SNUTP 95-032
HYUPT-95/2

Lie algebra cohomology and group structure of gauge theories

Hyun Seok Yang and Bum-Hoon Lee

Department of Physics, Hanyang University, Seoul 133-791, Korea

Abstract

We explicitly construct the adjoint operator of coboundary operator and obtain the Hodge decomposition theorem and the Poincaré duality for the Lie algebra cohomology of the infinite-dimensional gauge transformation group. We show that the adjoint of the coboundary operator can be identified with the BRST adjoint generator $Q^\dagger$ for the Lie algebra cohomology induced by BRST generator Q. We also point out an interesting duality relation - Poincaré duality - with respect to gauge anomalies and Wess-Zumino-Witten topological terms. We consider the consistent embedding of the BRST adjoint generator $Q^\dagger$ into the relativistic phase space and identify the noncovariant symmetry recently discovered in QED with the BRST adjoint Nöther charge $Q^\dagger$.

PACS numbers: 11.15.-q, 02.40.Re, 11.30.Ly

I. INTRODUCTION

The theory of gauge fields is based on symmetry principles and the hypothesis of locality of fields. The principle of local gauge invariance determines all the forms of the interactions and allows the geometrical description of the interactions [1]. However the quantization of gauge fields leads to difficulties due to the constraints arising from the gauge symmetry. These difficulties of the quantization of constrained systems can be circumvented by the extension of phase space including the anticommuting ghost variables [2]. In this approach, the original gauge symmetry is transformed into the so-called BRST symmetry in the extended phase space [3,4]. The BRST symmetry will determine all the forms of the interactions and the algebraic and topological properties of the fields in the quantum theory [5].

The question that comes naturally to mind is how we recover the original gauge invariant space consisting of only physical degrees of freedom from the extended phase space with ghosts [4–6] and what is the physical spectrum with the group invariant structure. In order to study the algebraic and topological structures of gauge theories, we follow the point of view of Ref. [7] about the ghost fields and the BRST transformation. That is, we identify the ghost field with the Cartan-Maurer form on an infinite-dimensional Lie group G_∞ - the group of gauge transformation - and the BRST generator Q with the coboundary operator s on its Lie algebra $\mathcal{G}$. Through these identifications, we have the natural framework to construct the Lie algebra cohomology induced by the BRST generator Q. This Lie algebra cohomology will be related to the group invariants of the configuration space of gauge fields and matter fields.

The organization of this paper is as follows. In Sec. II, we construct the cochain complex on $\mathcal{G}$ with values in a $\mathcal{G}$-module [8–10]. With the pairing between Lie algebra $\mathcal{G}$ and its dual space $\mathcal{G}^*$, we define a chain as an element of the dual space to the cochain and a dual operation s_* of s. We define a positive-definite inner product and construct an adjoint operator $s^\dagger$ of s using the Hodge duality operation. We obtain the Hodge decomposition theorem, Poincaré duality, and Künneth formula analogous to the de Rham cohomology

[11]. In Sec. III, we show that the adjoint of the coboundary operator can be identified with the BRST adjoint generator $Q^\dagger$ for the Lie algebra cohomology induced by BRST generator Q and each cohomology class on a polynomial space is characterized by the gauge invariant polynomials with a particular group invariant structure imposed on the cochain (or chain) space. We discuss the physical implications of the Lie algebra cohomology in the contexts of gauge anomaly and the effective action with the symmetry group G spontaneously broken to a subgroup H. The Lie algebra cohomology allows us algebraic and topological characterization of them and provides an interesting duality relation - Poincaré duality - between them. In Sec. IV, we apply this cohomology to QED and QCD. In order to consider the consistent embedding of the BRST adjoint generator $Q^\dagger$ into the relativistic phase space, we introduce the nonminimal sector of BRST generator [4]. Through this procedure, we find the BRST-like Nöther charge $Q^\dagger$ corresponding to the adjoint of the BRST generator Q, which generates a new kind of noncovariant symmetry in QED in Refs. [12,13]. Section V contains discussion and some comments.

II. LIE ALGEBRA COHOMOLOGY

Let P be a principal bundle with a structure group G (a compact Lie group with the invariant inner product defined on its Lie algebra g) over a differentiable manifold M (flat Minkowski space or Euclidean space $\mathbf{R}^n$). The gauge transformation group G_∞ - an automorphism of P- and its Lie algebra $\mathcal{G}$ can be identified with the set of C^∞-functions on M taking values in the structure group G and its Lie algebra g, respectively. One defines the dual spaces g^* of g and $\mathcal{G}^*$ of $\mathcal{G}$ as follows [10]:

$$< x,\ X >= \sum_{a=1}^{dimG} X^a x_a, \ \ \text{for } X \in g \ \ (\mathcal{G}), \ \ x \in g^* \ \ (\mathcal{G}^*). \tag{2.1}$$

The spacetime dependence of the elements of G_∞, $\mathcal{G}$, and $\mathcal{G}^*$ will be suppressed unless otherwise explicitly indicated and an L^2-norm will be assumed in the inner product (2.1) between $\mathcal{G}$ and $\mathcal{G}^*$ [14]. Using the pairing between Lie algebra g (or $\mathcal{G}$) and its dual space g^* (or $\mathcal{G}^*$), the coadjoint action of G (or G_∞) on g^* (or $\mathcal{G}^*$) is defined by

$$< X,\ Ad^*_g x >=< Ad_{g^{-1}}X,\ x > \text{for } g \in G\ \ (G_\infty),\ \ x \in g^*\ \ (\mathcal{G}^*). \tag{2.2}$$

Consider a p-cochain w^p, an element of $C^p(\mathcal{G}; R)$, where C^p is an antisymmetric p-linear map on $\mathcal{G}$ with values in a left $\mathcal{G}$-module R with the ring structure [8–11]. The space of cochains on $\mathcal{G}$ is the direct sum of the spaces of p-cochains:

$$C^* = \oplus_{p=0}^{dimG} C^p. \tag{2.3}$$

We introduce on C^* the operators $i(\vartheta)(x)$ and $\epsilon(\vartheta^*)(x)$ on a point $x \in M$ defined as follows:

$$i(\vartheta) : C^p \to C^{p-1}, \qquad \forall \vartheta \in \mathcal{G}$$

by

$$(i(\vartheta)(x)w^p)(\vartheta_1, \cdots, \vartheta_{p-1})(y) = w^p(\vartheta, \vartheta_1, \cdots, \vartheta_{p-1})(y)\delta(x-y), \quad w^p \in C^p; \tag{2.4}$$

and

$$\epsilon(\vartheta^*) : C^p \to C^{p+1}, \qquad \forall \vartheta^* \in \mathcal{G}^*$$

by

$$(\epsilon(\vartheta^*)(x)w^p)(\vartheta_1, \cdots, \vartheta_{p+1})(y) = \sum_{l=1}^{p+1}(-1)^{l+1} < \vartheta^*(x),\ \vartheta_l(y) > w^p(\vartheta_1, \cdots, \hat{\vartheta}_l, \cdots, \vartheta_{p+1})(y), \tag{2.5}$$

where $\hat{}$ indicates omission. Denote by $\{\theta_a\}, a = 1, \cdots, N \equiv dimG$, a basis of $\mathcal{G}$ and by $\{\theta^{*a}\}$ the basis of $\mathcal{G}^*$ such that

$$< \theta^{*a}(x),\ \theta_b(y) >= \delta^a_b \delta(x-y). \tag{2.6}$$

Then straightforward calculations using the definitions (2.4) and (2.5) lead to the following relations [10]

$$\begin{aligned}
&\{i(\theta_a), i(\theta_b)\} \equiv i(\theta_a) \circ i(\theta_b) + i(\theta_b) \circ i(\theta_a) = 0, \\
&\{\epsilon(\theta^{*a}), \epsilon(\theta^{*b})\} = 0, \\
&\{\epsilon(\theta^{*a}), i(\theta_b)\} =< \theta^{*a},\ \theta_b > \mathbf{1} = \delta^a_b \mathbf{1},
\end{aligned} \tag{2.7}$$

where $\circ$ denotes the map composition. Then, for example, the p-cochain $w^p \in C^p$ can be constructed using the operator $\epsilon(\theta^*)$ as follows

$$w^p = \sum \frac{1}{p!} \underbrace{\epsilon(\theta^{*a}) \circ \epsilon(\theta^{*b}) \circ \cdots \circ \epsilon(\theta^{*c})}_{p\ elements} \phi^{(p)}_{ab\cdots c}, \quad \text{where} \quad \phi^{(p)}_{ab\cdots c} \in R. \tag{2.8}$$

It must be kept in mind that the operations in Eqs. (2.4)-(2.8) must be understood as defined on a point $x \in M$ and we have omitted delta-function on M in Eq. (2.7). This shorthand notation will be used throughout this paper if it raises no confusion.

Let $s : C^p \to C^{p+1}$ be the coboundary operator, i.e., $s^2 = 0$ [7–10] defined on $C^*(\mathcal{G}; R)$ by

$$\begin{aligned}(sw^p)(\theta_1, \cdots, \theta_{p+1})(x) &= \sum_{l=1}^{p+1} (-1)^{l+1} \theta_l \cdot w^p(\theta_1, \cdots, \hat{\theta}_l, \cdots, \theta_{p+1})(x) \\ &+ \sum_{l<n} (-1)^{l+n} w^p([\theta_l, \theta_n], \theta_1, \cdots, \hat{\theta}_l, \cdots, \hat{\theta}_n, \cdots, \theta_{p+1})(x),\end{aligned} \tag{2.9}$$

where a dot means the linear transformation of R defined by an element of $\mathcal{G}$. The coboundary operator s can then be expressed in terms of $\epsilon(\theta^*)$ and $i(\theta)$ as follows

$$s = \sum_{a=1}^{N} \int_M \theta_a \cdot \epsilon(\theta^{*a}) - \sum_{a<b}^{N} \int \int_M i([\theta_a, \theta_b]) \circ \epsilon(\theta^{*a}) \circ \epsilon(\theta^{*b}), \tag{2.10}$$

where the integrations are defined over M.

Now we define a chain complex C as the dual space of the cochain complex C^* using the duality (2.1) [9,11], namely,

$$< \, , \, >: C^p \times C_p \to R$$

by

$$(w^p, \ v_p) \mapsto < \ w^p, \ v_p >= \int_{v_p} w^p, \qquad w^p \in C^p \text{ and } v_p \in C_p, \tag{2.11}$$

where we set $< \ w^p, \ v_q \ >= 0$ if $p \neq q$, and C^* and C are augmented compleces, that is, $C^p = C_p = 0$ for $p < 0$ [8,11]. The duality (2.11) allows us to define an operator $s_* : C_p(\mathcal{G}^*; R) \to C_{p-1}(\mathcal{G}^*; R)$ dual to s:

$$< \ sw^{p-1}, \ v_p >=< \ w^{p-1}, \ s_* v_p >, \qquad w^{p-1} \in C^{p-1} \text{ and } v_p \in C_p. \tag{2.12}$$

Obviously, Eq. (2.12) shows us $s^2 = 0$ implies $s_*^2 = 0$. Thus we will identify s_* with the boundary operator acting on the chains $\{v_p\}$. Of course, the above procedures defining the chain complex is completely analogous to the ordinary homology theory [8,9,11].

Let us introduce the Hodge star duality operation whose action on the cochain space is defined as follows

$$* : C^p \to C^{N-p} \tag{2.13}$$

by

$$(*w^p)(\theta_{a_{p+1}}, \cdots, \theta_{a_N}) = \sum \frac{1}{p!} w^p(\theta_{b_1}, \cdots, \theta_{b_p})\ \varepsilon^{b_1 \cdots b_p}{}_{a_{p+1} \cdots a_N}. \tag{2.14}$$

As the de Rham cohomology, we want to define the adjoint operator $s^\dagger$ of s [9,15] under the new nondegenerate inner product defined by

$$(w_1,\ w_2) = \int_{u_N} w_1 \wedge *w_2 \tag{2.15}$$

with the N-chain u_N satisfying $s_* u_N = 0$. Then

$$(sw_1,\ w_2) = (w_1,\ s^\dagger w_2), \tag{2.16}$$

and $s^\dagger : C^p \to C^{p-1}$ is given by

$$s^\dagger = (-1)^{Np+N+1} * \circ s \circ *. \tag{2.17}$$

For convenience, we have taken the Cartan-Killing metric g_{ab} of the semi-simple Lie subalgebra as positive definite:

$$g_{ab} = -\frac{1}{2} c^l_{ad} c^d_{bl} = \delta_{ab},$$

where $[\theta_a(x), \theta_b(y)] = c^l_{ab} \theta_l(x) \delta(x-y)$. The operator $s^\dagger$ is nilpotent since $s^{\dagger 2} \propto *s^2* = 0$. Using the definitions in Eqs. (2.17), (2.9), and (2.14), one can determine the action of $s^\dagger$ on a p-cochain w^p:

$$(s^\dagger w^p)(\theta_1, \cdots, \theta_{p-1})(x) = -\sum_{l=p}^{N} \theta_l \cdot w^p(\theta_l, \theta_1, \cdots, \theta_{p-1})(x)$$
$$- \sum_{l=1}^{p-1} \sum_{a<b} (-1)^{l+1} c_{ab}^l w^p(\theta_a, \theta_b, \theta_1, \cdots, \hat{\theta}_l, \cdots, \theta_{p-1})(x). \qquad (2.18)$$

Similarly, the adjoint operator $s^\dagger$ can be expressed in terms of $\epsilon(\theta^*)$ and $i(\theta)$ as follows

$$s^\dagger = -\sum_{a=1}^{N} \int_M \theta_a \cdot i(\theta_a) + \sum_{a<b}^{N} \int_M c_{ab}{}^c\, \epsilon(\theta^{*c}) \circ i(\theta_a) \circ i(\theta_b). \qquad (2.19)$$

Let us define an operator $\delta \equiv s \circ s^\dagger + s^\dagger \circ s$ corresponding to the Laplacian, which clearly takes p-cochains back into p-cochains as

$$\delta : C^p \to C^p.$$

The straightforward calculation using the Eq. (2.7) and the Jacobi identity for c_{ab}^c leads to the following expression for the Laplacian δ

$$\delta = -\int_M (\sum \theta_a \cdot \theta_a + \sum c_{ab}^c \theta_a \cdot \epsilon(\theta^{*c}) \circ i(\theta_b) + \frac{1}{2} \sum c_{ab}^c c_{ae}^d \epsilon(\theta^{*c}) \circ i(\theta_b) \circ \epsilon(\theta^{*d}) \circ i(\theta_e)). \qquad (2.20)$$

Considering the formal resemblance to the de Rham cohomology, it will be sufficient to state, without proof, only the important results which are necessary for later applications. For mathematical details of homology and cohomology theory, see Refs. [8,9,11].

We define the p-th cohomology group of the Lie algebra $\mathcal{G}$ by the equivalence class of the p-cochains $C^p(\mathcal{G}; R)$, that is, the kernel of s modulo its image:

$$H^p(\mathcal{G}; R) \equiv Ker^p s / Im^p s, \quad p = 0, \cdots, N. \qquad (2.21)$$

Then the nondegenerating inner product (2.11) provides a natural pairing between p-th cohomology group $H^p(\mathcal{G}; R)$ and p-th homology group $H_p(\mathcal{G}^*; R)$

$$H^p(\mathcal{G}; R) \otimes H_p(\mathcal{G}^*; R) \to R,$$

so that *the inner product (2.11) establishes the duality of the vector spaces* $H^p(\mathcal{G}; R)$ *and* $H_p(\mathcal{G}^*; R)$, the de Rham theorem [11].

The following result is the direct consequence of the positive definiteness of the inner product (2.15):

The "harmonic" p-cochain $w^p \in Harm(\mathcal{G}; R)$, i.e. $\delta w^p = 0$ is satisfied if and only if it is exact, i.e. $sw^p = 0$ and co-exact, i.e. $s^\dagger w^p = 0$.

The adjointness of the operator s and $s^\dagger$ under the nondegenerate inner product (2.15) and their nilpotency lead to the so-called Hodge decomposition theorem in the cochain space in a unique way [9,15]:

Any p-cochain w^p can be uniquely decomposed as a sum of exact, co-exact, and harmonic forms, i.e.,

$$w^p = \delta_H^p \oplus sw^{p-1} \oplus s^\dagger w^{p+1}, \quad p = 0, \cdots, N, \tag{2.22}$$

where δ_H^p is a harmonic p-cochain. The Hodge decomposition theorem (2.22) implies *the isomorphism between the p-th cohomology space $H^p(\mathcal{G}; R)$ and the p-th harmonic space $Harm^p(\mathcal{G}; R)$.*

The Hodge star operator $*$ maps $C^p \to C^{N-p}$ and commute with the Laplacian δ. Thus $*$ induces an isomorphism

$$Harm^p(\mathcal{G}; R) \approx Harm^{N-p}(\mathcal{G}; R).$$

Consequently, *$H^{N-p}(\mathcal{G}; R)$ and $H^p(\mathcal{G}; R)$ are isomorphic as vector spaces*,

$$H^{N-p}(\mathcal{G}; R) \approx H^p(\mathcal{G}; R). \tag{2.23}$$

This is just the Poincaré duality [11].

If the Lie algebra $\mathcal{G}$ is a direct sum of semi-simple Lie algebras and/or Abelian $u(1)$ algebras, that is, $\mathcal{G} = \mathcal{G}_1 \oplus \mathcal{G}_2$ and thus each of these algebras $\mathcal{G}_\alpha$ is an ideal of $\mathcal{G}$, then a total p-cochain C^p will be a sum of a tensor product of cochains corresponding to each Lie algebra $\mathcal{G}_\alpha$

$$C^p = \oplus_{q+r=p} C_1^q \otimes C_2^r$$

and $w^p \in C^p$ will be given by

$$w^p = \sum_{q=0}^{p} w_1^q \times w_2^{p-q}, \quad w_1^q \in C_1^q, \;\; w_2^{p-q} \in C_2^{p-q}.$$

The map $w^p \in C^p$ on $\mathcal{G}$ is defined by

$$w^p(\theta_1, \cdots, \theta_q; \xi_1, \cdots, \xi_{p-q}) = w_1^q(\theta_1, \cdots, \theta_q) w_2^{p-q}(\xi_1, \cdots, \xi_{p-q}), \quad \theta_i \in \mathcal{G}_1, \;\; \xi_i \in \mathcal{G}_2.$$

Then $H^p(\mathcal{G}; R)$ *can be decomposed into a sum of a product of each* $H^q(\mathcal{G}_1; R)$ *and* $H^{p-q}(\mathcal{G}_2; R)$:

$$H^p(\mathcal{G}; R) = \oplus_{q=0}^{p} [H^q(\mathcal{G}_1; R) \otimes H^{p-q}(\mathcal{G}_2; R)]. \tag{2.24}$$

This is known as the Künneth formula for a product space (in our case, a product group $G_1 \otimes G_2$) [11,15].

III. GROUP STRUCTURE OF GAUGE THEORIES

In this section we will show that the group invariant structure of constrained system can be described by the Lie algebra cohomology induced by the BRST generator Q in the algebra of invariant polynomials on $\mathcal{G}$ with the generalized Poisson bracket [4], taking the complete correspondence with the results of Sec. II. It will provide the algebraic and the topological characterization with respect to group invariant structures in the gauge theory.

Consider any physical system with gauge transformation group G_∞ and its compact Lie algebra $\mathcal{G}$ with N generators $G_a, a = 1, \cdots, N$, satisfying the following Lie algebra:

$$[G_a(x), G_b(y)] = g f_{ab}^c G_c(x) \delta(x - y), \;\; a, b, c = 1, \cdots, N. \tag{3.1}$$

Corresponding to each generator, we introduce a ghost $\eta^a(x)$ and an antighost $\rho_a(x)$ which satisfy the following Poisson bracket relations

$$\{\eta^a, \eta^a\} = \{\rho_a, \rho_b\} = 0, \;\; \{\eta^a, \rho_b\} = \delta_b^a. \tag{3.2}$$

Then we can construct the nilpotent BRST generator [4]

$$Q = \int_M G_a \eta^a - \frac{1}{2} g \int_M f^c_{ab} \rho_c \eta^a \eta^b, \tag{3.3}$$

and its nilpotency

$$Q^2 = 0 \tag{3.4}$$

follows from the Lie algebra (3.1) together with the Jacobi identity.

If one identifies the operators $\epsilon(\theta^{*a})(x)$ and $i(\theta_a)(x)$ in Sec. II with the ghost $\eta^a(x)$ and the antighost $\rho_a(x)$ respectively [10], the expression (2.10) about the coboundary operator s exactly agrees with the BRST generator Q, where structure constants $c^l_{ab} = g f^l_{ab}$ and G_a is any representation for θ_a. Rewrite the BRST generator as

$$Q = \int_M (J_a \eta^a - \frac{1}{2} \tau_a \eta^a), \tag{3.5}$$

where $J_a = G_a + \tau_a$. $\tau_a = g \rho_m f^m_{al} \eta^l$ satisfies the same algebra as G_a and commutes with it. Then BRST s-transformation law with respect to a field $\mathcal{F}(x)$ is defined as follows,

$$s\mathcal{F}(x) = [Q, \mathcal{F}(x)\}, \tag{3.6}$$

where the symbol $[\ ,\ \}$ is the generalized Poisson bracket. Thus the s-transformations with respect to the ghost fields η and ρ by Q are

$$s\eta^a = -\frac{1}{2} g f^a_{bc} \eta^b \eta^c, \quad s\rho_a = J_a. \tag{3.7}$$

According to the Ref. [7], we identify the ghost field $\eta(x)$ with a left-invariant Cartan-Maurer form on the group G_∞. With this interpretation of the ghost field $\eta(x)$, the first equation in Eq. (3.7) is just the Cartan-Maurer equation with respect to "exterior derivative" s for forms $\eta(x)$ on G_∞. It is also obvious that the adjoint operator $s^\dagger$ of s introduced in Sec. II can be constructed in terms of η and ρ. We define the corresponding generator by $Q^\dagger$ and it is given by

$$\begin{aligned} Q^\dagger &= - \int_M (G^a \rho_a - \frac{1}{2} g f^{ab}{}_c \eta^c \rho_a \rho_b), \\ &= - \int_M (J^a \rho_a - \frac{1}{2} \tau^a \rho_a). \end{aligned} \tag{3.8}$$

One can easily check this generator is also nilpotent, i.e. $Q^{\dagger 2} = 0$ as stated in Sec. II.

The generator $Q^\dagger$ first appeared in Ref. [16] to find the gauge invariant interactions in string theory and then in Ref. [17] to construct the BRST complex and the cohomology of compact Lie algebra. The Lie algebra cohomology in this paper is quite different from the BRST cohomology constructed in the paper [18], so we use the nomenclature, Lie algebra cohomology, in order to avoid confusion with the BRST cohomology since these two cohomologies have been often confused in the literatures. In fact, the cohomology of Ref. [17] corresponds to the Lie algebra cohomology in this paper as long as the spacetime dependences of the Lie group G_∞ and the Lie algebra $\mathcal{G}$ are fixed. However, it is necessary to consider the infinite-dimensional Lie group and Lie algebra in order that the BRST generator may be viewed as the coboundary operator for the Lie algebra cohomology [7].

The $s^\dagger$-transformation with respect to a field $\mathcal{F}(x)$ is defined by

$$s^\dagger \mathcal{F}(x) = [Q^\dagger, \mathcal{F}(x)\}. \tag{3.9}$$

Then the $s^\dagger$-transformations with respect to the ghost fields η and ρ are

$$s^\dagger \eta^a = -\, J^a, \quad s^\dagger \rho_a = \frac{1}{2} g f_a{}^{bc} \rho_b \rho_c. \tag{3.10}$$

The above equations show that one can identify the antighost ρ_a with the Cartan-Maurer form with respect to the "exterior derivative" $s^\dagger$ as well.

Since Q and $Q^\dagger$ are nilpotent, it follows that Q and $Q^\dagger$ are invariant by G_∞, i.e.

$$[Q, J_a] = 0, \quad [Q^\dagger, J_a] = 0. \tag{3.11}$$

One finds that Q and $Q^\dagger$ satisfy the supersymmetrylike algebra that closes into a Laplacian generator Δ

$$\{Q, Q^\dagger\} = -\Delta, \;\; [\Delta, Q] = 0, \;\; [\Delta, Q^\dagger] = 0, \tag{3.12}$$

where the Laplacian Δ can be computed in terms of the Casimir generators [16]

$$\Delta = \frac{1}{2} \int_M (J^a J_a + G^a G_a). \tag{3.13}$$

The operator $\delta : C^p \to C^p$ in Sec. II corresponds to this generator and it has the exactly same expression as Δ if it is rewritten in terms of Casimir operators.

Following the same scheme as those in the Refs. [19,20], we construct the cochains on $\mathcal{G}$ spanned by the polynomial $\omega_{(p)} = Tr\ \eta^p$, where $\eta = \eta^a T_a$ and T_a is a generator of g. That is, a p-dimensional cochain $C^p(\mathcal{G}; R)$ corresponding to the Eq. (2.8) is spanned by elements of the space of $w^p = \wedge^r \omega_{(p_r)} \cdot \phi$ ($\sum p_r = p$), where ϕ is an element of R, i.e. $\mathcal{G}$-module of symmetric polynomials on $\mathcal{G}$ without (anti-)ghosts. Then $\omega_{(p)} = 0$ if p is even and $\omega_{(p)}$ is a "closed" p-form - a p-cocycle, i.e. $s\omega_{(p)} = 0$ by Eq. (3.7). Notice, for semi-simple groups G, $\omega_{(1)} = 0$ [15]. Let us reexpress the p-cochain w^p as the following form:

$$w^p = \sum \frac{1}{p!} \eta^{a_1} \eta^{a_2} \cdots \eta^{a_p} \cdot \phi^{(p)}_{a_1 a_2 \cdots a_p}. \tag{3.14}$$

Note that the results such as Hodge decomposition theorem, Poincaré duality, and Künneth formula in Sec. II will be reproduced here in the same manner as well. In Sec. II, we stated the isomorphism between the p-th cohomology space $H^p(\mathcal{G}; R)$ and the p-th harmonic polynomial space $Harm^p(\mathcal{G}; R)$. Therefore, the BRST invariant polynomial space can be summarized as the *harmonic* polynomial space $\delta w^p = 0$, whose solutions are represented by

$$[G_a, w^p] = 0, \tag{3.15}$$

and

$$[\tau_a, w^p] = [g \rho_m f^m_{al} \eta^l, w^p] = 0. \tag{3.16}$$

The second condition reads, in components,

$$f^m_{a[a_1} \phi^{(p)}_{a_2 \cdots a_p]m} = 0, \tag{3.17}$$

where the square bracket denotes complete antisymmetrization over the enclosed indices [17]. The first condition (3.15) imposes the G-invariance - G-singlet - on the polynomial and the second one imposes very important constraints about the group invariant structures.

For the $p = 0$ and $p = N$, the condition (3.16) is always satisfied trivially as long as they are associated with the G-invariant polynomials, which leads to the conclusion that the zeroth and the N-th cohomology spaces require only the space of G-singlet. For semi-simple groups G, there are no solutions satisfying the condition (3.16) for $p = 1,\ 2,\ 4$ since there is no cohomology basis $\wedge^r \omega_{(p_r)}$ to be closed and for $p = N-1,\ N-2,\ N-4$ by Poincaré duality (2.23), so that their cohomologies $H^p(\mathcal{G}; R)$ vanish. Note that the gauge group $SU(2)$ is cohomologically trivial so that the group invariant structure in the $SU(2)$ gauge theory is similar to eletrodynamics. In this respect, we would like to refer the interesting analysis [21] which arrives at the same conclusion under the different approach. If one $U(1)$ factor is present (for example, $SU(2) \times U(1),\ U(2)$, etc.), then $H^1(\mathcal{G}; R)$ is non-trivial since $\omega_{(1)}$ is nonzero [15,19]. For $G = SU(N),\ N \geq 3$, there exist nontrivial cohomologies $H^3(\mathcal{G}; R)$ and $H^5(\mathcal{G}; R)$ whenever the symmetric polynomials $\phi^{(3)}$ and $\phi^{(5)}$ are proportional to the structure constants as follows, respectively:

$$\phi^{(3)}_{abc} = f_{abc} \cdot \phi, \quad \phi^{(5)}_{abcde} = d_{amn} f_{mbc} f_{nde} \cdot \phi, \tag{3.18}$$

where $d_{abc} = \frac{1}{2} Tr T_a \{T_b, T_c\}$ and ϕ is any G-singlet. These follow directly from the expansion (3.14) [7,22] or the Eq. (3.17) with the Jacobi identity.

It is worth mentioning, for $G = SU(3)$, the nontrivial cohomologies $H^3(\mathcal{G}; R)$ and $H^5(\mathcal{G}; R)$ are related with each other by Poincaré duality (2.23). The solution of the descent equations corresponding to the Wess-Zumino consistency conditions in gauge theories [23] shows that the polynomials $\omega_{(3)}$ and $\omega_{(5)}$ corresponding to the third and the fifth cohomologies (3.18) respectively generate the two dimensional and the four dimensional gauge anomaly [7,19] (see also recent analysis [24] by Sorella, where the cohomology basis $\omega_{(3)}$ and $\omega_{(5)}$ have a fundamental importance on solving the descent equations). Thus, from the results of these literatures, we can conclude that 2 and 4 dimensional $SU(3)$ anomalies are related with each other by the Poincaré duality; in other words, the gauge anomaly in two dimensional QCD implies the anomaly in four dimensional QCD as long as d-cohomology is trivial [7,19,20]. This observation is also applied to the problem yielding the general

G-invariant effective action [25] with the symmetry group G spontaneously broken to the subgroup H since the G-invariant effective actions for homogeneous spaces G/H can be understood as the Lie algebra cohomology problem of the manifold G/H. For example, in the case for $SU(3) \times SU(3)$ spontaneously broken to the subgroup $SU(3)$, the two dimensional correspondence of the Wess-Zumino-Witten term in four dimensional theory is the Goldstone-Wilczek topological current [26].

IV. COHOMOLOGY IN QED AND QCD

In this section, we want to see whether it is possible to find a corresponding adjoint generator $Q^\dagger$ of the nilpotent Nöther charge Q in relativistic theories and what is the role of the adjoint $Q^\dagger$ in the Lagrangian formulation. That is, the solution we want to find out is how to embed the adjoint $Q^\dagger$ of Q into the relativistic phase space. We showed in Ref. [13] the consistent nilpotent Nöther charge $Q^\dagger$ exists for Abelian gauge theories and the generator $Q^\dagger$ generates new noncovariant symmetry and imposes strong constraint on state space.

In order to consider the consistent embedding of the BRST adjoint generator $Q^\dagger$ into the relativistic phase space, it is necessary to introduce the nonminimal sector of BRST generator [4,27]. First, consider the BRST (and anti-BRST) invariant effective QED Lagrangian. (Our BRST treatments are parallel with those of Baulieu's paper [5].)

$$\begin{aligned}\mathcal{L}_{eff} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{2}\bar{s}s(A_\mu^2 + \alpha\bar{c}c) \\ = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi + A_\mu\partial^\mu b + \frac{\alpha}{2}b^2 - \partial_\mu\bar{c}\partial^\mu c,\end{aligned} \tag{4.1}$$

where $D_\mu = \partial_\mu + ieA_\mu$ is the covarint derivative with the metric $g_{\mu\nu} = (1, -1, -1, -1)$. The explicit BRST transformations are

$$\begin{aligned} &sA_\mu = \partial_\mu c, \quad sc = 0, \\ &s\bar{c} = b, \qquad sb = 0, \\ &s\psi = -iec\psi. \end{aligned} \tag{4.2}$$

We introduced an auxiliary field b to achieve off-shell nilpotency of the BRST (and the anti-BRST) transformation. Then the nilpotent Nöther charge generated by the BRST symmetry reads as

$$Q = \int d^3x \{(\partial_i F^{io} - J_0)c + b\dot{c}\}, \tag{4.3}$$

where J_0 is a charge density defined by

$$J_0 = e\bar{\psi}\gamma_0\psi. \tag{4.4}$$

The constraint functions G^i consist of two commuting groups, $G^i = (\Phi, b),\ i = 1, 2,$ where $\Phi = \partial_i F^{io} - J_0$ is a Gauss law constraint in the theory and b is the momentum canonically conjugate to the Lagrange multiplier A_0, so that it generate a gauge transformation, δA_0. Thus adding the nonminimal sector in the BRST generator, the Lie algebra $\mathcal{G}$ is composed of a direct sum of two Abelian ideals $\mathcal{G}_1$ and $\mathcal{G}_2$ corresponding to the $u(1)$ generators Φ and b, respectively. In the similar fashion, let the ghost fields split as follows:

$$\eta^i = (c,\ \ \pi_{\bar{c}} = \dot{c}),\ \ \ \rho^i = (\pi_c = -\dot{\bar{c}},\ \ \bar{c}). \tag{4.5}$$

Then the BRST charge Q can be written as

$$Q = G^i \alpha_{ij} \eta^j, \tag{4.6}$$

where $\alpha_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Since the constraints in the relativistic phase space for Abelian gauge theories impartially generate $u(1)$ Lie algebras and the Künneth formula (2.24) shows $H(\mathcal{G}; R)$ is the product of each $H(\mathcal{G}_1; R)$ and $H(\mathcal{G}_2; R)$, we expect it is trivial to embed the adjoint $Q^\dagger$ of the BRST generator Q corresponding to the total Lie algebra $\mathcal{G}$ including the nonminimal sector into the relativistic phase space. According to the Eq. (3.8), one can guess the form of the generator $Q^\dagger$ must be the following: $Q^\dagger = G^i \beta_{ij} \rho^j$. Note that we have a degree of freedom to the extent of multiplicative factor in defining the BRST generator Q or the its adjoint generator $Q^\dagger$ for a given Lie algebra $\mathcal{G}$ as long as it does not affect the nilpotency of Q or $Q^\dagger$. Using this degree of freedom either in the Lie algebra $\mathcal{G}_2$ or in

$\mathcal{G}_1$ sector in defining the adjoint generator $Q^\dagger$, we take the following choices for the matrix β_{ij} which will allow the well-defined canonical mass dimension for $Q^\dagger$: $\beta_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & -\nabla^2 \end{pmatrix}$ or $\begin{pmatrix} \nabla^{-2} & 0 \\ 0 & -1 \end{pmatrix}$. These choices make the BRST adjoint $Q^\dagger$ the symmetry generator of the Lagrangian (4.1) and so complete the consistent embedding of $Q^\dagger$ into the relativistic phase space. The former type corresponds to the generator in Ref. [13] and the latter to the generator in Ref. [12].

The explicit form of the BRST adjoint generator $Q^\dagger$ for the former type is

$$Q^\dagger = \int d^3x \{(\partial_i F^{io} - J_0)\dot{\bar{c}} + b\nabla^2 \bar{c}\}. \tag{4.7}$$

Then the explicit transformations defined by (3.9) are that

$$\begin{aligned}
&s^\dagger A_0 = -\nabla^2 \bar{c}, && s^\dagger A_i = -\partial_0 \partial_i \bar{c}, \\
&s^\dagger c = (\partial_i F^{io} - J_0), && s^\dagger \bar{c} = 0, \\
&s^\dagger \psi = ie\dot{\bar{c}}\psi, && s^\dagger b = 0.
\end{aligned} \tag{4.8}$$

In the Ref. [13], it has shown that this noncovariant transformation is a symmetry of the Lagrangian (4.1) and that there also exists the same kind of symmetry in the Landau-Ginzburg and the Chern-Simons theories. As discussed in the Ref. [13], the symmetry generated by $Q^\dagger$ is realized in quite different way compared to the BRST symmetry: while the gauge-fixing term in the effective QED Lagrangian (4.1), i.e. $A_\mu \partial^\mu b + \frac{\alpha}{2} b^2 \to -\frac{1}{2\alpha}(\partial_\mu A^\mu)^2$, remains invariant under the transformation (4.8), the variation from the ghost term is canceled up to the total derivative by the variation from the original gauge-invariant classical Lagrangian which remains invariant under the BRST transformation (4.2). These differences in the way of realizing the symmetries imply that the BRST adjoint symmetry can give the different superselection sector from the BRST symmetry [28] (as it is also seen from the Hodge decomposition theorem (2.22) which is a canonical decomposition into a direct sum of linearly independent subspaces) unlike the recent comment [29].

If we choose, instead, the matrix $\beta_{ij} = \begin{pmatrix} \frac{1}{\nabla^2} & 0 \\ 0 & -1 \end{pmatrix}$ in the Eq. (4.7), we will obtain the nonlocal symmetry in Ref. [12]. Of course, in this case, we must impose the good boundary conditions on fields. But there is no reason to introduce the nonlocality and it seems unnatural since the generator $Q^\dagger$ must be the adjoint of the generator Q of the *local* gauge transformation.

The adjoint generator in the configuration space can be understood as the generator of transformation consistent with the gauge fixing condition [12,13]. Thus, in the configuration space, there may not exist the global expression of the adjoint generator $Q^\dagger$ of non-Abelian gauge theory compatible with the gauge fixing condition on account of the topological obstructions such as Gribov ambiguity [30]. But it does not imply that there can not exist the local expression of $Q^\dagger$, because the difficulty posed by the Gribov ambiguity can be avoided [31] by finding a local cross section on a finite local covering and using the Faddeev-Popov trick locally. Nevertheless, it seems a nontrivial problem to find the solution for the consistent embedding into the relativistic phase space for the non-Abelian gauge theory such as QCD. This problem remains to be future work. We want to focus our attention about the construction of $su(3)$ Lie algebra cohomology in QCD.

Consider the BRST (and anti-BRST) invariant effective QCD Lagrangian:

$$\begin{aligned}
\mathcal{L}_{eff} =& -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - M)\Psi - \frac{1}{2}\bar{s}s(A^a_\mu A^{a\mu} + \alpha\bar{C}^a C^a) \\
=& -\frac{1}{4}F^2_{\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - M)\Psi + A_\mu\partial^\mu B + \frac{\alpha}{2}B^2 + \frac{\alpha}{2}gB[C,\bar{C}] \\
& -\partial_\mu\bar{C}D^\mu C + \frac{\alpha}{2}g^2[\bar{C},C]^2,
\end{aligned} \tag{4.9}$$

where quark fields Ψ are taken to transform according to the fundamental $SU(3)$ representation, the Yang-Mills vector potential A_μ, a pair of anticommuting ghosts C, $\bar{C}$ and the auxiliary field B take values in the adjoint representation of a $SU(3)$ Lie group. The QCD Lagrangian (4.9) is invariant with respect to the following BRST transformations [5]:

$$
\begin{aligned}
& sA_\mu = D_\mu C, \quad sC = -\tfrac{g}{2}[C, C], \\
& s\bar{C} = B, \qquad\quad sB = 0, \\
& s\Psi = -gC\Psi.
\end{aligned} \tag{4.10}
$$

D_μ defines the covariant derivatives of $SU(3)$ Yang-Mills symmetry group. The corresponding conserved nilpotent BRST generator is given by

$$
Q = \int d^3x \{(D_i F^{io} - J_0 + g[\dot{\bar{C}}, C])^a C^a + B^a (D_0 C)^a - \frac{1}{2} g [\dot{\bar{C}}, C]^a C^a\}, \tag{4.11}
$$

where J_0^a is a matter color charge density defind by

$$
J_0^a = -ig\bar{\Psi}\gamma_0 T^a \Psi. \tag{4.12}
$$

The constraint functions G^A are composed of two commuting groups, $G^A = (\Phi^a, B^a)$, where $\Phi^a = (D_i F^{io} - J_0)^a$ is the original Gauss-law constraints in the theory generating $su(3)$ Lie algebra:

$$
[\Phi_a, \Phi_b] = g f_{ab}^c \Phi_c, \tag{4.13}
$$

and B^as are the momenta canonically conjugate to the Lagrange multipliers A_0^a and generate $u(1)$ Lie algebras. In the similar fashion as QED, one can split the ghosts as follows:

$$
\eta^A = (C^a, \ \Pi_{\bar{C}}^a = (D_0 C)^a), \ \ \rho^A = (\Pi_C^a = -\dot{\bar{C}}^a, \ \ \bar{C}^a). \tag{4.14}
$$

Note that $s\Pi_{\bar{C}}^a = 0$, so that we can identify the ghost $\Pi_{\bar{C}}^a$ with the Cartan-Maurer form on $U(1)$ group. Of course, the BRST generator Q in Eq. (4.11) is exactly same form of the Eq. (3.3). Let us rewrite the BRST generator Q as the form of the Eq. (3.5)

$$
Q = \int d^3x \{J_a C^a + B_a \Pi_{\bar{C}}^a - \frac{1}{2} \tau_a C^a\}, \tag{4.15}
$$

where the generator J^a and the generator of the ghost representation τ_a [16] are given by

$$
J^a = (D_i F^{io} - J_0 + g[\dot{\bar{C}}, C])^a = \Phi^a + \tau^a, \quad \tau^a = g[\dot{\bar{C}}, C]^a. \tag{4.16}
$$

The generators J_a and τ_a satisfy the same $su(3)$ algebra:

$$[J_a, J_b] = gf^c_{ab}J_c, \quad [\tau_a, \tau_b] = gf^c_{ab}\tau_c. \tag{4.17}$$

Since the two groups of the constraint functions $G^A = (\Phi^a, B^a)$ commute with each other, the total Lie algebra $\mathcal{G}$ including the nonminimal sectors B^a is composed of the $su(3)$ non-Abelian ideal and the eight $u(1)$ Abelian ideals:

$$\mathcal{G} = \oplus su(3) \oplus^8_{\alpha=1} u(1)_\alpha. \tag{4.18}$$

In order to construct only the cohomology of the color $su(3)$ Lie algebra for the reason explained above, we drop the Abelian sectors from the BRST generator Q through the direct restriction on the cochain space (3.14), in other words, considering only $su(3)$ sub-cochain complex. The BRST adjoint $Q^\dagger$ defined on the cochain $C^*(su(3); R)$ is equal to

$$Q^\dagger = -\int d^3x\{J^a\Pi^a_C - \frac{1}{2}\tau^a\Pi^a_C\}. \tag{4.19}$$

Then the Laplacian Δ of the $su(3)$ subalgebra sector can be represented in terms of the generators J_a and the original constraints Φ_a

$$\Delta = \frac{1}{2}\int d^3x\{J^aJ_a + \Phi^a\Phi_a\}, \tag{4.20}$$

which is equal to the expression given by Eq. (3.13) for $su(3)$ cohomology. Thus the harmonic polynomials of the $su(3)$ algebra sector must satisfy the following conditions,

$$[\Phi^a, w^p] = [(D_iF^{io} - J_0)^a, w^p] = 0, \;\; a = 1, \cdots, 8, \tag{4.21}$$

and

$$[\tau^a, w^p] = [gf^a_{bc}\dot{\bar{C}}^bC^c, w^p] = 0, \;\; a = 1, \cdots, 8. \tag{4.22}$$

From the arguments in Sec. III, we see that the solutions of Eqs. (4.21) and (4.22) exist trivially for p=0 and p=8 as long as they are given by the gauge invariant polynomials because they are singlets under the adjoint representation of the $su(3)$ Lie algebra. But the cohomologies $H^p(su(3); R)$ for $p = 1,\ 2,\ 4,\ 6,$ and 7 vanish. For p=3 and 5, there always exist non-trivial cohomologies $H^3(su(3); R)$ and $H^5(su(3); R)$ whose structures are given by Eq. (3.18) and they are related with each other by the Poincaré duality (2.23). Since the Lie algebra cohomology proves the nontrivial property of group invariant structures, the nonvanishing Lie algebra cohomologies $H^p(su(3); R)$ can be related to the gauge invariants in $SU(3)$ gauge theory. It remains to investigate the deep relation between the gauge invariant configuration of gauge and matter fields in the spacetime and the Lie algebra cohomology.

V. DISCUSSION

We have constructed the Lie algebra cohomology of the group of gauge transformation and obtained the Hodge decomposition theorem and the Poincaré duality. As long as a Lie algebra has a nondegenerate Cartan-Killing metric so that the underlying manifold is orientable, we can always define a unique (up to a multiplicative factor) adjoint of the coboundary operator under a nondegenerate inner product using a Hodge duality. However, for Lie algebras such as the Virasoro algebra for which no Cartan-Killing metric exists, the adjoint can not be unique. Indeed, for the Virasoro algebra, the adjoint of BRST generator defined by Niemi [32] is different from ours and that in Ref. [16].

We also considered the consistent extension of the Lie algebra cohomology into the relativistic phase space in order to obtain the Lagrangian formulation. In order to do that, we extended the Lie algebra by including the nonminimal sector of BRST generator. The adjoint $Q^\dagger$ constructed through this procedure generates the noncovariant local or nonlocal symmetry in QED in Refs. [12,13]. We have pointed that there is no reason to introduce the nonlocality necessarily and it seems unnatural since the generator $Q^\dagger$ must be the adjoint of the BRST generator Q generating local gauge transformation. But, in the configuration space, the adjoint $Q^\dagger$ compatible with the gauge fixing condition can not exist globally for the non-Abelian gauge theory due to the topological obstructions such as Gribov ambiguity. As explained in Sec. IV, the adjoint $Q^\dagger$ in the non-Abelian gauge theory can exist locally (or perturbatively), so that it can generate new symmetry at least locally (or perturbatively). So it will be interesting to study the role of the symmetry transformation generated by the generator $Q^\dagger$ and the Ward identity of this symmetry in the local (or perturbative) sense.

Note that the Lie algebra cohomology constructed here is quite different from the BRST cohomology in Refs. [6,18,33]. In the two cohomologies, the role of ghost fields is quite different and each inner product to obtain Hodge theory is defined by the definitely different schemes. It can be shown [34] that there is no paired singlet in the BRST cohomology so that higher cohomologies with nonzero ghost number vanish as long as the asymptotic completeness is assumed. Therefore the ghost number characterizing cohomology classes in this paper has different meaning from the ghost number of state space. The distinction between the BRST cohomology and the Lie algebra cohomology will be further clarified [34].

In QCD, there are nontrivial cohomologies $H^p(su(3); R)$ for $p = 0,\ 8$ and $p = 3,\ 5$ and they are, respectively, related to each other by the Poincaré duality. Since the Lie algebra cohomology proves the nontrivial property of group invariant structures, the nonvanishing Lie algebra cohomologies $H^p(su(3); R)$ may be deeply related to the colorless combination of $SU(3)$ color charges which satisfy the $su(3)$ Lie algebra. Then it will be very interesting to investigate the relation between the color confinement and the $su(3)$ cohomology.

ACKNOWLEDGEMENTS

This work was supported by the Korean Science and Engeneering Foundation (94-1400-04-01-3 and Center for Theoretical Physics) and by the Korean Ministry of Education (BSRI-94-2441).

REFERENCES

[1] R. Utiyama, Phys. Rev. **101**, 157 (1956).

[2] L. D. Faddeev and V. N. Popov, Phys. Lett. **25**B, 30 (1967).

[3] C. Becchi, A. Rouet, and R. Stora, Comm. Math. Phys. **42**, 127 (1975); Ann. Phys. (N.Y.) **98**, 287 (1976) ; I. V. Tyutin, Lebedev preprint, FIAN No.**39** (1975), unpublished.

[4] M. Henneaux and C. Teitelboim, *Quantization of Gauge Systems* (Princeton University Press, Princeton, 1992).

[5] L. Baulieu, Phys. Rep. **129**, 1 (1985).

[6] N. Nakanishi and I. Ojima, *Covariant Operator Formalism of Gauge Theories and Quantum Gravity* (World Scientific, Singapore, 1990).

[7] L. Bonora and P. Cotta-Ramusino, Comm. Math. Phys. **87** (1983).

[8] H. Cartan and S. Eilenberg, *Homological Algebra* (Princeton University Press, Princeton, 1956).

[9] S. I. Goldberg, *Curvature and Homology* (Dover Publications, New York, 1962).

[10] Y. Choquet-Bruhat and C. DeWitt-Morette, *Analysis, Manifolds and Physics; Part II: 92 Applications* (North-Holland, Amsterdam, 1989).

[11] E. H. Spanier, *Algebraic Topology* (McGraw-Hill, New York, 1966).

[12] M. Lavelle and D. McMullan, Phys. Rev. Lett. **71**, 3758 (1993).

[13] H. S. Yang and B. -H. Lee, J. Korean Phys. Soc. **28**, 572 (1995).

[14] D. McMullan, J. Math. Phys. **28**, 428 (1987).

[15] T. Eguchi, P. B. Gilkey, and A. J. Hanson, Phys. Rep. **66**, 213 (1980).

[16] J. -L. Gervais, Nucl. Phys. **B276**, 339 (1986); I. Bars and S. Yankielowicz, Phys. Rev. D**35**, 3878 (1987).

[17] J. W. van Holten, Phys. Rev. Lett. **64**, 2863 (1990); Nucl. Phys. **B339**, 158 (1990).

[18] H. S. Yang and B. -H. Lee, J. Korean Phys. Soc. **28**, 138 (1995).

[19] M. Dubois-Violette, M. Talon, and C. M. Viallet, Phys. Lett. **158**B, 231 (1985); Comm. Math. Phys. **102**, 105 (1985).

[20] M. Dubois-Violette, M. Henneaux, M. Talon, and C. M. Viallet, Phys. Lett. **289**B, 361 (1992)

[21] L. V. Prokhorov and S. V. Shabanov, Int. J. Mod. Phys. A**7**, 7815 (1992).

[22] G. Bandelloni, J. Math. Phys. **27**, 2551 (1986).

[23] S. B. Treiman, R. Jackiw, B. Zumino, and E. Witten, *Current Algebra and Anomalies* (Princeton University Press, Princeton, 1985).

[24] S. P. Sorella, Comm. Math. Phys. **157**, 231 (1993).

[25] E. D'Hoker and S. Weinberg, Phys. Rev. D**50**, R6050 (1994); E. D'Hoker, Report No. UCLA/95/TEP/5, 1995 (unpublished); Y. -S. Wu, Phys. Lett. **153**B, 70 (1985).

[26] J. Goldstone and F. Wilczek, Phys. Rev. Lett. **47**, 986 (1981); E. Witten, Comm. Math. Phys. **92**, 483 (1984).

[27] E. S. Fradkin and G. A. Vilkovsky, Phys. Lett. **55**B, 224 (1975);I. A. Batalin and E. S. Fradkin, Phys. Lett. **122**B, 157 (1983).

[28] M. Lavelle and D. McMullan, preprint UAB-FT-666, hep-th/9509159.

[29] V. O. Rivelles, preprint IFUSP-P/1137, hep-th/9509028.

[30] V. N. Gribov, Nucl. Phys. **B139**, 1 (1978).

[31] I. M. Singer, Comm. Math. Phys. **60**, 7 (1978).

[32] A. J. Niemi, Phys. Rev. D**36**, 3731 (1987).

[33] M. Spiegelglas, Nucl. Phys. **B283**, 205 (1987); A. V. Razumov and G. N. Rybkin, *ibid.* **B332**, 209 (1989); W. Kalau and J. W. van Holten, *ibid.* **B361**, 233 (1991).

[34] H. S. Yang and B. -H. Lee, in preparation.