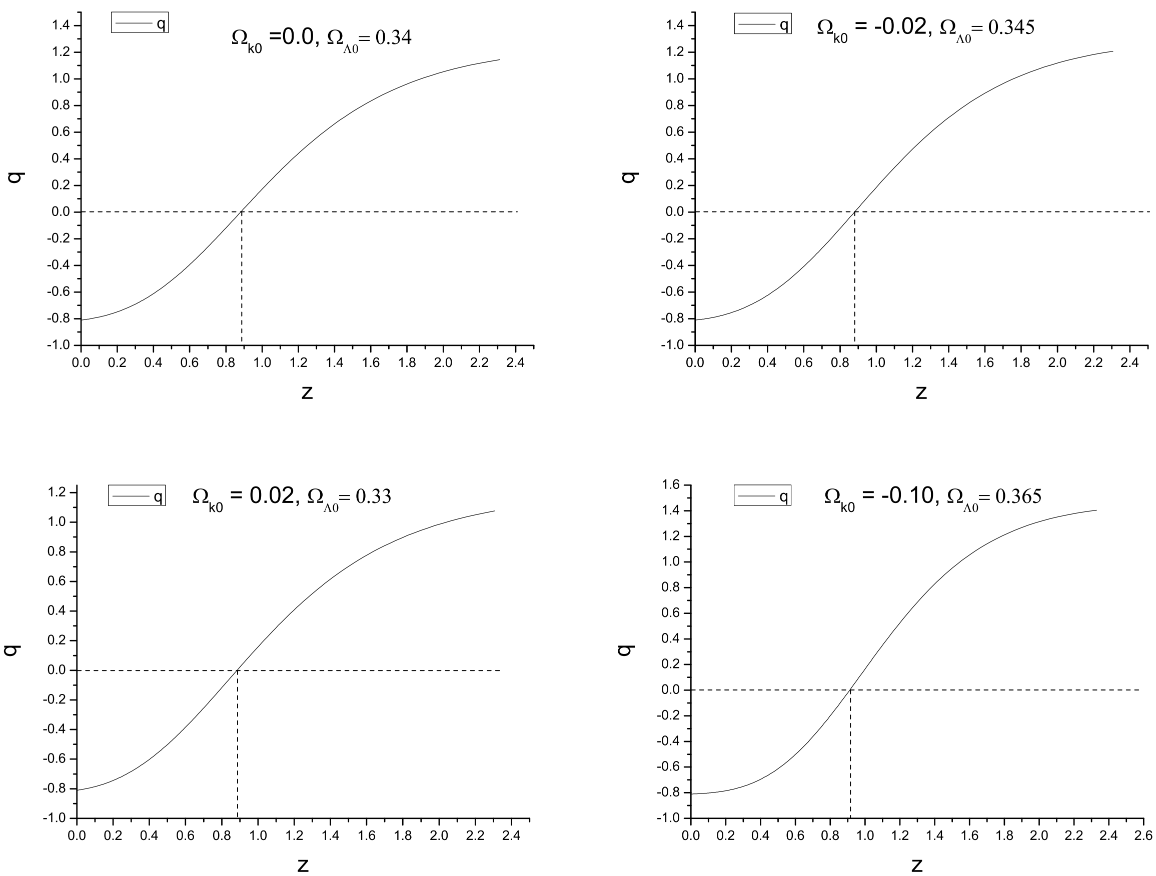

FIG. 2: Plots of the deceleration parameter versus red shift z. The transit from the decelerating expansion to the accelerating expansion occurs at $z < 1$ for all models plotted.

Figure 2 plots the behavior of deceleration parameter versus red shift z. We can see that the transit from the decelerating expansion to the accelerating expansion happen around at $z = 0.9$, which is qualitatively consistent with the analysis on the SN Ia observation [24].

V. CONCLUDING REMARKS

The astronomical observations show that the universe is probably asymptotically dS. It suggests that there is a need to analyze the observation data based on a theory with local dS symmetry.

We have shown that the torsion is vitally important in the explanation of the evolution of the universe not only for the model of dS gauge theory of gravity first proposed in 1970s, but also in a large class of gravitational theories containing quadratic terms of curvature and torsion. In a wide parameter range in the model of dS gauge theory of gravity, the spin-current-free cosmological solutions with homogenous and isotropic torsion may explain the SN Ia observation and supply a natural transit from decelerating expansion to accelerating expansion, without introducing other fields such as quintessence, K-essence, phantom, etc.

The transit occurs around at $z = 0.9$, which is qualitatively consistent with the analysis on the SN Ia observation. The reason that the redshift of the transit is systematically greater than the previous analysis is that the relation between q and z is obviously not linear one in our model, while the previous analysis is based on the assumption $q = q_0 + q_1 z$ [24]. If we make the linear fitting for the $q - z$ curve and then parallel transport the line so that it goes through q_0 at $z = 0$, then we shall get smaller redshift for the transit.

In the cosmological solutions with torsion we considered, the effects of torsion could not be ignored on the large scale, which is even greater than that of matter density or cosmological constant. Even though, it is very difficult to directly measure the energy density of torsion by local experiments because its order of magnitude is the same as that of cosmological constant. It is worthwhile to study the method of detecting torsion and place the upper limit for the torsion.

Needless to say, to check whether the model can really explain the gravitational phenomena in the universe, much work is needed. In particular, we should perturb the FRW metric and compare the anisotropic spectrum with WMAP data.

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