BRST Quantization of Gauge Theory in Noncommutative Geometry: Matrix Derivative Approach

Chang-Yeong Lee and Dae Sung Hwang

Department of Physics, Sejong University, Seoul 133-747, Korea

Yuval Ne'eman[#]

Wolfson Distinguished Chair of Theoretical Physics, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv 69978, Israel

Abstract

The BRST quantization of a gauge theory in noncommutative geometry is carried out in the "matrix derivative" approach. BRST/anti-BRST transformation rules are obtained by applying the horizontality condition, in the superconnection formalism. A BRST/anti-BRST invariant quantum action is then constructed, using an adaptation of the method devised by Baulieu and Thierry-Mieg for the Yang-Mills case. The resulting quantum action turns out to be the same as that of a gauge theory in the 't Hooft gauge with spontaneously broken symmetry. Our result shows that only the even part of the supergroup acts as a gauge symmetry, while the odd part effectively provides a global symmetry. We treat the general formalism first, then work out the $SU(2/1)$ and $SU(2/2)$ cases explicitly.

PACS number(s): 02.40.-k, 12.10.-g, 12.15.Cc

[#] Also on leave from: Center for Particle Physics, University of Texas, Austin, Tx 78712, USA

I. Introduction

The Higgs mechanism makes it possible to give masses to gauge bosons, while preserving the gauge symmetry. In this construction, some of the original scalar particle fields 'mutate' into the longitudinal components of the (now massive) gauge bosons. This fact may reflect the existence of an underlying structure, in which the gauge bosons and the original scalar particles belong to the same multiplet of a larger group. It is, therefore, natural to search for such a larger symmetry group and a suitable multiplet. As a matter of fact, this idea was implemented many years ago, using the supergroup $SU(2/1)$ [1]; it was also shown that this use of a supergroup could be extended to a large class of spontaneously broken symmetries [2]. More recently, the idea has further mathematically evolved within the *superconnection* construct [3, 4, 5, 6].

Another recent advance in mathematical pysics has consisted [7] in A. Connes' *noncommutative geometry*. In this formalism, the Dirac K-cycle on a star algebra acting on a Hilbert space, plays an important role, with possible applications to particle physics. Connes and Lott [8] then showed in particular that the *standard model* could be obtained in noncommutative geometry, as a gauge theory with a built-in spontaneous symmetry breakdown mechanism. Their work has been further extended to GUT (grand unified theories) [9], to gravity [10], and to supersymmetric theories [11].

Soon after the work of Connes and Lott, Coquereaux and other workers [12, 13] showed that the Connes-Lott approach is equivalent to a theory based on the superconnection concept [5, 14], rediscovering SU(2/1) in the process. In Coquereaux *et al.*'s formulation, a Z_2 graded space of matrix-valued forms is constructed, with a generalized derivative; 0-form and 1-form fields together represent a superconnection. The generalized derivative consists of the usual Cartan exterior differential operator, raising the form degree by one unit and thus also changing its Grassmann grading (which we denote as 'w-grading', i.e. $\mathbf{d}$ has odd w-grading) plus a graded

discrete operator consisting in a (graded) commutator with a constant matrix and satisfying certain algebraic conditions (including odd grading in a supergroup's generating superalgebra, 'g-odd' in our nomenclature). This graded commutator (or supercommutator) with a constant matrix is the *matrix derivative* [13]. We shall denote the Coquereaux *et al.* approach as *the matrix derivative* approach.

The equivalence between the Connes-Lott and Coquereaux *et al.* approaches has been stressed by Scheck and collaborators [15]. In both approaches, the 0-form scalar field is interpreted geometrically as an object interconnecting a two-sheeted world, whereas the 1-form field plays the usual role of a gauge field. The end-product is equivalent to an extension of the internal supersymmetry method in its superconnection formulation, completing, as we shall see, its geometric generation of a spontaneous symmetry breakdown mode for a local gauge symmetry.

We have recently quantized the SU(2/1) electro-weak theory in the superconnection formalism [16]. As an extension of this work, we now include in the present paper the quantization of the noncommutative geometry version of this "supergauge theory", by adjoining the matrix derivative approach to the superconnection formulation. Actually, this formulation goes beyond the internal supersymmetry method in one aspect, namely the emergence of the *negative squared mass* term for the scalar (Higgs) field from the geometry; in our previous treatment, most terms in the spontaneous symmetry breakdown Lagrangian emerged geometrically, namely (aside from the usual Yang-Mills term) the 'free' Higgs field Lagrangian plus its interaction with the gauge bosons – and the quartic Higgs field potential; the exception, which had to be put in 'by hand' (and thus also broke the symmetry explicitly) was this negative squared mass term, which is now provided by the matrix derivative.

We obtain the BRST/anti-BRST transformation rules of the theory, applying our *horizontality* condition, extending Thierry-Mieg's ansatz [6, 17, 18]. We construct the quantum action by adapting the Baulieu/Thierry-Mieg method [19] for

the Yang-Mills theory.

There are two important features deriving from our result. The first is the fact that we obtain the most appropriate gauge condition for a spontaneously broken gauge theory with scalar field, the 't Hooft gauge [20, 21], simply by adapting the method of Ref. [19], which would give the Landau gauge for the unbroken Yang-Mills theory, to the noncommutative geometry framework. The other relates to the physical content of a gauge theory in the noncommutative setting. Our quantization reveals that only the even part of the supergroup indeed acts as a gauge symmetry; the odd part simply produces a global symmetry. The resulting BRST transformation rules for the fields are thus the same as those of the spontaneously broken gauge theory with a Higgs mechanism, except that the scalar field transformation rule is changed by the addition of a constant shift (a vacuum shift), due to the action of the matrix derivative, thereby implementing geometrically the triggering of the spontaneous breakdown. Other fields are not affected by the appearance of the matrix derivative.

In section 2, we study the BRST quantization in the matrix derivative approach for the general case. In section 3, we treat the $SU(2/1)$ gauge theory, effectively an algebraically constrained standard model $SU(2) \times U(1)$ gauge theory of the electro-weak interaction. In section 4, we consider an $SU(2/2)$ gauge theory, which reduces to the spontaneously broken symmetry of an $SU(2) \times SU(2)$ σ-model. Section 5 contains a discussion and conclusions.

II. BRST/anti-BRST symmetry and quantum action

In the matrix derivative approach of a noncommutative geometrical gauge theory, the 0-form scalar field and 1-form gauge field together form a superconnection, with w-odd forms in the g-even part and w-even forms in the g-odd part of the

supergroup. We write the superconnection $\mathcal{J}$ as

$$\mathcal{J} = \mathcal{J}_{ev} + \mathcal{J}_{od} = \begin{pmatrix} \omega_0 & 0 \\ 0 & \omega_1 \end{pmatrix} + \begin{pmatrix} 0 & L_{01} \\ L_{10} & 0 \end{pmatrix}. \tag{1}$$

The overall Z_2 grading is given by the sum of the supermatrix grading (Z_2 'g'-grading) and the differential form grading (Z_2 'w'-grading). The total grading of the superconnection is therefore odd, in this Z_2 graded space [16]. Multiplication in this superspace is given by [5, 12]

$$(h \otimes W) \cdot (h' \otimes W') = (-1)^{|W||h'|}(hh') \otimes (WW'), \tag{2}$$

where $W,\ W'$ are differential forms of fixed Grassmannian Z_2 w-gradings $|W|,\ |W'|$, and $h,\ h'$ are supermatrices of fixed Z_2 g-grading $|\ h\ |,\ |\ h'\ |$. With this convention, we obtain the product rule for any two elements in our total Z_2 graded space, assuming A, B, C, D to be matrix-valued differential forms, which have fixed Z_2 w-gradings of 0 or 1, depending on whether they are even or odd forms, respectively, [5, 12]

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} = \begin{pmatrix} A \wedge A' + (-1)^{|\mathrm{B}|} B \wedge C' & (-1)^{|\mathrm{A}|} A \wedge B' + B \wedge D' \\ C \wedge A' + (-1)^{|\mathrm{D}|} D \wedge C' & (-1)^{|\mathrm{C}|} C \wedge B' + D \wedge D' \end{pmatrix}. \tag{3}$$

Once the superconnection is given, the supercurvature $\mathcal{F}_t$ is defined in the usual manner, with the generalized derivative $\mathbf{d}_t$, consisting of the usual 1-form differential operator $\mathbf{d}$ and the *matrix derivative* $\mathbf{d}_M$ [12, 13]:

$$\mathcal{F}_t = \mathbf{d}_t\, \mathcal{J} + \mathcal{J} \cdot \mathcal{J}, \tag{4}$$

$$\mathbf{d}_t = \mathbf{d} + \mathbf{d}_M, \tag{5}$$

$$\mathbf{d} = \begin{pmatrix} d & 0 \\ 0 & d \end{pmatrix}, \text{ where } d = 1 \otimes dx^\mu \frac{\partial}{\partial x^\mu}. \tag{6}$$

The matrix derivative is given by

$$\mathbf{d}_M = i[\eta, \quad]_\pm, \text{ where } \eta = \begin{pmatrix} 0 & \zeta \\ \overline{\zeta} & 0 \end{pmatrix}. \tag{7}$$

Here ζ and $\overline{\zeta}$ are constant matrices of zero forms, satisfying

$$\overline{\zeta}\zeta = \zeta\overline{\zeta} \propto 1, \tag{8}$$

so that the matrix derivative satisfies the nilpotency condition, $\mathbf{d}_M{}^2 = 0$. Note that the total grading of the matrix derivative $\mathbf{d}_M$ is odd. Thus the matrix derivative is a supercommutator, i.e. it acts as a commutator for objects of even total grading and as an anticommutator for objects of odd total grading, where by 'total', we mean the product of the gradings of 'g' and 'w'.

We now write the classical action of the gauge theory in noncommutative geometry as

$$\mathcal{S}_{cl} = -\frac{1}{4}\int \mathrm{Tr}\ \mathcal{F}_t{}^{\star}\cdot\mathcal{F}_t\ , \tag{9}$$

where $\star$ denotes taking the Hermitian conjugate for supermatrices and taking the Hodge dual for differential forms. In order to find the BRST/anti-BRST transformation rules, we use the so-called *horizontality* condition[3, 17, 18, 19], which is another description of the Maurer-Cartan equation:

$$\widetilde{\mathcal{F}_t} = \mathcal{F}_t\ , \tag{10}$$

where $\widetilde{\mathcal{F}_t}$ is the supercurvature, defined in the extended space of the *doubled* fiber bundle [16],

$$\widetilde{\mathcal{F}_t} = \widetilde{\mathbf{d}_t}\,\widetilde{\mathcal{J}} + \widetilde{\mathcal{J}}\cdot\widetilde{\mathcal{J}}\ . \tag{11}$$

'Doubling' implies the extension of the base manifold through doubling the fiber, from $\{\mathcal{G}\}$ to $\{\mathcal{G}\}\otimes\{\mathcal{G}\}$, so that we have a gauge fiber coordinate y and its dual $\bar{y}$ [6, 17, 18, 19]. In this extended space, the generalized derivative and superconnection are given by

$$\widetilde{\mathbf{d}_t} = \mathbf{d}_t + \mathbf{s} + \bar{\mathbf{s}}\ , \tag{12}$$

$$\widetilde{\mathcal{J}} = \mathcal{J} + \mathcal{C} + \bar{\mathcal{C}}\ . \tag{13}$$

Here, $\mathbf{s}$ and $\bar{\mathbf{s}}$ are 1-form differential operators acting respectively on the coordinates of the fiber and of its dual:

$$\mathbf{s} = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} \quad \text{where} \quad s = 1\otimes dy^N\frac{\partial}{\partial y^N},$$

$$\bar{\mathbf{s}} \;=\; \begin{pmatrix} \bar{s} & 0 \\ 0 & \bar{s} \end{pmatrix} \;\; \text{where} \;\; \bar{s} \;= 1 \otimes d\bar{y}^M \frac{\partial}{\partial \bar{y}^M}. \tag{14}$$

$\mathcal{C}$ and $\bar{\mathcal{C}}$ are obtained from $\mathcal{J}$ by replacing dx^μ by dy^N and $d\bar{y}^M$, and represent the ghost and anti-ghost fields, respectively:

$$\begin{aligned} \mathcal{C} &= \begin{pmatrix} c_{0N} dy^N & 0 \\ 0 & c_{1N} dy^N \end{pmatrix} \equiv \begin{pmatrix} c_0 & 0 \\ 0 & c_1 \end{pmatrix}, \\ \bar{\mathcal{C}} &= \begin{pmatrix} \bar{c}_{0M} d\bar{y}^M & 0 \\ 0 & \bar{c}_{1M} d\bar{y}^M \end{pmatrix} \equiv \begin{pmatrix} \bar{c}_0 & 0 \\ 0 & \bar{c}_1 \end{pmatrix}. \end{aligned} \tag{15}$$

After applying the horizontality condition we obtain the BRST/anti-BRST transformation rules:

$$\begin{aligned} (dy)^1 \;&:\; \mathbf{s}\, \mathcal{J} \;= -\mathbf{d}_t\, \mathcal{C} \;- \mathcal{J} \;\cdot \mathcal{C} \;- \mathcal{C} \;\cdot \mathcal{J}\;, \\ (d\bar{y})^1 \;&:\; \bar{\mathbf{s}}\, \mathcal{J} \;= -\mathbf{d}_t\, \bar{\mathcal{C}} \;- \mathcal{J} \;\cdot \bar{\mathcal{C}} \;- \bar{\mathcal{C}} \;\cdot \mathcal{J}\;, \\ (dy)^2 \;&:\; \mathbf{s}\, \mathcal{C} \;= -\mathcal{C} \;\cdot \mathcal{C}\;, \\ (d\bar{y})^2 \;&:\; \bar{\mathbf{s}}\, \bar{\mathcal{C}} \;= -\bar{\mathcal{C}} \;\cdot \bar{\mathcal{C}}\;, \\ (dy)^1 (d\bar{y})^1 \;&:\; \mathbf{s}\, \bar{\mathcal{C}} \;+ \bar{\mathbf{s}}\, \mathcal{C} \;+ \mathcal{C} \;\cdot \bar{\mathcal{C}} \;+ \bar{\mathcal{C}} \;\cdot \mathcal{C} \;= 0. \end{aligned} \tag{16}$$

By introducing an auxiliary field $\mathcal{E}$ such that

$$\mathbf{s}\, \bar{\mathcal{C}} \;\equiv \mathcal{E}\;, \;\; \text{i.e.,} \;\; \begin{pmatrix} s\bar{c}_0 & 0 \\ 0 & s\bar{c}_1 \end{pmatrix} \equiv \begin{pmatrix} b_0 & 0 \\ 0 & b_1 \end{pmatrix}, \tag{17}$$

we can fix the remaining BRST/anti-BRST transformation rules,

$$\begin{aligned} \bar{\mathbf{s}}\, \mathcal{C} &= -\mathcal{E} \;- \mathcal{C} \;\cdot \bar{\mathcal{C}} \;- \bar{\mathcal{C}} \;\cdot \mathcal{C}\;, \\ \mathbf{s}\, \mathcal{E} &= 0, \\ \bar{\mathbf{s}}\, \mathcal{E} &= -\bar{\mathbf{s}}\, (\mathcal{C} \;\cdot \bar{\mathcal{C}} \;+ \bar{\mathcal{C}} \;\cdot \mathcal{C}\;) = -\bar{\mathcal{C}} \;\cdot \mathcal{E} \;+ \mathcal{E} \;\cdot \bar{\mathcal{C}}\;. \end{aligned} \tag{18}$$

One can easily check the nilpotency property of the BRST/anti-BRST transformations, $\mathbf{s}^{\,2} = \bar{\mathbf{s}}^{\,2} = 0$, for the above transformation rules (16), (17) and (18).

Decomposing $\mathcal{J}$ into $\mathcal{J}_{\,ev} + \mathcal{J}_{\,od}$ as in (1), we can write the even and odd parts of the first two equations in (16) separately as follows, by noting that $\mathbf{d}$, $\mathbf{s}$

and $\bar{\mathbf{s}}$ are even matrices, whose entries are one-form differential operators.

$$\begin{aligned}
\text{even part} \;:\; \mathbf{s}\,\mathcal{J}_{ev} &= -\mathbf{d}\,\mathcal{C} - \mathcal{J}_{ev}\cdot\mathcal{C} - \mathcal{C}\cdot\mathcal{J}_{ev}\,, \\
\bar{\mathbf{s}}\,\mathcal{J}_{ev} &= -\mathbf{d}\,\bar{\mathcal{C}} - \mathcal{J}_{ev}\cdot\bar{\mathcal{C}} - \bar{\mathcal{C}}\cdot\mathcal{J}_{ev}\,, \\
\text{odd part} \;:\; \mathbf{s}\,\mathcal{J}_{od} &= -\mathbf{d}_M\,\mathcal{C} - \mathcal{J}_{od}\cdot\mathcal{C} - \mathcal{C}\cdot\mathcal{J}_{od}\,, \\
\bar{\mathbf{s}}\,\mathcal{J}_{od} &= -\mathbf{d}_M\,\bar{\mathcal{C}} - \mathcal{J}_{od}\cdot\bar{\mathcal{C}} - \bar{\mathcal{C}}\cdot\mathcal{J}_{od}\,.
\end{aligned} \tag{19}$$

Note that the even parts are the usual BRST/anti-BRST transformation rules of a one-form gauge field [19], while the odd parts are those of a matter field, plus the additional terms caused by the matrix derivative. These additional terms represent a translation of the scalar field and correspond to the vacuum shift in the usual Higgs mechanism. The difference, however, is that this is a built-in property of a gauge theory in the noncommutative geometry setting, in contradistinction to the conventional Higgs construction. The system's 'ordinary' gauge symmetry is thereby broken explicitly through that geometrical setting.

Adapting the Baulieu/Thierry-Mieg method for a BRST/anti-BRST invariant quantum action, which yields the Landau gauge for the usual Yang-Mills theory[19], we write the quantum action as

$$\mathcal{S}_Q = -\frac{1}{4}\int Tr\,\{\mathcal{F}_t\,^\star\cdot\mathcal{F}_t - \mathbf{s}\,\bar{\mathbf{s}}\,(\mathcal{J}\,^\star\cdot\mathcal{J}\,) + \alpha\;\mathbf{s}\,(\bar{\mathcal{C}}^\star\cdot\mathcal{E}\,)\}, \tag{20}$$

where, α is a parameter. Using the transformation rules (16), (17), (18) and (19), we obtain

$$Tr\{\mathbf{s}\,\bar{\mathbf{s}}\,(\mathcal{J}_{ev}\,^\star\cdot\mathcal{J}_{ev}\,)\} = 2\,Tr\{(\mathcal{J}_{ev}\,)^\star\cdot(\mathbf{d}\,\mathcal{E}\,)+(\mathbf{d}\,\bar{\mathcal{C}}\,)^\star\cdot(\mathbf{d}\,\mathcal{C}+\mathcal{J}_{ev}\cdot\mathcal{C}+\mathcal{C}\cdot\mathcal{J}_{ev}\,)\}, \tag{21}$$

$$Tr\{\mathbf{s}\,\bar{\mathbf{s}}\,(\mathcal{J}_{od}\,^\star\cdot\mathcal{J}_{od}\,)\} = 2\,Tr\{(\mathcal{J}_{od}\,)^\star\cdot(\mathbf{d}_M\,\mathcal{E}\,)+(\mathbf{d}_M\,\bar{\mathcal{C}}\,)^\star\cdot(\mathbf{d}_M\,\mathcal{C}+\mathcal{J}_{od}\cdot\mathcal{C}+\mathcal{C}\cdot\mathcal{J}_{od}\,)\}, \tag{22}$$

and

$$Tr\left\{\alpha\mathbf{s}\,(\bar{\mathcal{C}}\,^\star\cdot\mathcal{E}\,)\right\} = Tr\,\{\alpha\mathcal{E}\,^\star\cdot\mathcal{E}\,\}\,. \tag{23}$$

Thus, the quantum action $\mathcal{S}_Q$ can be written as

$$\begin{aligned}\mathcal{S}_Q &= -\frac{1}{4}\int Tr\,\{\mathcal{F}_t{}^\star\cdot\mathcal{F}_t + \alpha\ \mathcal{E}^\star\cdot\mathcal{E} \\ &- 2(\mathcal{J}_{ev})^\star\cdot(\mathbf{d}\,\mathcal{E}) - 2(\mathbf{d}\,\bar{\mathcal{C}})^\star\cdot(\mathbf{d}\,\mathcal{C} + \mathcal{J}_{ev}\cdot\mathcal{C} + \mathcal{C}\cdot\mathcal{J}_{ev}) \\ &- 2(\mathcal{J}_{od})^\star\cdot(\mathbf{d}_M\,\mathcal{E}) - 2(\mathbf{d}_M\,\bar{\mathcal{C}})^\star\cdot(\mathbf{d}_M\,\mathcal{C} + \mathcal{J}_{od}\cdot\mathcal{C} + \mathcal{C}\cdot\mathcal{J}_{od})\}. \end{aligned} \tag{24}$$

One can check that this quantum action is BRST/anti-BRST invariant.

In the above quantum action (24), the terms with the auxiliary field $\mathcal{E}$ are the gauge fixing terms and give rise to the 't Hooft gauge condition [20, 21] as we shall see in the next two sections. The first term is the classical action, and the remaining terms constitute the kinetic and interaction terms of the ghost fields. In the following two sections we calculate the quantum action (24) for the $SU(2/1)$ and $SU(2/2)$ cases explicitly.

III. BRST quantization of the $SU(2/1)$ case

The generators of $SU(2/1)$ are the same as those of $SU(3)$, namely the conventional λ - matrices, except for t_8, which is given by

$$t_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \tag{25}$$

in order to satisfy $STr(t_i) = 0$. We write the $SU(2/1)$ superconnection as

$$\begin{aligned}\mathcal{J} &= it_iJ_i\ \ (i = 1, 2, \cdots, 8) \\ &= \mathcal{J}_{ev} + \mathcal{J}_{od} = i\begin{pmatrix} \tau_aW_a - \frac{1}{\sqrt{3}}B & 0 \\ 0 & -\frac{2}{\sqrt{3}}B \end{pmatrix} + i\begin{pmatrix} 0 & \sqrt{2}\Phi \\ \sqrt{2}\Phi^\dagger & 0 \end{pmatrix}, \end{aligned} \tag{26}$$

where we identified the gauge and Higgs fields W_a, B, Φ, and $\Phi^\dagger$ with the components; $W_a = J_a\ (a = 1, 2, 3)$, $B = J_8$, $\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} J_4 - iJ_5 \\ J_6 - iJ_7 \end{pmatrix}$, and $\Phi^\dagger = \frac{1}{\sqrt{2}}\begin{pmatrix} J_4 + iJ_5 \\ J_6 + iJ_7 \end{pmatrix}$.

We now introduce the ghost, anti-ghost, and auxiliary fields, in the doubled-fiber bundle space.

$$\begin{aligned}
\mathcal{C} &= i\begin{pmatrix} \tau_a c_a - \frac{1}{\sqrt{3}} c_8 & 0 \\ 0 & -\frac{2}{\sqrt{3}} c_8 \end{pmatrix}, \quad \bar{\mathcal{C}} = i\begin{pmatrix} \tau_a \bar{c_a} - \frac{1}{\sqrt{3}} \bar{c_8} & 0 \\ 0 & -\frac{2}{\sqrt{3}} \bar{c_8} \end{pmatrix}, \\
\mathcal{E} &= i\begin{pmatrix} \tau_a b_a - \frac{1}{\sqrt{3}} b_8 & 0 \\ 0 & -\frac{2}{\sqrt{3}} b_8 \end{pmatrix} \quad (a = 1, 2, 3). \qquad (27)
\end{aligned}$$

In order to derive the BRST/anti-BRST transformation rules, we apply eqs.(16)-(19) of the previous section. In calculating the $SU(2/1)$ case, we encounter the following difficulty. With the 3×3 matrix representation, it is not possible to choose a constant matrix $\eta = \begin{pmatrix} 0 & \zeta \\ \overline{\zeta} & 0 \end{pmatrix}$ for the matrix derivative, satisfying the condition (8), $\overline{\zeta}\zeta = \zeta\overline{\zeta} \propto 1$, which is essential for the nilpotency of the matrix derivative. In order to resolve this difficulty, we first extend all 3×3 matrix representations of fields into 4×4 matrices, simply by adjoining a 4th row and a 4th column, with all components vanishing. We then choose the η matrix in this extended 4×4 matrix representation space, in which it does satisfy the nilpotency condition. This 4×4 η matrix, enables us to perform all calculations involving the η matrix, such as evaluating the supercurvature, etc. After this is done, we project back onto the 3×3 matrix representation space, simply discarding the 4th row and column. Note that this construction reflects the fact that the true fundamental representation of SU(2/1) is 4-dimensional [3], reflecting the homomorphism with OSp(2/2) and fitting the internal quantum numbers for quarks, i.e. $(u_R/u_L, d_L/d_R)$ where the order follows descending weak hypercharges $(4/1, 1/-2)$ (in units of $(1/3)$). However, for integer charges, the upper state trivializes and disconnects (e.g. the ν_R) and we are left with the 3-dimensional representation. As a matter of fact, the procedure we use here also corresponds to the *projective module* method of Connes and Lott [8]. We thus perform the actual calculation with

$$\zeta = \overline{\zeta} = \sqrt{2}k\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad k : \text{real},$$

and obtain the following BRST/anti-BRST transformation rules.

$$sA_{II} = -dc_{II} - A_{II}c_{II} - c_{II}A_{II},$$

$$
\begin{aligned}
\bar{s}A_{II} &= -dc\bar{}_{II} - A_{II}c\bar{}_{II} - c\bar{}_{II}A_{II}, \\
sA_I &= -dc_I, \quad \bar{s}A_I = -d\bar{c}_I, \\
s\Phi &= -c_{II}(\Phi+\xi) - \frac{1}{\sqrt{3}}c_I(\Phi+\xi), \\
\bar{s}\Phi &= -c\bar{}_{II}(\Phi+\xi) - \frac{1}{\sqrt{3}}\bar{c}_I(\Phi+\xi), \\
s\Phi^\dagger &= (\Phi^\dagger+\xi^\dagger)c_{II} + \frac{1}{\sqrt{3}}(\Phi^\dagger+\xi^\dagger)c_I, \\
\bar{s}\Phi^\dagger &= (\Phi^\dagger+\xi^\dagger)c\bar{}_{II} + \frac{1}{\sqrt{3}}(\Phi^\dagger+\xi^\dagger)\bar{c}_I, \\
sc_{II} &= -c_{II}c_{II}, \quad \bar{s}c\bar{}_{II} = -c\bar{}_{II}c\bar{}_{II}, \\
sc_I &= \bar{s}\bar{c}_I = 0, \\
sc\bar{}_{II} &= b_{II}, \quad \bar{s}c_{II} = -b_{II} - c_{II}c\bar{}_{II} - c\bar{}_{II}c_{II}, \\
sb_{II} &= 0, \quad \bar{s}b_{II} = -c\bar{}_{II}b_{II} + b_{II}c\bar{}_{II}, \\
s\bar{c}_I &= -\bar{s}c_I = b_I, \quad sb_I = \bar{s}b_I = 0,
\end{aligned}
\tag{28}
$$

where

$$
\begin{aligned}
A_{II} &= i\tau_a W_a, \quad A_I = iB, \quad c_{II} = i\tau_a c_a, \quad c_I = ic_8, \\
c\bar{}_{II} &= i\tau_a \bar{c}_a, \quad \bar{c}_I = i\bar{c}_8, \quad b_{II} = i\tau_a b_a \ \ (a=1,2,3), \quad b_I = ib_8, \\
\xi &= k\begin{pmatrix} 0 \\ 1 \end{pmatrix}.
\end{aligned}
\tag{29}
$$

Note that the transformation rules of Φ and $\Phi^\dagger$ correspond to those of the Higgs fields with a shifted vacuum. For the supercurvature we obtain

$$
\mathcal{F}_t = \begin{pmatrix} F_W - \frac{1}{\sqrt{3}}F_B - 2(\Phi\Phi^\dagger + \xi\Phi^\dagger + \Phi\xi^\dagger) & -i\sqrt{2}(D\Phi + (i\vec{W}\cdot\vec{\tau} + \frac{i}{\sqrt{3}}B)\xi) \\ -i\sqrt{2}(D\Phi^\dagger - \xi^\dagger(i\vec{W}\cdot\vec{\tau} + \frac{i}{\sqrt{3}}B)) & -\frac{2}{\sqrt{3}}F_B - 2(\Phi^\dagger\Phi + \xi^\dagger\Phi + \Phi^\dagger\xi) \end{pmatrix},
\tag{30}
$$

where

$$
\begin{aligned}
F_W &= \frac{1}{2}F_{W\mu\nu}dx^\mu \wedge dx^\nu = d(i\vec{W}\cdot\vec{\tau}) + (i\vec{W}\cdot\vec{\tau})(i\vec{W}\cdot\vec{\tau}), \\
F_B &= \frac{1}{2}F_{B\mu\nu}dx^\mu \wedge dx^\nu = d(iB), \\
\Phi &= \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_3 + i\phi_4 \\ \phi_1 + i\phi_2 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} J_4 - iJ_5 \\ J_6 - J_7 \end{pmatrix},
\end{aligned}
\tag{31}
$$

$$D\Phi = (D\Phi)_\mu dx^\mu = d\Phi + (i\vec{W}\cdot\vec{\tau} + \frac{i}{\sqrt{3}}B)\Phi,$$
$$D\Phi^\dagger = (D\Phi^\dagger)_\mu dx^\mu = d\Phi^\dagger - \Phi^\dagger(i\vec{W}\cdot\vec{\tau} + \frac{i}{\sqrt{3}}B).$$

We use $d^4x = dx^0\wedge dx^1\wedge dx^2\wedge dx^3$, $\epsilon_{0123} = 1$, and adopt the convention of Ref.[22] for the dual of a differential form in n dimension, required for (24),

$$*\,(dx^{i_1}\wedge dx^{i_2}\wedge\cdots\wedge dx^{i_p}) = \frac{1}{(n-p)!}\epsilon^{i_1 i_2\cdots i_p}{}_{i_{p+1}\cdots i_n} dx^{i_{p+1}}\wedge\cdots\wedge dx^{i_n} \qquad (32)$$

satisfying $**\omega_p = (-1)^{p(n-p)}\omega_p$ for a p-form ω_p.

Selecting the metric $g_{\mu\nu} = (-1,+1,+1,+1)$, the first term in (24), the classical action, is given by

$$\begin{aligned}
\mathcal{L}_C &= \frac{1}{4}F_{Wa\mu\nu}F_{Wa}^{\mu\nu} + \frac{1}{4}F_{B\mu\nu}F_B^{\mu\nu} \\
&\quad -(D\Phi^\dagger - \xi^\dagger(i\vec{W}\cdot\vec{\tau} + \frac{i}{\sqrt{3}}B))_\mu(D\Phi + (i\vec{W}\cdot\vec{\tau} + \frac{i}{\sqrt{3}}B)\xi)^\mu \\
&\quad -2((\Phi^\dagger + \xi^\dagger)(\Phi+\xi) - \xi^\dagger\xi)^2 \\
&= \frac{1}{4}F_{Wa\mu\nu}F_{Wa}^{\mu\nu} + \frac{1}{4}F_{B\mu\nu}F_B^{\mu\nu} \\
&\quad -(D\Phi^\dagger - ik(\sqrt{2}W_- + \frac{2}{\sqrt{3}}Z))_\mu(D\Phi + ik\begin{pmatrix}\sqrt{2}W_+\\ \frac{2}{\sqrt{3}}Z\end{pmatrix})^\mu \qquad (33)\\
&\quad -2(\Phi^\dagger\Phi + k(\phi^0 + \bar{\phi^0}))^2,
\end{aligned}$$

where $W_\pm^\mu = \frac{1}{\sqrt{2}}(W_1^\mu \mp iW_2^\mu)$, $Z^\mu = -\frac{\sqrt{3}}{2}W_3^\mu + \frac{1}{2}B^\mu$, $A^\mu = \frac{1}{2}W_3^\mu + \frac{\sqrt{3}}{2}B^\mu$.

In order to see the physical spectrum of the theory, we now write the above expression in the *unitary gauge*, which is given by $\Phi = \begin{pmatrix}0\\ \frac{1}{\sqrt{2}}\chi\end{pmatrix}$ with real χ.

$$\begin{aligned}
\mathcal{L}_C^{UG} &= \frac{1}{4}F_{Wa\mu\nu}F_{Wa}^{\mu\nu} + \frac{1}{4}F_{B\mu\nu}F_B^{\mu\nu} \\
&- \{\frac{1}{2}\partial_\mu\chi\partial^\mu\chi + g^2W_{+\mu}W_-^\mu(\chi + \frac{\sqrt{2}k}{g})^2 + \frac{2}{3}g^2Z_\mu Z^\mu(\chi+\frac{\sqrt{2}k}{g})^2\} \qquad (34)\\
&- \frac{1}{2}(g\chi^2 + 2\sqrt{2}k\chi)^2
\end{aligned}$$

The coupling constant g is introduced by scaling the superconnection as $\mathcal{J} \to g\mathcal{J}$. In this unitary gauge we see that only one scalar field remains as a physical (and

massive) Higgs field χ, whereas the other three scalars have been 'mutated', now providing the longitudinal components of $W_\pm$ and Z. The masses of the massive particles are $M_\chi = 2\sqrt{2}k, M_W = \sqrt{2}k, M_Z = \frac{2\sqrt{2}}{\sqrt{3}}k$, and we see the relations $\frac{M_W^2}{M_Z^2} = \frac{3}{4} = cos^2\theta_W,\ M_\chi = 2M_W$. We shall return to the latter ratio $\frac{M_\chi}{M_W}$ in section 5, when discussing possible quantum corrections.

We now write the quantum Lagrangian of (24) as

$$\mathcal{L}_Q = \mathcal{L}_C + \mathcal{L}_1 + \mathcal{L}_2, \tag{35}$$

where $\mathcal{L}_C$ is the classical Lagragian, $\mathcal{L}_1$ stands for the ghost terms, and $\mathcal{L}_2$ for the gauge fixing terms. After some calculations, we obtain $\mathcal{L}_1$,

$$\begin{aligned} \mathcal{L}_1 &= \frac{1}{2}\ tr[\partial_\mu \bar{c_{II}} D^\mu c_{II} + 2\partial_\mu \bar{c_I} \partial^\mu c_I \\ &+ \{(\bar{c_{II}} + \frac{1}{\sqrt{3}}\bar{c_I})\xi(\Phi^\dagger + \xi^\dagger)(c_{II} + \frac{1}{\sqrt{3}}c_I)\} \\ &+ \{\xi^\dagger(\bar{c_{II}} + \frac{1}{\sqrt{3}}\bar{c_I})(c_{II} + \frac{1}{\sqrt{3}}c_I)(\Phi + \xi)\}], \end{aligned} \tag{36}$$

where $D^\mu c_{II} = \partial^\mu c_{II} + [A^\mu_{II}, c_{II}]$.

For $\mathcal{L}_2$, we obtain

$$\begin{aligned} \mathcal{L}_2 &= \frac{\alpha}{2}\ \{[(b_1)^2 + (b_2)^2 + (b_Z)^2 + (b_A)^2] + \frac{2}{\alpha}\ [b_1(\partial_\mu W_1^\mu - \sqrt{2}k\phi_4) \\ &+ b_2(\partial_\mu W_2^\mu - \sqrt{2}k\phi_3) + b_Z(\partial_\mu Z^\mu - \frac{2\sqrt{2}}{\sqrt{3}}k\phi_2) + b_A(\partial_\mu A^\mu)]\}, \end{aligned} \tag{37}$$

where $b_Z = -\frac{\sqrt{3}}{2}b_3 + \frac{1}{2}b_8, b_A = \frac{1}{2}b_3 + \frac{\sqrt{3}}{2}b_8$. After integrating out the auxiliary fields b_1, b_2, b_Z, and b_A, $\mathcal{L}_2$ becomes

$$\mathcal{L}_2 = -\frac{1}{2\alpha}\ \{(\partial_\mu W_1^\mu - \sqrt{2}k\phi_4)^2 + (\partial_\mu W_2^\mu - \sqrt{2}k\phi_3)^2 + (\partial_\mu Z^\mu - \frac{2\sqrt{2}}{\sqrt{3}}k\phi_2)^2 + (\partial_\mu A^\mu)^2\}. \tag{38}$$

This expression clearly shows that we obtain the gauge-fixed quantum Lagrangian of the 't Hooft gauge [20, 21], as we claimed in the previous section:

$$\partial_\mu W_1^\mu - M_W\phi_4 = 0,$$

$$\begin{aligned} \partial_\mu W_2^\mu - M_W \phi_3 &= 0, \\ \partial_\mu Z^\mu - M_Z \phi_2 &= 0, \\ \partial_\mu A^\mu &= 0. \end{aligned} \tag{39}$$

IV. BRST quantization of $SU(2/2)$ case

We now calculate the $SU(2/2)$ case. The generators of $SU(2/2)$ are the same as those of $SU(4)$, except for t_8 and t_{15}, which are replaced by

$$t_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad t_{15} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}, \tag{40}$$

to conforming with the super-tracelessness of the $SU(2/2)$ generators. The super-connection for the $SU(2/2)$ case can be written as

$$\mathcal{J} = it_i J_i \quad (i = 1, 2, \cdots, 15) = \begin{pmatrix} A_L + \frac{1}{\sqrt{2}} B & i\Phi \\ i\Phi^\dagger & A_R + \frac{1}{\sqrt{2}} B \end{pmatrix} \tag{41}$$

with one-forms in the even part and zero-forms in the odd part, given as

$$A_L = i\tau_a A_{La}, \quad A_R = i\tau_a A_{Ra}, \quad B = iIY, \quad \Phi = I\phi_0 + i\tau_a \phi_a, \tag{42}$$

where $\tau_a (a = 1, 2, 3)$ are Pauli matrices, and I is 2×2 identity matrix. A_{La}, A_{Ra}, Y are real, whereas ϕ_0, ϕ_a are complex, the fields being assigned to the components of J's according to

$$\begin{aligned} A_{La} &= J_a \ (a = 1, 2, 3), \quad A_{R_1} = J_{13}, \quad A_{R_2} = J_{14}, \\ A_{R_3} &= -\frac{1}{\sqrt{3}}(J_8 + \sqrt{2} J_{15}), \quad Y = -\frac{1}{\sqrt{3}}(\sqrt{2} J_8 - J_{15}), \\ \phi_0 &= \frac{1}{2}[(J_4 - iJ_5) + (J_{11} - iJ_{12})], \quad \phi_1 = -\frac{i}{2}[(J_6 - iJ_7) + (J_9 - iJ_{10})], \\ \phi_2 &= -\frac{1}{2}[(J_6 - iJ_7) - (J_9 - iJ_{10})], \quad \phi_3 = -\frac{i}{2}[(J_4 - iJ_5) - (J_{11} - iJ_{12})]. \end{aligned}$$

A_L and A_R are thus the $SU(2)$ gauge fields, B is the $U(1)$ gauge field, and Φ is the complex scalar field (with its four components).

We now introduce the ghost and anti-ghost fields,

$$\mathcal{C} = \begin{pmatrix} c_L & 0 \\ 0 & c_R \end{pmatrix} + \begin{pmatrix} \frac{1}{\sqrt{2}}c_I & 0 \\ 0 & \frac{1}{\sqrt{2}}c_I \end{pmatrix}, \ \bar{\mathcal{C}} = \begin{pmatrix} \bar{c_L} & 0 \\ 0 & \bar{c_R} \end{pmatrix} + \begin{pmatrix} \frac{1}{\sqrt{2}}\bar{c_I} & 0 \\ 0 & \frac{1}{\sqrt{2}}\bar{c_I} \end{pmatrix}, \tag{43}$$

where $c_L = i\tau_a c_{La}, c_R = i\tau_a c_{Ra} (a = 1, 2, 3), c_I = iIc_{Ir}$, with real c_{La}, c_{Ra}, and c_{Ir} and similarly for $\bar{\mathcal{C}}$. $\{c_L, \bar{c_L}\}$ and $\{c_R, \bar{c_R}\}$ are the ghost and antighost fields for the $SU(2)$ gauge fields A_L and A_R, respectively, and $\{c_I, \bar{c_I}\}$ are those of the $U(1)$ gauge field B.

The BRST/anti-BRST transformation rules are obtained from (16)-(19). Choosing

$$\eta = \begin{pmatrix} 0 & \xi \\ \xi^\dagger & 0 \end{pmatrix}, \text{ where } \xi = k \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ k: \text{ real}, \tag{44}$$

we get

$$\begin{aligned}
sA_L &= -dc_L - A_L c_L - c_L A_L, \\
\bar{s}A_L &= -d\bar{c_L} - A_L \bar{c_L} - \bar{c_L} A_L, \\
sA_R &= -dc_R - A_R c_R - c_R A_R, \\
\bar{s}A_R &= -d\bar{c_R} - A_R \bar{c_R} - \bar{c_R} A_R, \\
sB &= -dc_I, \quad \bar{s}B = -d\bar{c_I}, \\
s\Phi &= (\Phi + \xi)c_R - c_L(\Phi + \xi), \\
\bar{s}\Phi &= (\Phi + \xi)\bar{c_R} - \bar{c_L}(\Phi + \xi), \\
s\Phi^\dagger &= (\Phi^\dagger + \xi^\dagger)c_L - c_R(\Phi^\dagger + \xi^\dagger), \\
\bar{s}\Phi^\dagger &= (\Phi^\dagger + \xi^\dagger)\bar{c_L} - \bar{c_R}(\Phi^\dagger + \xi^\dagger), \\
sc_L &= -c_L c_L, \quad \bar{s}\bar{c_L} = -\bar{c_L}\bar{c_L}, \\
sc_R &= -c_R c_R, \quad \bar{s}\bar{c_R} = -\bar{c_R}\bar{c_R}, \\
sc_I &= \bar{s}\bar{c_I} = 0, \\
s\bar{c_L} &= b_L, \quad \bar{s}c_L = -b_L - c_L\bar{c_L} - \bar{c_L}c_L, \\
s\bar{c_R} &= b_R, \quad \bar{s}c_R = -b_R - c_R\bar{c_R} - \bar{c_R}c_R, \\
sb_L &= 0, \quad \bar{s}b_L = -\bar{c_L}b_L + b_L\bar{c_L},
\end{aligned} \tag{45}$$

$$sb_R = 0, \quad \bar{s}b_R = -\bar{c_R}b_R + b_R\bar{c_R},$$

$$s\bar{c_I} = -\bar{s}c_I = b_I, \quad sb_I = \bar{s}b_I = 0.$$

We have introduced the auxiliary fields $\mathcal{E} = \begin{pmatrix} b_L & 0 \\ 0 & b_R \end{pmatrix} + \begin{pmatrix} \frac{1}{\sqrt{2}}b_I & 0 \\ 0 & \frac{1}{\sqrt{2}}b_I \end{pmatrix}$ with $b_L = i\tau_a b_{La}$, $b_R = i\tau_a b_{Ra}$ $(a = 1, 2, 3)$, and $b_I = iIb_{Ir}$, where b_{La}, b_{Ra}, and b_{Ir} are real. For the supercurvature, we obtain

$$\mathcal{F}_t == \begin{pmatrix} F_L + \frac{1}{\sqrt{2}}F_B - (\Phi\Phi^\dagger + \xi\Phi^\dagger + \Phi\xi^\dagger) & -i(D\Phi + A_L\xi - \xi A_R) \\ -i(D\Phi^\dagger - \xi^\dagger A_L + A_R\xi^\dagger) & F_R + \frac{1}{\sqrt{2}}F_B - (\Phi^\dagger\Phi + \xi^\dagger\Phi + \Phi^\dagger\xi) \end{pmatrix}, \tag{46}$$

where $F_L = dA_L + A_L A_L$, $F_R = dA_R + A_R A_R$, $F_B = dB$, $D\Phi = d\Phi + A_L\Phi - \Phi A_R$, $D\Phi^\dagger = d\Phi^\dagger - \Phi^\dagger A_L + A_R\Phi^\dagger$.

The classical Lagrangian, the first term in (24), is given by [23],

$$\begin{aligned} \mathcal{L}_C &= tr[\frac{1}{4}F_{+\mu\nu}F_+^{\mu\nu} + \frac{1}{4}F_{-\mu\nu}F_-^{\mu\nu} + \frac{1}{8}F_{B\mu\nu}F_B^{\mu\nu} \\ &- \frac{1}{2}(D\Phi^\dagger + 2kA_-)_\mu(D\Phi - 2kA_-)^\mu - \frac{1}{2}(\Phi^\dagger\Phi + k(\Phi + \Phi^\dagger))^2], \end{aligned} \tag{47}$$

where $A_\pm$ are respectively the vector and axial vector gauge fields, as defined by $A_\pm = \frac{1}{2}(\pm A_L + A_R) = i\tau_a A_{\pm a}$, and $F_+^{\mu\nu} = \partial^\mu A_+^\nu - \partial^\nu A_+^\mu + [A_+^\mu, A_+^\nu] + [A_-^\mu, A_-^\nu]$, $F_-^{\mu\nu} = \partial^\mu A_-^\nu - \partial^\nu A_-^\mu + [A_+^\mu, A_-^\nu] + [A_-^\mu, A_+^\nu]$. The above expression tells us that the three axial vector gauge fields A_{-a} have acquired the mass $2k$, whereas the three vector gauge fields A_{+a} and the $U(1)$ gauge field Y remain massless.

For the quantum Lagrangian $\mathcal{L}_Q$, we again write, as in (35),

$$\mathcal{L}_Q = \mathcal{L}_C + \mathcal{L}_1 + \mathcal{L}_2.$$

The ghost part $\mathcal{L}_1$ is given by

$$\begin{aligned} \mathcal{L}_1 &= \frac{1}{2}tr[(\partial_\mu\bar{c_L}D^\mu c_L + \partial_\mu\bar{c_R}D^\mu c_R + \partial_\mu\bar{c_I}\partial^\mu c_I) - 2k^2(\bar{c_L} - \bar{c_R})(c_L - c_R) \\ &+ k(\{(\bar{c_L} - \bar{c_R})c_R - c_L(\bar{c_L} - \bar{c_R})\}\Phi^\dagger + \{c_R(\bar{c_L} - \bar{c_R}) - (\bar{c_L} - \bar{c_R})c_L\}\Phi)], \end{aligned} \tag{48}$$

where $D^\mu c_L = \partial^\mu c_L + [A_L^\mu, c_L]$, $D^\mu c_R = \partial^\mu c_R + [A_R^\mu, c_R]$.

The gauge fixing part $\mathcal{L}_2$ is given by

$$\mathcal{L}_2 = \alpha\ \{((b_{-a})^2 + (b_{+a})^2 + \frac{1}{2}(b_{Ir})^2) + \frac{2}{\alpha}(b_{-a}(\partial_\mu A_{-a}^\mu - 2k\varphi_a) + b_{+a}(\partial_\mu A_{+a}^\mu) + \frac{1}{2}b_{Ir}(\partial_\mu Y^\mu))\}, \tag{49}$$

where $b_\pm = \frac{1}{2}(\pm b_L + b_R) = i\tau_a b_{\pm a}$, $\varphi = \frac{1}{2}(\Phi - \Phi^\dagger) = \tau_a \varphi_a$. Integrating out the auxiliary fields $b_\pm$, $\mathcal{L}_2$ becomes

$$\mathcal{L}_2 = -\frac{1}{\alpha} \{(\partial_\mu A^\mu_- - 2k\varphi)^2 + (\partial_\mu A^\mu_+)^2 + \frac{1}{2}(\partial_\mu Y^\mu)^2\}. \tag{50}$$

This expression again displays the quantum Lagrangian in the 't Hooft gauge:

$$\begin{aligned} \partial_\mu A^\mu_{-a} - M_{A_-}\varphi_a &= 0, \\ \partial_\mu A^\mu_{+a} &= 0, \\ \partial_\mu Y^\mu &= 0, \end{aligned} \tag{51}$$

where $M_{A_-} = 2k$ is used. If we write Φ as $(\sigma + i\vec{\pi}\cdot\vec{\tau}) + i(\eta + \vec{\rho}\cdot\vec{\tau})$ with real $\sigma, \vec{\pi}, \eta, \vec{\rho}$ fields, then φ in $\mathcal{L}_2$ can be identified with $\vec{\pi}$. This is consistent with the fact that the $\vec{\pi}$ fields are gauged away and mutate into the longitudinal components of the axial vector fields A_- in the unitary gauge. This is also related to the fact that the $SU(2/2)$ case corresponds to the gauged $SU(2)\times SU(2)$ σ-model [23].

V. Conclusion

In the matrix derivative approach, derived from noncommutative geometrical gauge theory and adjoined to internal supersymmetry, in its superconnection version, the vector gauge fields and the scalar fields are combined together, constituting the superconnection. The two sets of fields are thus related as a supermultiplet from the very beginning. This provides for an elegant geometrical realization of the Higgs mechanism. The entire Lagrangian is geometrical, even including the negative mass term for the scalar field, needed to trigger the spontaneous symmetry breakdown for the (g-even) gauge subgroup. That symmetry-breaking quadratic term for the scalar field is provided by the matrix derivative, beyond the unification achieved by the supergroup by itself. Summarizing, the unification is complete, within the limitations set by the broken symmetry actual content. We return to these limitations in our last paragraph.

Another advantage of the formalism touches upon the quantum action, namely in the gauge in which it appears, as a result of the construction. This turns out to be the 't Hooft gauge, most convenient for a spontaneously broken symmetry with Higgs field and suitable for renormalization [20, 21]. We obtained this action just by adapting the Baulieu/Thierry-Mieg method [19], which would yield the Landau gauge for the unbroken Yang-Mills theory, to the matrix derivative approach.

For the calculation of the ${\mathcal{F}_t}^\star \cdot \mathcal{F}_t$ term in the classical and some of the other parts of the quantum Lagrangian we have used the definition of (32) for the dual form. This definition gives the kinetic terms of both the vector and scalar fields automatically in their canonical form, also providing the relation $M_\chi = 2M_W$.[1] This ratio is also due to the fact that we have only one overall supergauge coupling constant g for the superconnection $\mathcal{J}$ in section 3, due to universality. Without the assumption of universality for the supergroup we would have independent couplings for fields corresponding to forms of different degrees - in our case the even and odd parts of the superconnection, i.e. two independent couplings. One might then obtain a different mass ratio for the Higgs and gauge bosons [24].

Lastly, we note that only the even part of the supergroup is gauged in the sense of Relativistic Quantum Field Theory - even though the entire supergroup is used as a structure group for the theory and provides the geometrical framework for the quantization procedure, including the 't Hooft gauge. As a result, there is no guarantee of non-renormalization of the theory's couplings beyond those of the g-even gauge subgroup.

Acknowledgements

CYL would like to thank Hoil Kim for helpful discussions on the mathematics of noncommutative geometry. This work was supported in part by NON DIRECTED

[1] This is a classical relationship. Including quantum correction should modify this result. For a related result, see Ref. [16], in an application of the superconnection approach without introducing the notion of a matrix derivative.

RESEARCH FUND, Korea Research Foundation, in part by the KOSEF through the SRC program of SNU-CTP, and in part by the BSRI program of the Ministry of Education.

References

[1] Y. Ne'eman, Phys. Lett. B81 (1979) 190. For a different approach, see also D. B. Fairlie, Phys. Lett. B82 (1979) 97.

[2] Y. Ne'eman and J. Thierry-Mieg, Proc. Nat. Acad. Sci. USA 77 (1980) 720.

[3] Y. Ne'eman and J. Thierry-Mieg, in Proceedings of the Salamanca(1979) International Conference on Differential Geometric Methods in Mathematical Physics, ed. A. Perez-Rendon, Lecture Notes in Mathematics No. 836 (Springer-Verlag, Berlin, 1980).

[4] Y. Ne'eman and J. Thierry-Mieg, Phys. Lett. B108 (1982) 399.

[5] Y. Ne'eman and S. Sternberg, Proc. Natl. Acad. Sci. USA 87 (1990) 7875.

[6] C.Y. Lee and Y. Ne'eman, Phys. Lett. B264 (1991) 389; *ibid.* B269 (1991) 477.

[7] A. Connes, in "The interface of mathematics and particle physics", ed. D. Quillen, G. Segal and S. Tsou (Oxford Univ. Press, Oxford, 1990).

[8] A. Connes and J. Lott, Nucl. Phys. B (Proc. Suppl.) 18B (1990) 29.

[9] A. H. Chamseddine, G. Felder and J. Fröhlich, Nucl. Phys. B395 (1993) 672.

[10] A. H. Chamseddine, G. Felder and J. Fröhlich, Comm. Math. Phys. 155 (1993) 205.

[11] A. H. Chamseddine, "Connection between space-time supersymmetry and non-commutative geometry", hepth-9404138.

[12] R. Coquereaux, G. Esposito-Farèse and G. Vaillant, Nucl. Phys. B353 (1991) 689.

[13] R. Häußling, N.A. Papadopoulos and F. Scheck, Phys. Lett. B260 (1991) 125.

[14] D. Quillen, Topology 24 (1985) 89.

[15] N.A. Papadopoulos, J. Plass, and F. Scheck, Preprint MZ-TH/93-26 (1993).

[16] D.S. Hwang, C.Y. Lee and Y. Ne'eman, "BRST quantization of SU(2/1) electro-weak theory in the superconnection approach - and the Higgs meson mass", to be published in Int. J. Mod. Phys. A.

[17] J. Thierry-Mieg, J. Math. Phys. 21 (1980) 2834. For a doubled fiber bundle structure, see M. Quiros, F. J. De Urries, J. Hoyos, M. L. Mazou, and E. Rodriguez, *ibid.* 22 (1981) 1767 and the next reference.

[18] J. Thierry-Mieg and Y. Ne'eman, Proc. Natl. Acad. Sci. USA 79 (1982) 7068.

[19] L. Baulieu and J. Thierry-Mieg, Nucl. Phys. B197 (1982) 477.

[20] G. 't Hooft, Nucl. Phys. B33 (1971) 173; *ibid.* B35 (1971) 167.

[21] K. Fujikawa, B. W. Lee and A. I. Sanda, Phys. Rev. D6 (1972) 2923.

[22] T. Eguchi, P. Gilkey and A. Hanson, Phys. Rep. 66 (1980) 213.

[23] D. S. Hwang and T. Lee, Int. J. Mod. Phys. A9 (1994) 5531.

[24] R. Coquereaux, G. Esposito-Farèse and F. Scheck, Int. J. Mod. Phys. A7 (1992) 6555.